

# Calculus 1301b

Note Title

28/01/2011

- Prof Shuen-yakover is not available today
  - My name is Rob Corless
  - today's lecture notes will be made available on [www.apmaths.umd.edu/~rcorless](#) under "Teaching"
- Today: more Section 9.3 - separable eq'ns.

Mixing problems (eg Example 6, p 585  
problems 41-44 p 587).

blah blah blah ← this is the hard bit  
(interworking bit)

$$\frac{dy}{dt} = 0.75 - \frac{y(t)}{200} = \frac{150 - y(t)}{200}$$

$$\frac{1}{150 - y} \frac{dy}{dt} = \frac{1}{200}$$

$$\int \frac{1}{150-y} \frac{dy}{dt} dt = \int \frac{1}{200} dt = \frac{1}{200} t + C$$

change of variables

$$u = y(t) \quad du = \frac{dy}{dt} dt$$

$$\int \frac{1}{150-u} du = -\ln(150-u) + C_1$$

$$\ln |150-u|$$

but watch out if  $u=150$ )

$$- \ln(150 - u) = \frac{1}{200} \cdot t + \underbrace{C - C}_R$$

$$- \ln(150 - y(t)) = \frac{1}{200} t + K$$

1<sup>st</sup> order eq'n  $\Rightarrow$  One constant of integration

$$\ln(150 - y(t)) = -\frac{1}{200} t + K$$

$$e^{\ln(150 - y(t))} = e^{-\frac{1}{200} t + K}$$

$$\text{or } 150 - y(t) = C e^{-\frac{t}{200}} \quad (C = e^k \text{ here})$$

$$\text{or } 150 - C e^{-\frac{t}{200}} = y(t)$$

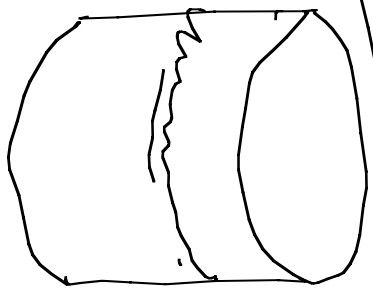
blabla: "Let  $y(t)$  be the amount of salt in kg at time  $t$ "

$$y(t) = 150 - C e^{-\frac{t}{200}}$$

$$\lim_{t \rightarrow \infty} y(t) = 150$$

Problem set-up in more detail (modelling)

A tank



contains 20 kg of  
salt dissolved  
in 1000 L of water

at  $t=0$   $y(t) = 20$

$$y(t) = 150 - C \cdot e^{t/1000}$$

$$20 = 150 - C \cdot 1$$

$$\therefore C = 150 - 20 = 130$$

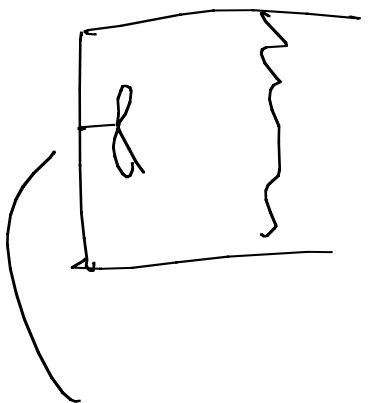
$$\therefore y(t) = 150 - 130e^{-t/200}$$

Why does  $\lim_{t \rightarrow \infty} 150 - 130e^{-t/200} = 150$  ?

↪ goes to 0.

" Brine that contains 0.03 kg/L of salt enters the tank at a rate of 25 L/min. "

"The solution is kept thoroughly mixed



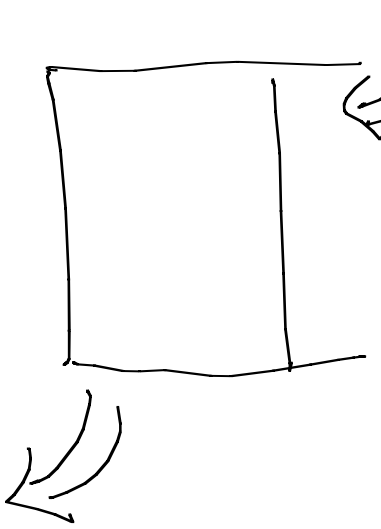
idea mixer, instantly brings  
bore to evenly  
distributed.

and draws from the tank at the same  
rate (25 L/min)."



How much salt remains in the tank after 1 hour (60 min).

(Salt in)  $(0.03 \text{ kg/L} \times 25 \text{ L}) / \text{min}$



(Salt out)  $(y(t) / 5000 \text{ L}) \times 25 \text{ L} / \text{min}$

$$\text{Concentration} = \frac{\text{Amount of salt}}{\text{total volume}}$$

rate of change of salt = salt in - salt out  
per minute per minute

$$\frac{dy}{dt} = (0.03)(25) - y(t) \cdot \frac{25}{5000}$$
$$= 0.75 - \frac{y(t)}{200}$$

AAAA!

A similar problem: #41 in text

A tank contains 1000 L of brine, 15 kg salt

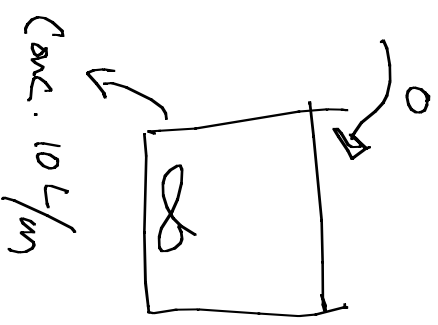
initially dissolved.

Pure water enters @ 10 L/min

(drains @ same rate, thoroughly mixed)

Concentration  $\frac{y(t)}{1000}$

$y(t)$  = amount of salt at  $t$  minutes)



rate of  
change  
of salt

$$\frac{dy}{dt}$$

= amount in — amount out  
per minute — per minute

$$= 0 - \frac{y(t)}{1000} \cdot 10$$

$$= - \frac{y(t)}{100}$$

$$\frac{dy}{y} = -\frac{1}{100} dt$$

$$\frac{1}{y} dy = -\frac{1}{100}$$

$$\int \frac{1}{y} dy = \int -\frac{1}{100} dt$$

$$\ln y = -\frac{1}{100} t + C$$

$$\therefore y = K e^{-t/100}$$

$$y(0) = 15 \text{ kg} \quad (\text{initial amount of salt})$$

$$\therefore 15 \text{ kg} = K \cdot e^{-0/100} = K$$

$$\therefore y(t) = 15 e^{-t/100}$$

b) amount after 20 minutes?

$$y(20) = 15 e^{-20/100} = 15 e^{-1/5}$$

In general

$$\frac{dy}{dx} = f(y) g(x)$$

$$\frac{1}{f(y)} \frac{dy}{dx} = g(x)$$

$$\therefore \int \frac{1}{f(y)} \frac{dy}{dx} dx = \int g(x) dx$$

$$= \int \frac{1}{f(u)} du$$

Change of variables :  $u = y(x)$      $du = \frac{dy}{dx} dx$

$$\int 2x \sin(x^2) dx$$

$$u = x^2 \quad du = 2x dx$$

$$\Rightarrow \int \sin(u) du$$