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**AM375a. Midterm, October 30, 2006. Instructor: Dr. Karttunen.**

Write out *all* the steps in your calculations.

Return this exam paper.

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**Problem 1.** Answer the following questions briefly and precisely. If needed, use figures.

- Define piecewise continuity.
- What is the Gibbs phenomenon? Give at least two examples of practical situations when it is of importance.
- Define exponential order.
- Give a definition for a general integral transform (write out a general formula and define the various terms in there).

**Problem 2.** Solve

- a)  $\mathcal{L}^{-1} \left\{ \frac{1}{s^5} \right\}$    b)  $\mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 7} \right\}$    c)  $\mathcal{L}^{-1} \left\{ \frac{-2s + 6}{s^2 + 4} \right\}$    d) Expand  $f(x) = x, -2 < x < 2$  in Fourier series

**Problem 3.**

Solve

$$f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau)e^{t-\tau} d\tau$$

for  $f(t)$ .

**Problem 4.**

Find the Fourier series of

$$f(x) = e^x, -\pi < x < \pi$$

**Problem 5.**

Show that the set  $\{1, \cos x, \cos 2x, \dots\}$  is orthogonal on the interval  $[-\pi, \pi]$ .

**Problem 6.**

Solve the boundary value problem

$$y'' + \lambda y = 0, 0 < x < L, y(0) = 0 \text{ and } y(L) = 0.$$

Give your answer in terms of the eigenvalues and eigenfunctions. Hint: self-adjoint form.