A General Dimension Reduction Technique
For Derivative Pricing

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Abstract

This paper develops a powerful new approach for the numerical valuation of stochastic integrals which occur in many finance applications. We illustrate the method in the context of derivative pricing. In recent years, the application of quasi-Monte Carlo method to high-dimensional derivative pricing has been gaining popularity over other competitive approaches such as the Monte Carlo methods. Such success can be, in part, attributed to the notion of effective dimension associated with finance problems. Two simulation techniques, which aim to reduce effective dimensions of the problems, have been proposed to further enhance the underlying quasi-Monte Carlo methods. These approaches are based on principal component analysis and Brownian bridge discretization. In this paper, we provide additional insights to these two techniques in connection with effective dimensions. We discuss the pros and cons of these methods and we also propose a general dimension reduction technique in the context of derivative pricing. The proposed method is not only efficient when compared to other competitive approaches, it also has the advantages that it is general and is applicable to a wide range of problems. These are illustrated by valuing high-dimensional derivative securities including multi-factor path-dependent options and options with stochastic volatility. The nominal dimensions in some of these applications can be as high as 2500.