AM375a. In-class exam, November 14, 2006. Instructor: Dr. Karttunen.

Write out *all* the steps in your calculations.
Return this exam paper.

Problem 1.
Show that the given functions are orthogonal on the indicated interval.

$$a) f_1(x) = x, f_2(x) = x^2, [-2, 2] \quad \text{b) } f_1(x) = e^x, f_2(x) = xe^{-x} - e^{-x}, [0, 2]$$

Problem 2.
Put the differential equation

$$x^2y'' + xy' + \lambda y = 0$$

in the self-adjoint form.

Problem 3.
Expand the given function in an appropriate sine or cosine series.

$$f(x) = x^2, \ -1 < x < 1.$$

Problem 4.
A rod of length L coincides with the interval $[0, L]$ on the x-axis. Set up the boundary value problem for the temperature $u(x, t)$. The left end is held at temperature zero, and the right end is insulated. The initial temperature is $f(x)$ throughout.

Problem 5.
Expand $f(x) = e^{-x}$, $-\pi < x < \pi$ in complex Fourier series, that is, show that

$$f(x) = \frac{\sinh \pi}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \frac{1 - in}{n^2 + 1} e^{inx}.$$