AM375a. Problem set 9. Instructor: Dr. Karttunen.

Write out *all* the steps in your calculations.

Problem 1. (5.3.10)
Verify that $y = x^n J_n(x)$ is a particular solution of

$$xy'' + (1-2n)y' + xy = 0, x > 0$$

Problem 2. (5.3.11)
Verify that $y = x^{-n} J_n(x)$ is a particular solution of

$$xy'' + (1+2n)y' + xy = 0, x > 0$$

Problem 3. (5.3.33)
It can be shown by a change of variables that the general solution of the differential equation $xy'' + x\lambda = 0$ on interval $(0, \infty)$ is

$$y = c_1\sqrt{x}J_1(2\sqrt{\lambda x}) + c_2\sqrt{x}Y_1(2\sqrt{\lambda x}).$$

Verify by direct substitution that $y = \sqrt{x}J_1(2\sqrt{\lambda x})$ is a particular solution of the equation in the case $\lambda = 1$.

Problem 4. (12.6.3)
Expand $f(x) = 1, 0 < x < 2$, in a Fourier-Bessel series using Bessel functions of order zero that satisfy the given boundary condition.

$$J_0(2\lambda) = 0$$

Problem 5. (12.6.5)
Expand $f(x) = 1, 0 < x < 2$, in a Fourier-Bessel series using Bessel functions of order zero that satisfy the given boundary condition.

$$J_0(2\lambda) + 2\lambda J_0'(2\lambda) = 0$$

Problem 6. (12.6.7)
Expand the given function in a Fourier-Bessel series using Bessel functions of the same order as in the indicated boundary condition.

$$f(x) = 5x, \;\; 0 < x < 4, \;\; 3J_1(4\lambda) + 4\lambda J_1'(4\lambda) = 0$$