Theoretical Condensed Matter Physics.
Problem set 5 (2 pages). Colloids, Poisson-Boltzmann.
Instructor: Dr. Karttunen.

Write out *all* the steps in your calculations.
Due: By 6:00 pm, Monday Mar. 23, 2009.

Problem 1. Find the energies (from literature) characteristic for the following interactions (when needed, assume room temperature): 1) hydrogen bond, 2) ion-ion interactions, 3) van der Waals interactions, 4) covalent bond, thermal energy (Brownian motion). Describe the differences in terms of their importance for experiments, biological systems and practical applications.

Problem 2. The density profile of dispersed colloidal particles follows the barometric law, i.e., $C(h) = C_0 e^{mgh/k_B T}$, where C is the concentration and h is the height.

Let us now assume that we have colloidal gold particles dispersed in water. The data below (originally obtained by the Swedish Nobel Laureate Svedberg in 1928) describes the number of particles as a function of height as:

height (μm)	number of particles
0	889
100	692
200	572
300	426
400	357
500	253
600	217
700	185
800	152
900	125
1000	108
1100	78

Using the above data calculate an estimate for the Boltzmann constant k_B. Assume that the radius of the gold particles is 21 nm and that the density is 19.3 g cm^{-3}. Let the temperature be 20 C. Write out all the details of your calculations.

Problem 3. Using the Poisson-Boltzmann equation as your starting point, show that for a system consisting of a planar surface and contaning symmetric electrolyte the PB equation can be written as

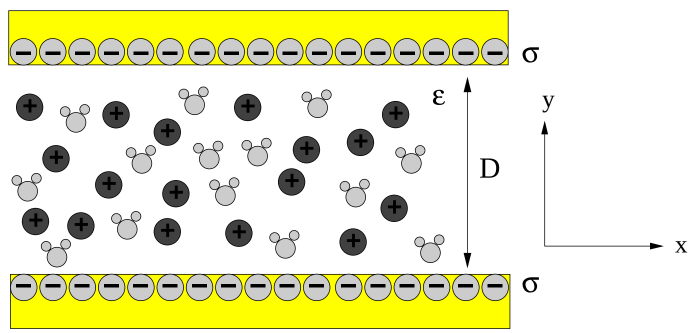
$$\frac{d^2\psi}{dx^2} = \frac{2ez}{\varepsilon_0 \varepsilon_r} n_0 \sinh(ez\psi/k_B T),$$

where n_0 is the 'number density' bulkissa, z the valence, $\psi$the potential, e the electron charge, k_B the Boltzmannin constant, ε the dielectric constant and T on the temperature. Show all the steps in your calculation.

Problem 4. Let us assume that we have a system in which the counterions are in equilibrium. Assume further that we have the system as illustrated in the figure. The surface charge is σ for each surface. Let D be the distance between the surfaces and use a coordinate system in which $y = 0$ is in midplane.

Compute the ionic concentration $\rho(y)$ (let $\rho_0 = \rho(y = 0)$). Show all the steps in your calculation.

Assume now that we have the system in water and room temperature, and that $\sigma = 0.2$ C/m^2. Using your result, compute ionic concentration. Change σ to $\sigma = 0.02$C/m^2. Interpret your result.



Problem 5. Assume you live in a two-dimensional world. Calculate the electrostatic field by solving

$$\nabla \mathbf{E} = \frac{\mathbf{1}}{\epsilon_{\mathbf{0}}} \rho(\mathbf{r})$$

for a point charge placed into the origin of this two-dimensional world. Don't be shy and look up gradients or other operators in an appropriate coordinate system and then solve for the electric field. Interpret your result.

Problem 6. a) Show (with all details) that $\langle \Delta T \Delta S \rangle = k_B T$, where $\langle ... \rangle$ denotes an average and k_B is the Boltzmann constant.

b) Discuss and list different experimental situations and setups in which knowledge of thermal fluctuations is important.

Problem 7. Choose E ja V as independent variables and compute (show all details) $\langle \Delta E \Delta V \rangle$, where $\langle ... \rangle$ denotes an average. Interpret your result. Hint: Follow the same approach as discussed in the lectures and think what is the best way of choosing independent variables.