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**AM375a. Fall 2006**

Problem set 1. Write out *all* the steps in your calculations. Do not use Laplace transform tables in these problems.

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**Problem 1.**

- Calculate  $\mathcal{L}\{1/\sqrt{t}\}$ .
- Calculate  $\mathcal{L}\{\sin(at)\}$ , where  $a$  is a constant.
- Calculate  $\mathcal{L}\{t \cos(t)\}$ .

**Problem 2.**

- Calculate  $\mathcal{L}\{\sinh(kt)\}$ , where  $k$  is a constant.
- Calculate  $\mathcal{L}\{e^t \sinh(t)\}$ .
- Calculate  $\mathcal{L}\{(2t - 1)^3\}$ .

**Problem 3.**

- Calculate  $\mathcal{L}\{\cos^2(t)\}$ .
- Calculate  $\mathcal{L}\{\cos(2t) \sin(2t)\}$ .

**Problem 4.** One definition of the gamma function is given by the improper integral

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt, \quad \alpha > 0.$$

- Show that  $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ .
- Show that

$$\mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}, \quad \alpha > -1.$$

**Problem 5.**

Use the fact that  $\Gamma(1/2) = \sqrt{\pi}$  and the previous problem to find the Laplace transforms of:

- $f(t) = t^{-1/2}$
- $f(t) = t^{1/2}$
- $f(t) = t^{3/2}$

**Problem 6.**

Laplace transform of a function is

$$f(s) = \frac{k^2}{s(s^2 + k^2)}.$$

Find the original function.