

Rubi 4.11 IntegrationTest Results

This file shows the integration problems for which **Rubi** either failed to get an antiderivative *or* returned a nonoptimal one. Each entry consists of the following items:

1. a phrase indicating the type of deficiency;
2. a list of the integrand, integration variable, number of rule applications desired, and number required;
3. the optimal antiderivative as contained in the test suite; and
4. the antiderivative as generated by **Rubi**.

0 Independent test suites

```
IntegrationTest["0 Independent test suites\\Apostol Problems"];
```

```
Testing Rubi on 175 integration problems...
```

```
Test complete!
```

```
IntegrationTest["0 Independent test suites\\Moses Problems"];
```

```
Testing Rubi on 113 integration problems...
```

```
Test complete!
```

```
IntegrationTest["0 Independent test suites\\Timofeev Problems"];
```

```
Testing Rubi on 705 integration problems...
```

```
Problem #222: Valid but suboptimal antiderivative:
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$$\left\{ \frac{\sqrt{1-x} x (1+x)^{2/3}}{-(1-x)^{5/6} (1+x)^{1/3} + (1-x)^{2/3} \sqrt{1+x}}, x, -49, 49 \right\}$$

$$-\frac{1}{12} (1-3x) (1-x)^{2/3} (1+x)^{1/3} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4} (1-x) (3+x) + \frac{1}{12} (1-x)^{1/3} (1+x)^{2/3} (1+3x) + \frac{1}{12} (1-x)^{1/6} (1+x)^{5/6} (2+3x) -$$

$$\frac{1}{12} (1-x)^{5/6} (1+x)^{1/6} (10+3x) + \frac{1}{6} \operatorname{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right] - \frac{4 \operatorname{ArcTan}\left[\frac{(1-x)^{1/3} - 2(1+x)^{1/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{3\sqrt{3}} - \frac{5}{6} \operatorname{ArcTan}\left[\frac{(1-x)^{1/3} - (1+x)^{1/3}}{(1-x)^{1/6} (1+x)^{1/6}}\right] + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3} (1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} + (1+x)^{1/3}}\right]}{6\sqrt{3}}$$

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$$\begin{aligned} & \frac{x}{2} + \frac{x^2}{4} - \frac{7}{12} (1-x)^{5/6} (1+x)^{1/6} + \frac{1}{6} (1-x)^{2/3} (1+x)^{1/3} - \frac{1}{4} (1-x)^{5/3} (1+x)^{1/3} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} + \frac{1}{3} (1-x)^{1/3} (1+x)^{2/3} - \frac{1}{4} (1-x)^{4/3} (1+x)^{2/3} + \frac{5}{12} (1-x)^{1/6} (1+x)^{5/6} - \\ & \frac{1}{4} (1-x)^{7/6} (1+x)^{5/6} - \frac{1}{4} (1-x)^{5/6} (1+x)^{7/6} + \frac{\text{ArcSin}[x]}{4} - \frac{2}{3} \text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \frac{2 \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{3\sqrt{3}} + \frac{1}{3} \text{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{1}{3} \text{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \\ & \frac{2 \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}} - \frac{1}{9} \text{Log}[1-x] + \frac{1}{9} \text{Log}[1+x] + \frac{1}{3} \text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right] - \frac{\text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}} + \frac{\text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}} - \frac{1}{3} \text{Log}\left[1 + \frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right] \end{aligned}$$

Problem #226: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{((-1+x)^2(1+x))^{1/3}}, x, -3, 3 \right\}$$

$$\sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\right] - \frac{1}{2} \text{Log}[1+x] - \frac{3}{2} \text{Log}\left[1 - \frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\right]$$

$$-\frac{\sqrt{3}(-1+x)^{2/3}(1+x)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1+x)^{1/3}}{\sqrt{3}(-1+x)^{1/3}}\right]}{(1-x-x^2+x^3)^{1/3}} - \frac{(-1+x)^{2/3}(1+x)^{1/3} \text{Log}[-1+x]}{2(1-x-x^2+x^3)^{1/3}} - \frac{3(-1+x)^{2/3}(1+x)^{1/3} \text{Log}\left[-1 + \frac{(1+x)^{1/3}}{(-1+x)^{1/3}}\right]}{2(1-x-x^2+x^3)^{1/3}}$$

Problem #228: Valid but suboptimal antiderivative:

$$\left\{ \frac{((-1+x)^2(1+x))^{1/3}}{x^2}, x, -6, 6 \right\}$$

$$-\frac{((-1+x)^2(1+x))^{1/3}}{x} - \frac{\text{ArcTan}\left[\frac{1 - \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\right] + \frac{\text{Log}[x]}{6} - \frac{2}{3} \text{Log}[1+x] - \frac{3}{2} \text{Log}\left[1 - \frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\right] - \frac{1}{2} \text{Log}\left[1 + \frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\right]$$

$$-\frac{(1-x-x^2+x^3)^{1/3}}{x} - \frac{(1-x-x^2+x^3)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(-1+x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{\sqrt{3}(-1+x)^{2/3}(1+x)^{1/3}} - \frac{\sqrt{3}(1-x-x^2+x^3)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(-1+x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{(-1+x)^{2/3}(1+x)^{1/3}} +$$

$$\frac{(1-x-x^2+x^3)^{1/3} \text{Log}[x]}{6(-1+x)^{2/3}(1+x)^{1/3}} - \frac{(1-x-x^2+x^3)^{1/3} \text{Log}[1+x]}{2(-1+x)^{2/3}(1+x)^{1/3}} - \frac{3(1-x-x^2+x^3)^{1/3} \text{Log}\left[-1 + \frac{(-1+x)^{1/3}}{(1+x)^{1/3}}\right]}{2(-1+x)^{2/3}(1+x)^{1/3}} - \frac{(1-x-x^2+x^3)^{1/3} \text{Log}\left[-(-1+x)^{1/3} - (1+x)^{1/3}\right]}{2(-1+x)^{2/3}(1+x)^{1/3}}$$

Problem #232: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{(9+3x-5x^2+x^3)^{1/3}}, x, -2, 2 \right\}$$

$$\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(-3+x)}{(9+3x-5x^2+x^3)^{1/3}}}{\sqrt{3}}\right] - \frac{1}{2} \operatorname{Log}[1+x] - \frac{3}{2} \operatorname{Log}\left[1 - \frac{-3+x}{(9+3x-5x^2+x^3)^{1/3}}\right]$$

$$- \frac{\sqrt{3}(-3+x)^{2/3}(1+x)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1+x)^{1/3}}{\sqrt{3}(-3+x)^{1/3}}\right]}{(9+3x-5x^2+x^3)^{1/3}} - \frac{(-3+x)^{2/3}(1+x)^{1/3} \operatorname{Log}[-3+x]}{2(9+3x-5x^2+x^3)^{1/3}} - \frac{3(-3+x)^{2/3}(1+x)^{1/3} \operatorname{Log}\left[-1 + \frac{(1+x)^{1/3}}{(-3+x)^{1/3}}\right]}{2(9+3x-5x^2+x^3)^{1/3}}$$

Problem #288: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{x + \sqrt{1+x+x^2}}, x, -3, 3 \right\}$$

$$-x + \sqrt{1+x+x^2} - \frac{3}{2} \operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right] + 2 \operatorname{Log}\left[x + \sqrt{1+x+x^2}\right]$$

$$\frac{3}{2\left(1+2\left(x + \sqrt{1+x+x^2}\right)\right)} + 2 \operatorname{Log}\left[x + \sqrt{1+x+x^2}\right] - \frac{3}{2} \operatorname{Log}\left[1+2\left(x + \sqrt{1+x+x^2}\right)\right]$$

Problem #306: Valid but suboptimal antiderivative:

$$\left\{ (x(1-x^2))^{1/3}, x, -13, 13 \right\}$$

$$\frac{1}{2} x (x(1-x^2))^{1/3} + \frac{\operatorname{ArcTan}\left[\frac{2x-(x(1-x^2))^{1/3}}{\sqrt{3}(x(1-x^2))^{1/3}}\right]}{2\sqrt{3}} + \frac{\operatorname{Log}[x]}{12} - \frac{1}{4} \operatorname{Log}\left[x + (x(1-x^2))^{1/3}\right]$$

$$\frac{1}{2} x (x-x^3)^{1/3} - \frac{x^{2/3}(1-x^2)^{2/3} \operatorname{ArcTan}\left[\frac{1-\frac{2x^{2/3}}{(1-x^2)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}(x-x^3)^{2/3}} + \frac{x^{2/3}(1-x^2)^{2/3} \operatorname{Log}\left[1 + \frac{x^{4/3}}{(1-x^2)^{2/3}} - \frac{x^{2/3}}{(1-x^2)^{1/3}}\right]}{12(x-x^3)^{2/3}} - \frac{x^{2/3}(1-x^2)^{2/3} \operatorname{Log}\left[1 + \frac{x^{2/3}}{(1-x^2)^{1/3}}\right]}{6(x-x^3)^{2/3}}$$

Problem #401: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Tan}[x]}{(-1 + \sqrt{\operatorname{Tan}[x]})^2}, x, -21, 21 \right\}$$

$$-\frac{x}{2} + \frac{\operatorname{ArcTan}\left[\frac{1-\operatorname{Tan}[x]}{\sqrt{2}\sqrt{\operatorname{Tan}[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTanh}\left[\frac{1+\operatorname{Tan}[x]}{\sqrt{2}\sqrt{\operatorname{Tan}[x]}}\right]}{\sqrt{2}} + \frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x]] + \operatorname{Log}\left[1 - \sqrt{\operatorname{Tan}[x]}\right] + \frac{1}{1 - \sqrt{\operatorname{Tan}[x]}}$$

$$-\frac{x}{2} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{\operatorname{Tan}[x]}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{\operatorname{Tan}[x]}\right]}{\sqrt{2}} + \frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x]] + \operatorname{Log}\left[1 - \sqrt{\operatorname{Tan}[x]}\right] - \frac{\operatorname{Log}\left[1 - \sqrt{2}\sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x]\right]}{2\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \sqrt{2}\sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x]\right]}{2\sqrt{2}} + \frac{1}{1 - \sqrt{\operatorname{Tan}[x]}}$$

Problem #411: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[x]^3 (\cos[2x] - 3 \tan[x])}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}}, x, -128, 128 \right\}$$

$$\frac{33}{32} \operatorname{ArcTanh}\left[\frac{1}{2} \operatorname{Sec}[x] \sqrt{\sin[2x]}\right] - \frac{9 \cos[x]}{16 \sqrt{\sin[2x]}} - \frac{5 \cos[x] \cot[x]}{24 \sqrt{\sin[2x]}} + \frac{\cos[x] \cot[x]^2}{20 \sqrt{\sin[2x]}}$$

$$- \left(33 i \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{1}{2}(-1 - \sqrt{5}), i \operatorname{ArcSinh}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{1 - \tan\left[\frac{x}{2}\right]} \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 + \tan\left[\frac{x}{2}\right]} \right) / \left(32 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) -$$

$$\left(33 i \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), i \operatorname{ArcSinh}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{1 - \tan\left[\frac{x}{2}\right]} \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 + \tan\left[\frac{x}{2}\right]} \right) / \left(32 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) -$$

$$\frac{69 \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} - 9(2 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} +$$

$$\frac{80 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} - 80(1 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}$$

$$\left(15(2 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} \right) / \left(8(3 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) +$$

$$\frac{3(3 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} - 9(2 + \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} +$$

$$\frac{32(1 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} - 80(1 + \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}$$

$$\left(15(2 + \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} \right) / \left(8(3 + \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) +$$

$$\frac{3(3 + \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} -$$

$$32(1 + \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}$$

$$\frac{43 \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} - (1 - 3\sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} +$$

$$\frac{40 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} - 160(1 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}$$

$$\left(11(2 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2} \right) / \left(16(1 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) -$$

$$\left\{ \frac{\cos[2x] - \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}}, x, -31, 31 \right\}$$

$$-\sqrt{2} \operatorname{Log}\left[\cos[x] + \sin[x] - \sqrt{2} \operatorname{Sec}[x] \sqrt{\cos[x]^3 \sin[x]}\right] - \frac{\operatorname{ArcSin}[\cos[x] - \sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\operatorname{ArcTanh}[\sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\sin[2x]}{\sqrt{\cos[x]^3 \sin[x]}}$$

$$-2 \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]} - \sqrt{2} \operatorname{ArcSinh}[\tan[x]] \cot[x] (\operatorname{Sec}[x]^2)^{3/2} \sqrt{\cos[x] \sin[x]} \sqrt{\cos[x]^3 \sin[x]} - \frac{\sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[x]}\right] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]}}{\sqrt{\tan[x]}} +$$

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[x]}\right] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]}}{\sqrt{\tan[x]}} - \frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[x]} + \tan[x]\right] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]}}{\sqrt{2} \sqrt{\tan[x]}} + \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[x]} + \tan[x]\right] \operatorname{Sec}[x]^2 \sqrt{\cos[x]^3 \sin[x]}}{\sqrt{2} \sqrt{\tan[x]}}$$

Problem #427: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[3x]}{-\sqrt{-1 + 8 \cos[x]^2} + \sqrt{3 \cos[x]^2 - \sin[x]^2}}, x, -51, 51 \right\}$$

$$\frac{5 \operatorname{ArcSin}\left[2 \sqrt{\frac{2}{7}} \sin[x]\right]}{4 \sqrt{2}} + \frac{3}{4} \operatorname{ArcSin}\left[\frac{2 \sin[x]}{\sqrt{3}}\right] - \frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{-1 + 4 \cos[x]^2}}\right] - \frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{-1 + 8 \cos[x]^2}}\right] - \frac{1}{2} \sqrt{-1 + 4 \cos[x]^2} \sin[x] - \frac{1}{2} \sqrt{-1 + 8 \cos[x]^2} \sin[x]$$

$$\frac{5 \operatorname{ArcSin}\left[2 \sqrt{\frac{2}{7}} \sin[x]\right]}{4 \sqrt{2}} + \frac{3}{4} \operatorname{ArcSin}\left[\frac{2 \sin[x]}{\sqrt{3}}\right] + \frac{3}{8} \operatorname{ArcTan}\left[\frac{7 - 8 \sin[x]}{\sqrt{7 - 8 \sin[x]^2}}\right] - \frac{3}{8} \operatorname{ArcTan}\left[\frac{7 + 8 \sin[x]}{\sqrt{7 - 8 \sin[x]^2}}\right] +$$

$$\frac{3}{8} \operatorname{ArcTan}\left[\frac{3 - 4 \sin[x]}{\sqrt{3 - 4 \sin[x]^2}}\right] - \frac{3}{8} \operatorname{ArcTan}\left[\frac{3 + 4 \sin[x]}{\sqrt{3 - 4 \sin[x]^2}}\right] - \frac{1}{2} \sin[x] \sqrt{7 - 8 \sin[x]^2} - \frac{1}{2} \sin[x] \sqrt{3 - 4 \sin[x]^2}$$

Problem #447: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Sec}[x]^2 (-\cos[2x] + 2 \tan[x]^2)}{(\tan[x] \tan[2x])^{3/2}}, x, -20, 20 \right\}$$

$$2 \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right] - \frac{11 \operatorname{ArcTanh}\left[\frac{\sqrt{2} \tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right]}{4 \sqrt{2}} + \frac{\tan[x]}{2 (\tan[x] \tan[2x])^{3/2}} + \frac{2 \tan[x]^3}{3 (\tan[x] \tan[2x])^{3/2}} + \frac{3 \tan[x]}{4 \sqrt{\tan[x] \tan[2x]}}$$

$$\frac{3 \tan[x]}{4 \sqrt{2} \sqrt{\frac{\tan[x]^2}{1 - \tan[x]^2}}} + \frac{\cot[x] (1 - \tan[x]^2)}{4 \sqrt{2} \sqrt{\frac{\tan[x]^2}{1 - \tan[x]^2}}} + \frac{\tan[x] (1 - \tan[x]^2)}{3 \sqrt{2} \sqrt{\frac{\tan[x]^2}{1 - \tan[x]^2}}} - \frac{11 \operatorname{ArcTan}\left[\sqrt{-1 + \tan[x]^2}\right] \tan[x]}{4 \sqrt{2} \sqrt{\frac{\tan[x]^2}{1 - \tan[x]^2}} \sqrt{-1 + \tan[x]^2}} + \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-1 + \tan[x]^2}}{\sqrt{2}}\right] \tan[x]}{\sqrt{\frac{\tan[x]^2}{1 - \tan[x]^2}} \sqrt{-1 + \tan[x]^2}}$$

Problem #455: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]^6 \tan[x]}{\cos[2x]^{3/4}}, x, -16, 16 \right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{\cos[2x]}}{\sqrt{2}\cos[2x]^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{1+\sqrt{\cos[2x]}}{\sqrt{2}\cos[2x]^{1/4}}\right]}{\sqrt{2}} + \frac{71}{45}\cos[2x]^{1/4} + \frac{13}{45}\cos[2x]^{1/4}\sin[x]^2 + \frac{1}{9}\cos[2x]^{1/4}\sin[x]^4$$

$$\frac{\operatorname{ArcTan}\left[1-\sqrt{2}\left(-1+2\cos[x]^2\right)^{1/4}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1+\sqrt{2}\left(-1+2\cos[x]^2\right)^{1/4}\right]}{\sqrt{2}} + \frac{7}{4}\left(-1+2\cos[x]^2\right)^{1/4} - \frac{1}{5}\left(-1+2\cos[x]^2\right)^{5/4} +$$

$$\frac{1}{36}\left(-1+2\cos[x]^2\right)^{9/4} + \frac{\operatorname{Log}\left[1-\sqrt{2}\left(-1+2\cos[x]^2\right)^{1/4}+\sqrt{-1+2\cos[x]^2}\right]}{2\sqrt{2}} - \frac{\operatorname{Log}\left[1+\sqrt{2}\left(-1+2\cos[x]^2\right)^{1/4}+\sqrt{-1+2\cos[x]^2}\right]}{2\sqrt{2}}$$

Problem #456: Valid but suboptimal antiderivative:

$$\left\{ \sqrt{\tan[x] \tan[2x]}, x, -6, 6 \right\}$$

$$-\operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right]$$

$$\operatorname{ArcTan}\left[\frac{\sqrt{-1+\tan[x]^2}}{\sqrt{2}}\right] \cot[x] \sqrt{\frac{\tan[x]^2}{1-\tan[x]^2}} \sqrt{-1+\tan[x]^2}$$

Problem #493: Unable to integrate:

$$\left\{ \frac{x^2}{(x \cos[x] - \sin[x])^2}, x, -1, 0 \right\}$$

$$\frac{\cos[x] + x \sin[x]}{x \cos[x] - \sin[x]}$$

$$\operatorname{Int}\left[\frac{x^2}{(x \cos[x] - \sin[x])^2}, x\right]$$

Problem #567: Valid but suboptimal antiderivative:

$$\left\{ e^{x/2} x^2 \cos[x]^3, x, -31, 31 \right\}$$

$$-\frac{132}{125}e^{x/2}\cos[x] + \frac{18}{25}e^{x/2}x\cos[x] + \frac{48}{185}e^{x/2}x^2\cos[x] + \frac{2}{37}e^{x/2}x^2\cos[x]^3 - \frac{428e^{x/2}\cos[3x]}{50653} + \frac{70e^{x/2}x\cos[3x]}{1369}$$

$$\frac{24}{125}e^{x/2}\sin[x] - \frac{24}{25}e^{x/2}x\sin[x] + \frac{96}{185}e^{x/2}x^2\sin[x] + \frac{12}{37}e^{x/2}x^2\cos[x]^2\sin[x] - \frac{792e^{x/2}\sin[3x]}{50653} - \frac{24e^{x/2}x\sin[3x]}{1369}$$

$$\begin{aligned}
& -\frac{6\,687\,696\,e^{x/2}\cos[x]}{6\,331\,625} + \frac{24\,792\,e^{x/2}x\cos[x]}{34\,225} + \frac{48}{185}e^{x/2}x^2\cos[x] + \frac{16\,e^{x/2}\cos[x]^3}{50\,653} - \frac{8\,e^{x/2}x\cos[x]^3}{1369} + \frac{2}{37}e^{x/2}x^2\cos[x]^3 - \frac{432\,e^{x/2}\cos[3x]}{50\,653} + \frac{72\,e^{x/2}x\cos[3x]}{1369} - \frac{1\,218\,672\,e^{x/2}\sin[x]}{6\,331\,625} \\
& -\frac{32\,556\,e^{x/2}x\sin[x]}{34\,225} + \frac{96}{185}e^{x/2}x^2\sin[x] + \frac{96\,e^{x/2}\cos[x]^2\sin[x]}{50\,653} - \frac{48\,e^{x/2}x\cos[x]^2\sin[x]}{1369} + \frac{12}{37}e^{x/2}x^2\cos[x]^2\sin[x] - \frac{816\,e^{x/2}\sin[3x]}{50\,653} - \frac{12\,e^{x/2}x\sin[3x]}{1369}
\end{aligned}$$

Problem #592: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\cosh[x](-\cosh[2x] + \tanh[x])}{\sqrt{\sinh[2x]}(\sinh[x]^2 + \sinh[2x])}, x, -52, 52 \right\} \\
& \sqrt{2} \operatorname{ArcTan}\left[\operatorname{Sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{1}{6} \operatorname{ArcTan}\left[\frac{\sinh[x]}{\sqrt{\sinh[2x]}}\right] - \frac{1}{3} \sqrt{2} \operatorname{ArcTanh}\left[\operatorname{Sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{\cosh[x]}{\sqrt{\sinh[2x]}} \\
& \left((-1)^{1/4} \cosh\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{2}{i - \sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{\tanh\left[\frac{x}{2}\right]} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \right) / \\
& \left(\sqrt{\sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) + \\
& \left((-1)^{1/4} \cosh\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{2}{i + \sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{\tanh\left[\frac{x}{2}\right]} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \right) / \\
& \left(\sqrt{\sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) - \frac{i \operatorname{Csch}\left[\frac{x}{2}\right] \operatorname{Sech}\left[\frac{x}{2}\right] \sqrt{i \sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)}}{2 \sqrt{\sinh[2x]}} - \\
& \left(4 (-1)^{3/4} \operatorname{EllipticPi}\left[-i, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) / \\
& \left(\sqrt{\sinh[2x]} \sqrt{\tanh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) + \\
& \left(4 (-1)^{3/4} \operatorname{EllipticPi}\left[i, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) / \\
& \left(3 \sqrt{\sinh[2x]} \sqrt{\tanh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) + \\
& \left(5 (-1)^{3/4} \operatorname{EllipticPi}\left[\frac{2}{i - \sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) / \\
& \left(6 \sqrt{\sinh[2x]} \sqrt{\tanh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(5 (-1)^{3/4} \operatorname{EllipticPi}\left[\frac{2}{i + \sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \operatorname{Sinh}[2x]} \sqrt{-i\left(i - \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \sqrt{-i\left(i + \operatorname{Tanh}\left[\frac{x}{2}\right]\right)} \sqrt{i \operatorname{Cosh}\left[\frac{x}{2}\right]^3 \operatorname{Sinh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) / \\
& \left(6 \sqrt{\operatorname{Sinh}[2x]} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) - \\
& \left(\operatorname{Cosh}\left[\frac{x}{2}\right]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right]}\right], \frac{1}{2}\right] \sqrt{i \operatorname{Sinh}[2x]} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \sqrt{\frac{1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2}{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(2 \sqrt{\operatorname{Sinh}[2x]} \sqrt{i \operatorname{Cosh}\left[\frac{x}{2}\right]^3 \operatorname{Sinh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) + \\
& \left(3 i \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right]}\right], \frac{1}{2}\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \operatorname{Sinh}[2x]} \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{i \operatorname{Cosh}\left[\frac{x}{2}\right]^3 \operatorname{Sinh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \sqrt{\frac{1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2}{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(2 \sqrt{\operatorname{Sinh}[2x]} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) - \\
& \left(5 (3 i - \sqrt{3}) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right]}\right], \frac{1}{2}\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \operatorname{Sinh}[2x]} \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{i \operatorname{Cosh}\left[\frac{x}{2}\right]^3 \operatorname{Sinh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \sqrt{\frac{1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2}{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \\
& \left(24 \sqrt{\operatorname{Sinh}[2x]} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right) - \\
& \left(5 (3 i + \sqrt{3}) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right]}\right], \frac{1}{2}\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \operatorname{Sinh}[2x]} \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]\right) \sqrt{i \operatorname{Cosh}\left[\frac{x}{2}\right]^3 \operatorname{Sinh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \sqrt{\frac{1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2}{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \\
& \left(24 \sqrt{\operatorname{Sinh}[2x]} \sqrt{\operatorname{Tanh}\left[\frac{x}{2}\right] \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)} \right)
\end{aligned}$$

Problem #695: Valid but suboptimal antiderivative:

$$\left\{ \operatorname{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right], x, -8, 8 \right\}$$

$$-\frac{\sqrt{2} a \sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}} + (a+x) \operatorname{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

$$\begin{aligned}
& \frac{2a \sqrt{\frac{a}{a+x}} \sqrt{\frac{a-x}{a+x}} \sqrt{a+x}}{\sqrt{1+\frac{a-x}{a+x}} \left(1+\frac{a-x}{a+x}\right)} + x \operatorname{ArcSin}\left[\sqrt{\frac{a-x}{a+x}}\right] - \frac{a \sqrt{\frac{a}{a+x}} \sqrt{a+x} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{a-x}{a+x}}}{\sqrt{1-\frac{a-x}{a+x}}}\right]}{\sqrt{1-\frac{a-x}{a+x}} \sqrt{1+\frac{a-x}{a+x}}}
\end{aligned}$$

Problem #705: Valid but suboptimal antiderivative:

$\{e^x \operatorname{ArcSin}[\operatorname{Tanh}[x]], x, -5, 5\}$

$$e^x \operatorname{ArcSin}[\operatorname{Tanh}[x]] - \operatorname{Cosh}[x] \operatorname{Log}[1 + e^{2x}] \sqrt{\operatorname{Sech}[x]^2}$$

$$-e^x \operatorname{ArcSin}\left[\frac{1 - e^{2x}}{1 + e^{2x}}\right] - e^{-x} \sqrt{\frac{e^{2x}}{(1 + e^{2x})^2}} (1 + e^{2x}) \operatorname{Log}[1 + e^{2x}]$$

Test complete!

IntegrationTest["0 Independent test suites\Charlwood Problems"];

Testing Rubi on 50 integration problems...

Problem #3: Unable to integrate:

$\{-\operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}], x, -3, 3\}$

$$\frac{(\sqrt{x} + 3\sqrt{1+x}) \sqrt{-x + \sqrt{x} \sqrt{1+x}}}{4\sqrt{2}} - \left(\frac{3}{8} + x\right) \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]$$

$$-x \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}] + \frac{\operatorname{Subst}\left[\operatorname{Int}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}, x\right], x, \sqrt{1+x}\right]}{\sqrt{2}}$$

Problem #4: Valid but suboptimal antiderivative:

$\{\operatorname{Log}[1+x\sqrt{1+x^2}], x, -32, 32\}$

$$-2x + \sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{-2+\sqrt{5}}(x+\sqrt{1+x^2})\right] - \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{2+\sqrt{5}}(x+\sqrt{1+x^2})\right] + x \operatorname{Log}[1+x\sqrt{1+x^2}]$$

$$\begin{aligned}
& -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] + \\
& \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] + 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \\
& \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] - \sqrt{\frac{2}{5}(1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] + x \operatorname{Log}[1+x\sqrt{1+x^2}]
\end{aligned}$$

Problem #5: NOT valid or unverifiable antiderivative:

$$\left\{ \frac{\cos[x]^2}{\sqrt{1+\cos[x]^2+\cos[x]^4}}, x, -4, 4 \right\}$$

$$-\frac{x}{3} + \frac{1}{3} \operatorname{ArcTan}\left[\frac{\cos[x](1+\cos[x]^2)\sin[x]}{1+\cos[x]^2\sqrt{1+\cos[x]^2+\cos[x]^4}} \right]$$

$$\operatorname{ArcTan}\left[\frac{\tan[x]}{\sqrt{3+3\tan[x]^2+\tan[x]^4}} \right] \cos[x]^2 \sqrt{3+3\tan[x]^2+\tan[x]^4}$$

$$2\sqrt{\cos[x]^4(3+3\tan[x]^2+\tan[x]^4)}$$

$$\left(3^{1/4} \cos[x]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\tan[x]}{3^{1/4}}\right], \frac{1}{4}(2-\sqrt{3})\right] (\sqrt{3}+\tan[x]^2) \sqrt{\frac{3+3\tan[x]^2+\tan[x]^4}{(\sqrt{3}+\tan[x]^2)^2}} \right) / \left(2(3-\sqrt{3}) \sqrt{\cos[x]^4(3+3\tan[x]^2+\tan[x]^4)} \right) +$$

$$\left(3^{1/4} (1+\sqrt{3}) \cos[x]^2 \operatorname{EllipticPi}\left[\frac{1}{6}(3-2\sqrt{3}), 2 \operatorname{ArcTan}\left[\frac{\tan[x]}{3^{1/4}}\right], \frac{1}{4}(2-\sqrt{3})\right] (\sqrt{3}+\tan[x]^2) \sqrt{\frac{3+3\tan[x]^2+\tan[x]^4}{(\sqrt{3}+\tan[x]^2)^2}} \right) / \left(4(3-\sqrt{3}) \sqrt{\cos[x]^4(3+3\tan[x]^2+\tan[x]^4)} \right)$$

Problem #12: Valid but suboptimal antiderivative:

$$\left\{ \operatorname{ArcTan}\left[x + \sqrt{1-x^2}\right], x, -44, 44 \right\}$$

$$-\frac{\operatorname{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \operatorname{ArcTan}\left[\frac{-1+\sqrt{3}x}{\sqrt{1-x^2}}\right] + \frac{1}{4}\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\sqrt{3}x}{\sqrt{1-x^2}}\right] - \frac{1}{4}\sqrt{3} \operatorname{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right] + x \operatorname{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{4} \operatorname{ArcTanh}\left[x\sqrt{1-x^2}\right] - \frac{1}{8} \operatorname{Log}[1-x^2+x^4]$$

$$\begin{aligned}
& -\frac{\text{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right]}{\sqrt{3}} + \frac{1}{12}(3i-\sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right] + \\
& \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}}{\sqrt{1-x^2}}x\right]}{\sqrt{3}} - \frac{1}{12}(3i+\sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}}{\sqrt{1-x^2}}x\right] + x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{8}\text{Log}\left[1-x^2+x^4\right]
\end{aligned}$$

Problem #13: Valid but suboptimal antiderivative:

$$\left\{ \frac{x \text{ArcTan}\left[x + \sqrt{1-x^2}\right]}{\sqrt{1-x^2}}, x, -33, 33 \right\}$$

$$-\frac{\text{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{-1+\sqrt{3}x}{\sqrt{1-x^2}}\right] + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{1+\sqrt{3}x}{\sqrt{1-x^2}}\right] - \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right] - \sqrt{1-x^2} \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right] + \frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

$$\begin{aligned}
& -\frac{\text{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right]}{2\sqrt{3}} - \frac{1}{12}(3i-\sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right] + \\
& \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}}{\sqrt{1-x^2}}x\right]}{2\sqrt{3}} + \frac{1}{12}(3i+\sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}}{\sqrt{1-x^2}}x\right] - \sqrt{1-x^2} \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \frac{1}{8}\text{Log}\left[1-x^2+x^4\right]
\end{aligned}$$

Problem #42: Valid but suboptimal antiderivative:

$$\left\{ \frac{\text{Sec}[x]}{\sqrt{-1+\text{Sec}[x]^4}}, x, -6, 6 \right\}$$

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x] \text{Cot}[x] \sqrt{-1+\text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \sin[x]}{\sqrt{2 \sin[x]^2 - \sin[x]^4}}\right] \sqrt{1 - \cos[x]^4} \sec[x]^2}{\sqrt{2} \sqrt{-1 + \sec[x]^4}}$$

Problem #44: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]}{\sqrt{1 - \sin[x]^6}}, x, -5, 5 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{3} \cos[x] (1 + \sin[x]^2)}{2 \sqrt{1 - \sin[x]^6}}\right]}{2 \sqrt{3}}$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{3} (2 - \cos[x]^2)}{2 \sqrt{3 - 3 \cos[x]^2 + \cos[x]^4}}\right] \cos[x] \sqrt{3 - 3 \cos[x]^2 + \cos[x]^4}}{2 \sqrt{3} \sqrt{3 \cos[x]^2 - 3 \cos[x]^4 + \cos[x]^6}}$$

Problem #45: Unable to integrate:

$$\left\{ \sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}, x, -1, 0 \right\}$$

$$\sqrt{2} \left(\sqrt{-1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{-2 + 2\sqrt{2}} (-\sqrt{2} - \sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]})}{2 \sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}}\right] - \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2 + 2\sqrt{2}} (-\sqrt{2} - \sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]})}{2 \sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}}\right] \right) -$$

$$\left(\sqrt{1 + \sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{-2 + 2\sqrt{2}} \sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}}{\sqrt{2} - \sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}\right] + \sqrt{-1 + \sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{2 + 2\sqrt{2}} \sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}}{\sqrt{2} - \sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}\right] \right) \cot[x] \sqrt{-1 + \sec[x]} \sqrt{1 + \sec[x]}$$

$$\text{Int}\left[\sqrt{-\sqrt{-1 + \sec[x]} + \sqrt{1 + \sec[x]}}, x\right]$$

Test complete!

IntegrationTest["0 Independent test suites\\Stewart Problems"];

Testing Rubi on 376 integration problems...

Test complete!

IntegrationTest["0 Independent test suites\\Hearn Problems"];

Testing Rubi on 284 integration problems...

Problem #169: Unable to integrate:

$$\left\{ \frac{e^{1-e^{x^2}} x + 2 x^2 (x + 2 x^3)}{(1 - e^{x^2} x)^2}, x, -3, 3 \right\}$$

$$-\frac{e^{1-e^{x^2}} x}{-1 + e^{x^2} x}$$

$$\text{Int} \left[\frac{e^{1-e^{x^2}} x + 2 x^2 (x + 2 x^3)}{(1 - e^{x^2} x)^2}, x \right] + 2 \text{Int} \left[\frac{e^{1-e^{x^2}} x + 2 x^2 (x + 2 x^3)}{(1 - e^{x^2} x)^2}, x \right]$$

Problem #201: Unable to integrate:

$$\left\{ \frac{-2 \sqrt{1+x^3} + 5 x^4 \sqrt{1+x^3} - 3 x^2 \sqrt{1-2x+x^5}}{2 \sqrt{1+x^3} \sqrt{1-2x+x^5}}, x, -4, 4 \right\}$$

$$-\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

$$-\sqrt{1+x^3} - \text{Int} \left[\frac{1}{\sqrt{1-2x+x^5}}, x \right] + \frac{5}{2} \text{Int} \left[\frac{x^4}{\sqrt{1-2x+x^5}}, x \right]$$

Problem #274: Unable to integrate:

$$\left\{ \frac{e^{x^2}}{x} + 2 e^{x^2} x \text{Log}[x] + \frac{-2 + \text{Log}[x]}{(x + \text{Log}[x]^2)^2} + \frac{1 + \frac{1}{x} + \frac{2 \text{Log}[x]}{x}}{x + \text{Log}[x]^2}, x, -9, 9 \right\}$$

$$e^{x^2} \text{Log}[x] - \frac{\text{Log}[x]}{x + \text{Log}[x]^2} + \text{Log}[x + \text{Log}[x]^2]$$

$$e^{x^2} \text{Log}[x] - 2 \text{Int} \left[\frac{1}{(x + \text{Log}[x]^2)^2}, x \right] + \text{Int} \left[\frac{\text{Log}[x]}{(x + \text{Log}[x]^2)^2}, x \right] + \text{Int} \left[\frac{1}{x + \text{Log}[x]^2}, x \right] + \text{Int} \left[\frac{1}{x (x + \text{Log}[x]^2)}, x \right] + 2 \text{Int} \left[\frac{\text{Log}[x]}{x (x + \text{Log}[x]^2)}, x \right]$$

Problem #278: Unable to integrate:

$$\left\{ \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}}, x, -10, 10 \right\}$$

$$\frac{(1 + 2x) \sqrt{1 + 2x^2 + 4x^3 + x^4}}{2(-1 + 2x^2)} - \text{ArcTanh} \left[\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3) \sqrt{1+2x^2+4x^3+x^4}} \right]$$

$$\begin{aligned} & -\frac{9}{4} \operatorname{Int}\left[\frac{1}{\sqrt{1+2x^2+4x^3+x^4}}, x\right] - \frac{13}{4} \operatorname{Int}\left[\frac{1}{(\sqrt{2}-2x)^2 \sqrt{1+2x^2+4x^3+x^4}}, x\right] + \operatorname{Int}\left[\frac{x}{\sqrt{1+2x^2+4x^3+x^4}}, x\right] + \frac{1}{2} \operatorname{Int}\left[\frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}, x\right] - \\ & \frac{13}{4} \operatorname{Int}\left[\frac{1}{(\sqrt{2}+2x)^2 \sqrt{1+2x^2+4x^3+x^4}}, x\right] - \frac{13}{8} \operatorname{Int}\left[\frac{1}{(1-\sqrt{2}x) \sqrt{1+2x^2+4x^3+x^4}}, x\right] - \frac{1}{8} (15+\sqrt{2}) \operatorname{Int}\left[\frac{1}{(1-\sqrt{2}x) \sqrt{1+2x^2+4x^3+x^4}}, x\right] - \\ & \frac{13}{8} \operatorname{Int}\left[\frac{1}{(1+\sqrt{2}x) \sqrt{1+2x^2+4x^3+x^4}}, x\right] - \frac{1}{8} (15-\sqrt{2}) \operatorname{Int}\left[\frac{1}{(1+\sqrt{2}x) \sqrt{1+2x^2+4x^3+x^4}}, x\right] - \frac{17}{2} \operatorname{Int}\left[\frac{x}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}}, x\right] \end{aligned}$$

Problem #279: Unable to integrate:

$$\begin{aligned} & \left\{ \frac{(1+2y) \sqrt{1-5y-5y^2}}{y(1+y)(2+y) \sqrt{1-y-y^2}}, y, -2, 2 \right\} \\ & -\frac{1}{4} \operatorname{ArcTanh}\left[\frac{(1-3y) \sqrt{1-5y-5y^2}}{(1-5y) \sqrt{1-y-y^2}}\right] - \frac{1}{2} \operatorname{ArcTanh}\left[\frac{(4+3y) \sqrt{1-5y-5y^2}}{(6+5y) \sqrt{1-y-y^2}}\right] + \frac{9}{4} \operatorname{ArcTanh}\left[\frac{(11+7y) \sqrt{1-5y-5y^2}}{3(7+5y) \sqrt{1-y-y^2}}\right] \\ & \frac{1}{2} \operatorname{Int}\left[\frac{\sqrt{1-5y-5y^2}}{y \sqrt{1-y-y^2}}, y\right] + \operatorname{Int}\left[\frac{\sqrt{1-5y-5y^2}}{(1+y) \sqrt{1-y-y^2}}, y\right] - \frac{3}{2} \operatorname{Int}\left[\frac{\sqrt{1-5y-5y^2}}{(2+y) \sqrt{1-y-y^2}}, y\right] \end{aligned}$$

Problem #284: Unable to integrate:

$$\begin{aligned} & \left\{ \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x, -2, 2 \right\} \\ & \frac{1}{2} \left((1+\sqrt{2}) \operatorname{Log}\left[1+x+\sqrt{2}x+\sqrt{2}x^2-x^7\right] - (-1+\sqrt{2}) \operatorname{Log}\left[-1+(-1+\sqrt{2})x+\sqrt{2}x^2+x^7\right] \right) \\ & 3 \operatorname{Int}\left[\frac{1}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] + 3 \operatorname{Int}\left[\frac{x}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] - \\ & 4 \operatorname{Int}\left[\frac{x^2}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] - 4 \operatorname{Int}\left[\frac{x^3}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] - 7 \operatorname{Int}\left[\frac{x^6}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] + \\ & 4 \operatorname{Int}\left[\frac{x^7}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] + 10 \operatorname{Int}\left[\frac{x^8}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] + 7 \operatorname{Int}\left[\frac{x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x\right] \end{aligned}$$

Test complete!

IntegrationTest["0 Independent test suites\Jeffrey Problems"];

Testing Rubi on 9 integration problems...

Problem #2: Valid but suboptimal antiderivative:

$$\left\{ \frac{1 + \cos[x] + 2 \sin[x]}{3 + \cos[x]^2 + 2 \sin[x] - 2 \cos[x] \sin[x]}, x, -49, 49 \right\}$$

$$-\text{ArcTan}\left[\frac{2 \cos[x] - \sin[x]}{2 + \sin[x]}\right]$$

$$\text{ArcTan}\left[\frac{1}{4} \left(-1 + \left(1 + 2 \tan\left[\frac{x}{2}\right]\right)^2\right)\right] + \text{Cot}\left[\frac{x}{2}\right] - \frac{\sin[x]}{1 - \cos[x]}$$

Problem #3: Valid but suboptimal antiderivative:

$$\left\{ \frac{2 + \cos[x] + 5 \sin[x]}{4 \cos[x] - 2 \sin[x] + \cos[x] \sin[x] - 2 \sin[x]^2}, x, -29, 29 \right\}$$

$$-\text{Log}[1 - 3 \cos[x] + \sin[x]] + \text{Log}[3 + \cos[x] + \sin[x]]$$

$$-\text{Log}\left[1 - 2 \tan\left[\frac{x}{2}\right]\right] - \text{Log}\left[1 + \tan\left[\frac{x}{2}\right]\right] + \text{Log}\left[2 + \tan\left[\frac{x}{2}\right] + \tan\left[\frac{x}{2}\right]^2\right]$$

Problem #4: Valid but suboptimal antiderivative:

$$\left\{ \frac{3 + 7 \cos[x] + 2 \sin[x]}{1 + 4 \cos[x] + 3 \cos[x]^2 - 5 \sin[x] - \cos[x] \sin[x]}, x, -35, 35 \right\}$$

$$-\text{Log}[1 + \cos[x] - 2 \sin[x]] + \text{Log}[3 + \cos[x] + \sin[x]]$$

$$-\text{Log}\left[1 - 2 \tan\left[\frac{x}{2}\right]\right] + \text{Log}\left[2 + \tan\left[\frac{x}{2}\right] + \tan\left[\frac{x}{2}\right]^2\right]$$

Problem #5: Unable to integrate:

$$\left\{ \frac{-1 + 4 \cos[x] + 5 \cos[x]^2}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x, -2, 2 \right\}$$

$$x - 2 \text{ArcTan}\left[\frac{\sin[x]}{3 + \cos[x]}\right] - 2 \text{ArcTan}\left[\frac{3 \sin[x] + 7 \cos[x] \sin[x]}{1 + 2 \cos[x] + 5 \cos[x]^2}\right]$$

$$\text{Int}\left[\frac{1}{1 + 4 \cos[x] + 3 \cos[x]^2 - 4 \cos[x]^3}, x\right] + 4 \text{Int}\left[\frac{\cos[x]}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x\right] + 5 \text{Int}\left[\frac{\cos[x]^2}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x\right]$$

Problem #6: Unable to integrate:

$$\left\{ \frac{-5 + 2 \cos[x] + 7 \cos[x]^2}{-1 + 2 \cos[x] - 9 \cos[x]^2 + 4 \cos[x]^3}, x, -2, 2 \right\}$$

$$x - 2 \text{ArcTan}\left[\frac{2 \cos[x] \sin[x]}{1 - \cos[x] + 2 \cos[x]^2}\right]$$

$$-5 \operatorname{Int}\left[\frac{1}{-1 + 2 \operatorname{Cos}[x] - 9 \operatorname{Cos}[x]^2 + 4 \operatorname{Cos}[x]^3}, x\right] + 2 \operatorname{Int}\left[\frac{\operatorname{Cos}[x]}{-1 + 2 \operatorname{Cos}[x] - 9 \operatorname{Cos}[x]^2 + 4 \operatorname{Cos}[x]^3}, x\right] + 7 \operatorname{Int}\left[\frac{\operatorname{Cos}[x]^2}{-1 + 2 \operatorname{Cos}[x] - 9 \operatorname{Cos}[x]^2 + 4 \operatorname{Cos}[x]^3}, x\right]$$

Test complete!

IntegrationTest["0 Independent test suites\\Hebisch Problems"];

Testing Rubi on 7 integration problems...

Problem #2: Unable to integrate:

$$\left\{ \frac{e^{\frac{x}{2+x^2}} (2-x^2)}{2x+x^3}, x, -5, 5 \right\}$$

$$\operatorname{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

$$\operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2-x}}, x\right] + \operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right] - \operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2+x}}, x\right]$$

Problem #3: Unable to integrate:

$$\left\{ \frac{e^{\frac{x}{2+x^2}} (2+2x+3x^2-x^3+2x^4)}{2x+x^3}, x, -5, 5 \right\}$$

$$e^{\frac{x}{2+x^2}} (2+x^2) + \operatorname{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

$$-\operatorname{Int}\left[e^{\frac{x}{2+x^2}}, x\right] + (1+i\sqrt{2}) \operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2-x}}, x\right] + \operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right] + 2 \operatorname{Int}\left[e^{\frac{x}{2+x^2}} x, x\right] - (1-i\sqrt{2}) \operatorname{Int}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2+x}}, x\right]$$

Problem #5: Unable to integrate:

$$\left\{ \frac{e^{-\frac{1}{1-x^2}} (1-3x-x^2+x^3)}{1-x-x^2+x^3}, x, -6, 6 \right\}$$

$$e^{-\frac{1}{1-x^2}} (1+x)$$

$$\operatorname{Int}\left[e^{-\frac{1}{1-x^2}}, x\right] + \frac{1}{2} \operatorname{Int}\left[\frac{e^{-\frac{1}{1-x^2}}}{1-x}, x\right] - \operatorname{Int}\left[\frac{e^{-\frac{1}{1-x^2}}}{(-1+x)^2}, x\right] + \frac{1}{2} \operatorname{Int}\left[\frac{e^{-\frac{1}{1-x^2}}}{1+x}, x\right]$$

Problem #7: Unable to integrate:

$$\left\{ \frac{e^{\frac{x+1}{\log(x)}} (-1 + (1+x) \log(x)^2)}{\log(x)^2}, x, -2, 2 \right\}$$

$$e^{\frac{x+1}{\log(x)}} x$$

$$\text{Int} \left[e^{\frac{x+1}{\log(x)}}, x \right] + \text{Int} \left[e^{\frac{x+1}{\log(x)}} x, x \right] - \text{Int} \left[\frac{e^{\frac{x+1}{\log(x)}}}{\log(x)^2}, x \right]$$

Test complete!

IntegrationTest["0 Independent test suites\\Wester Problems"];

Testing Rubi on 8 integration problems...

Test complete!

IntegrationTest["0 Independent test suites\\Welz Problems"];

Testing Rubi on 63 integration problems...

Problem #9: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{\sqrt{-1+x^2} (\sqrt{x} + \sqrt{-1+x^2})^2}, x, -18, 18 \right\}$$

$$\frac{2-4x}{5(\sqrt{x} + \sqrt{-1+x^2})} + \frac{1}{25} \sqrt{-110+50\sqrt{5}} \text{ArcTan} \left[\frac{1}{2} \sqrt{2+2\sqrt{5}} \sqrt{x} \right] - \frac{1}{50} \sqrt{-110+50\sqrt{5}} \text{ArcTan} \left[\frac{\sqrt{-2+2\sqrt{5}} \sqrt{-1+x^2}}{2-(1-\sqrt{5})x} \right] -$$

$$\frac{1}{25} \sqrt{110+50\sqrt{5}} \text{ArcTanh} \left[\frac{1}{2} \sqrt{-2+2\sqrt{5}} \sqrt{x} \right] - \frac{1}{50} \sqrt{110+50\sqrt{5}} \text{ArcTanh} \left[\frac{\sqrt{2+2\sqrt{5}} \sqrt{-1+x^2}}{2-x-\sqrt{5}x} \right]$$

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \sqrt{\frac{2}{5}(-11+5\sqrt{5})} \text{ArcTan} \left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{x} \right] +$$

$$\sqrt{\frac{2}{5(-1+\sqrt{5})}} \text{ArcTan} \left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})} \sqrt{-1+x^2}} \right] - \frac{2}{5} \sqrt{\frac{1}{5}(-2+5\sqrt{5})} \text{ArcTan} \left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})} \sqrt{-1+x^2}} \right] -$$

$$\frac{1}{5} \sqrt{\frac{2}{5}(11+5\sqrt{5})} \text{ArcTanh} \left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x} \right] + \sqrt{\frac{2}{5(1+\sqrt{5})}} \text{ArcTanh} \left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})} \sqrt{-1+x^2}} \right] - \frac{2}{5} \sqrt{\frac{1}{5}(2+5\sqrt{5})} \text{ArcTanh} \left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})} \sqrt{-1+x^2}} \right]$$

Problem #10: Valid but suboptimal antiderivative:

$$\left\{ \frac{(\sqrt{x} - \sqrt{-1+x^2})^2}{(1+x-x^2)^2 \sqrt{-1+x^2}}, x, -25, 25 \right\}$$

$$\frac{2-4x}{5(\sqrt{x} + \sqrt{-1+x^2})} + \frac{1}{25} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{1}{2} \sqrt{2+2\sqrt{5}} \sqrt{x}\right] - \frac{1}{50} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}} \sqrt{-1+x^2}}{2-(1-\sqrt{5})x}\right] -$$

$$\frac{1}{25} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{1}{2} \sqrt{-2+2\sqrt{5}} \sqrt{x}\right] - \frac{1}{50} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}} \sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right]$$

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \sqrt{\frac{2}{5}(-11+5\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{x}\right] - \frac{1}{5} \sqrt{\frac{1}{10}(-11+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})} \sqrt{-1+x^2}}\right] -$$

$$\frac{1}{5} \sqrt{\frac{1}{5}(-2+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})} \sqrt{-1+x^2}}\right] + \frac{1}{5} \sqrt{\frac{1}{5}(2+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})} \sqrt{-1+x^2}}\right] - \frac{1}{5} \sqrt{\frac{2}{5}(11+5\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x}\right] -$$

$$\frac{1}{5} \sqrt{\frac{1}{5}(-2+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})} \sqrt{-1+x^2}}\right] - \frac{1}{5} \sqrt{\frac{1}{5}(2+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})} \sqrt{-1+x^2}}\right] + \frac{1}{5} \sqrt{\frac{1}{10}(11+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})} \sqrt{-1+x^2}}\right]$$

Problem #33: Unable to integrate:

$$\left\{ \frac{1}{(1+x)(1-x^3)^{1/3}}, x, -1, 0 \right\}$$

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{2(1-x)+2^{2/3}(1-x^3)^{1/3}}{2^{2/3}\sqrt{3}(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}} - \frac{\operatorname{Log}[1-x]}{4 \times 2^{1/3}} - \frac{\operatorname{Log}[1+x]}{2 \times 2^{1/3}} + \frac{3 \operatorname{Log}[-1+x+2^{2/3}(1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

$$\operatorname{Int}\left[\frac{1}{(1+x)(1-x^3)^{1/3}}, x\right]$$

Problem #34: Unable to integrate:

$$\left\{ \frac{x}{(1+x)(1-x^3)^{1/3}}, x, -10, 10 \right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{2x-(1-x^3)^{1/3}}{\sqrt{3}(1-x^3)^{1/3}}\right]}{\sqrt{3}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{2(1-x)+2^{2/3}(1-x^3)^{1/3}}{2^{2/3}\sqrt{3}(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}} + \frac{\operatorname{Log}[1-x]}{4 \times 2^{1/3}} + \frac{\operatorname{Log}[1+x]}{2 \times 2^{1/3}} + \frac{1}{2} \operatorname{Log}[x+(1-x^3)^{1/3}] - \frac{3 \operatorname{Log}[-1+x+2^{2/3}(1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{6} \text{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{3} \text{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] + \text{Int}\left[\frac{1}{(-1-x)(1-x^3)^{1/3}}, x\right]$$

Problem #35: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{x(2-3x+x^2)^{1/3}}, x, -2, 2\right\}$$

$$-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{2/3}(2-x)}{\sqrt{3}(2-3x+x^2)^{1/3}}\right]}{2 \times 2^{1/3}} - \frac{\text{Log}[2-x]}{4 \times 2^{1/3}} - \frac{\text{Log}[x]}{2 \times 2^{1/3}} + \frac{3 \text{Log}[2-x-2^{2/3}(2-3x+x^2)^{1/3}]}{4 \times 2^{1/3}}$$

$$-\frac{\sqrt{3}(-2+x)^{1/3}(-1+x)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2^{1/3}(-2+x)^{2/3}}{\sqrt{3}(-1+x)^{1/3}}\right]}{2 \times 2^{1/3}(2-3x+x^2)^{1/3}} + \frac{3(-2+x)^{1/3}(-1+x)^{1/3} \text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}} - 2^{1/3}(-1+x)^{1/3}\right]}{4 \times 2^{1/3}(2-3x+x^2)^{1/3}} - \frac{(-2+x)^{1/3}(-1+x)^{1/3} \text{Log}[x]}{2 \times 2^{1/3}(2-3x+x^2)^{1/3}}$$

Problem #36: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{(-5+7x-3x^2+x^3)^{1/3}}, x, -12, 12\right\}$$

$$\frac{1}{2} \sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(-1+x)}{\sqrt{3}(-5+7x-3x^2+x^3)^{1/3}}\right] + \frac{1}{4} \text{Log}[1-x] - \frac{3}{4} \text{Log}\left[1-x+(-5+7x-3x^2+x^3)^{1/3}\right]$$

$$\frac{\sqrt{3}(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{(4+(-1+x)^2)^{1/3}}}{\sqrt{3}}\right]}{2(-5+7x-3x^2+x^3)^{1/3}} - \frac{(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{Log}\left[1-\frac{(-1+x)^{2/3}}{(4+(-1+x)^2)^{1/3}}\right]}{2(-5+7x-3x^2+x^3)^{1/3}} + \frac{(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{Log}\left[1+\frac{(-1+x)^{2/3}}{(4+(-1+x)^2)^{1/3}}+\frac{(-1+x)^{4/3}}{(4+(-1+x)^2)^{2/3}}\right]}{4(-5+7x-3x^2+x^3)^{1/3}}$$

Problem #37: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{x(-q+x^2)^{1/3}}, x, -12, 12\right\}$$

$$\frac{1}{2} \sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2x}{\sqrt{3}(x(-q+x^2))^{1/3}}\right] + \frac{\text{Log}[x]}{4} - \frac{3}{4} \text{Log}\left[-x+(x(-q+x^2))^{1/3}\right]$$

$$\frac{\sqrt{3}x^{1/3}(-q+x^2)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2x^{2/3}}{(-q+x^2)^{1/3}}}{\sqrt{3}}\right]}{2(-qx+x^3)^{1/3}} - \frac{x^{1/3}(-q+x^2)^{1/3} \text{Log}\left[1-\frac{x^{2/3}}{(-q+x^2)^{1/3}}\right]}{2(-qx+x^3)^{1/3}} + \frac{x^{1/3}(-q+x^2)^{1/3} \text{Log}\left[1+\frac{x^{4/3}}{(-q+x^2)^{2/3}}+\frac{x^{2/3}}{(-q+x^2)^{1/3}}\right]}{4(-qx+x^3)^{1/3}}$$

Problem #38: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{((-1+x)(q-2x+x^2))^{1/3}}, x, -13, 13 \right\}$$

$$\frac{1}{2} \sqrt{3} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} + \frac{2(-1+x)}{\sqrt{3}((-1+x)(q-2x+x^2))^{1/3}} \right] + \frac{1}{4} \operatorname{Log}[1-x] - \frac{3}{4} \operatorname{Log} \left[1-x + ((-1+x)(q-2x+x^2))^{1/3} \right]$$

$$\frac{\sqrt{3}(-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{ArcTan} \left[\frac{1 + \frac{2(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}}}{\sqrt{3}} \right] - (-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{Log} \left[1 - \frac{(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}} \right]}{2(-q+(2+q)x-3x^2+x^3)^{1/3}} + \frac{(-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{Log} \left[1 + \frac{(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}} + \frac{(-1+x)^{4/3}}{(-1+q+(-1+x)^2)^{2/3}} \right]}{4(-q+(2+q)x-3x^2+x^3)^{1/3}}$$

Problem #39: Unable to integrate:

$$\left\{ \frac{1}{x(-1+x)(q-2qx+x^2)^{1/3}}, x, -2, 2 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} + \frac{2q^{1/3}(-1+x)}{\sqrt{3}((-1+x)(q-2qx+x^2))^{1/3}} \right]}{2q^{1/3}} + \frac{\operatorname{Log}[1-x]}{4q^{1/3}} + \frac{\operatorname{Log}[x]}{2q^{1/3}} - \frac{3 \operatorname{Log}[-q^{1/3}(-1+x) + ((-1+x)(q-2qx+x^2))^{1/3}]}{4q^{1/3}}$$

$$\frac{(-1+x)^{1/3}(-q+2qx-x^2)^{1/3} \operatorname{Int} \left[\frac{1}{(-1+x)^{1/3}x(-q+2qx-x^2)^{1/3}}, x \right]}{(-q+3qx-(1+2q)x^2+x^3)^{1/3}}$$

Problem #40: Unable to integrate:

$$\left\{ \frac{2-(1+k)x}{((1-x)x(1-kx))^{1/3}(1-(1+k)x)}, x, -6, 6 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan} \left[\frac{1 + \frac{2k^{1/3}x}{(1-x)x(1-kx)^{1/3}}}{\sqrt{3}} \right]}{k^{1/3}} + \frac{\operatorname{Log}[x]}{2k^{1/3}} + \frac{\operatorname{Log}[1-(1+k)x]}{2k^{1/3}} - \frac{3 \operatorname{Log}[-k^{1/3}x + ((1-x)x(1-kx))^{1/3}]}{2k^{1/3}}$$

$$- \left(3(1-x)^{1/3}x^{1/3}(kx)^{1/3}(1-kx) \left(1 - \frac{1-kx}{1-k} \right)^{1/3} \operatorname{AppellF1} \left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, 1-kx, \frac{1-kx}{1-k} \right] \right) / (2k((1-x)x(1-kx))^{1/3}(x-x^2)^{1/3}) +$$

$$\frac{(1-x)^{1/3}x^{1/3}(1-kx)^{1/3} \operatorname{Int} \left[\frac{1}{(1+(-1-k)x)(1-kx)^{1/3}(x-x^2)^{1/3}}, x \right]}{(1-x)x(1-kx)^{1/3}}$$

Problem #41: Unable to integrate:

$$\left\{ \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}, x, -1, 1 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{2/3}(1-kx)}{(1-k)^{1/3}((1-x)x(1-kx))^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}(1-k)^{1/3}} + \frac{\operatorname{Log}[1 - (2-k)x]}{2^{2/3}(1-k)^{1/3}} + \frac{\operatorname{Log}[1-kx]}{2 \times 2^{2/3}(1-k)^{1/3}} - \frac{3 \operatorname{Log}[-1+kx + 2^{2/3}(1-k)^{1/3}((1-x)x(1-kx))^{1/3}]}{2 \times 2^{2/3}(1-k)^{1/3}}$$

$$\frac{(1-x)^{2/3} x^{2/3} (1-kx)^{2/3} \operatorname{Int}\left[\frac{(1-kx)^{1/3}}{(1-x)^{2/3} x^{2/3} (1+(-2+k)x)}, x\right]}{((1-x)x(1-kx))^{2/3}}$$

Problem #42: Unable to integrate:

$$\left\{ \frac{a + bx + cx^2}{(1-x+x^2)(1-x^3)^{1/3}}, x, -12, 12 \right\}$$

$$-\frac{1}{6} c \left(2\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] + \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] - 2 \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] \right) + \frac{(a-b-2c) \left(-2\sqrt{3} \operatorname{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right] - 3 \operatorname{Log}\left[2^{1/3} - (1-x^3)^{1/3}\right] \right)}{12 \times 2^{1/3}} + \frac{1}{4 \times 2^{1/3}}$$

$$(a+b) \left(2\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \times 2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] + \operatorname{Log}\left[3 - 6x + 6x^2 - 3x^3\right] - 3 \operatorname{Log}\left[-2^{1/3}(-1+x) + (1-x^3)^{1/3}\right] \right) - \frac{(a-b-2c) \left(2\sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \times 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] - 3 \operatorname{Log}\left[2^{1/3}x + (1-x^3)^{1/3}\right] \right)}{12 \times 2^{1/3}}$$

$$-\frac{c \operatorname{ArcTan}\left[\frac{1 - \frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{6} c \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{3} c \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] +$$

$$\frac{1}{3} (3b - \sqrt{3} (i(2a+b-c) - \sqrt{3}c)) \operatorname{Int}\left[\frac{1}{(-1-i\sqrt{3}+2x)(1-x^3)^{1/3}}, x\right] + \frac{1}{3} (3b + \sqrt{3} (i(2a+b-c) + \sqrt{3}c)) \operatorname{Int}\left[\frac{1}{(-1+i\sqrt{3}+2x)(1-x^3)^{1/3}}, x\right]$$

Problem #49: Valid but suboptimal antiderivative:

$$\left\{ \frac{-a - \sqrt{1+a^2} + x}{(-a + \sqrt{1+a^2} + x) \sqrt{(-a+x)(1+x^2)}}, x, -6, 6 \right\}$$

$$-\sqrt{2} \sqrt{a + \sqrt{1+a^2}} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{-a + \sqrt{1+a^2}} (-a+x)}{\sqrt{(-a+x)(1+x^2)}}\right]$$

$$\frac{2i \sqrt{\frac{a-x}{i+a}} \sqrt{1+x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-ix}}{\sqrt{2}}\right], \frac{2}{1-ia}\right] - 4 \sqrt{\frac{i-x}{i-a}} \sqrt{\frac{i+x}{i+a}} \sqrt{-a+x} \operatorname{EllipticPi}\left[-\frac{i-a}{\sqrt{1+a^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1}{i-a}} \sqrt{-a+x}\right], -\frac{i-a}{i+a}\right]}{\sqrt{-(a-x)(1+x^2)} \sqrt{\frac{1}{i-a} \sqrt{-(a-x)(1+x^2)}}$$

Problem #52: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{x (4 - 6x + 3x^2)^{1/3}}, x, -1, 1 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{-2+x-2 \times 2^{1/3} (4-6x+3x^2)^{1/3}}{\sqrt{3} (-2+x)}\right]}{2^{2/3} \sqrt{3}} + \frac{\text{Log}\left[\frac{-4+2x+2 \times 2^{1/3} (4-6x+3x^2)^{1/3}}{x}\right]}{2 \times 2^{2/3}}$$

$$- \frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{2/3} (2-x)}{\sqrt{3} (4-6x+3x^2)^{1/3}}\right]}{2^{2/3} \sqrt{3}} - \frac{\text{Log}[x]}{2 \times 2^{2/3}} + \frac{\text{Log}\left[6 - 3x - 3 \times 2^{1/3} (4 - 6x + 3x^2)^{1/3}\right]}{2 \times 2^{2/3}}$$

Problem #55: Unable to integrate:

$$\left\{ \frac{(1-x^3)^{1/3}}{1+x}, x, -1, 0 \right\}$$

0

$$\text{Int}\left[\frac{(1-x^3)^{1/3}}{1+x}, x\right]$$

Problem #56: Unable to integrate:

$$\left\{ \frac{(1-x^3)^{1/3}}{1-x+x^2}, x, -2, 2 \right\}$$

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+2 \times 2^{1/3} (-1+x)}{\sqrt{3} (1-x^3)^{1/3}}\right]}{2^{2/3}} + \frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3} (1-x^3)^{1/3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1-2 \times 2^{1/3} x}{\sqrt{3} (1-x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+2^{2/3} (1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}$$

$$\frac{\text{Log}\left[-3(-1+x)(1-x+x^2)\right]}{2 \times 2^{2/3}} + \frac{\text{Log}\left[2^{1/3} - (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} + \frac{3 \text{Log}\left[-2^{1/3}(-1+x) + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} + \frac{1}{2} \text{Log}\left[x + (1-x^3)^{1/3}\right] - \frac{\text{Log}\left[2^{1/3}x + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

$$\frac{2i \text{Int}\left[\frac{(1-x^3)^{1/3}}{1+i\sqrt{3}-2x}, x\right]}{\sqrt{3}} + \frac{2i \text{Int}\left[\frac{(1-x^3)^{1/3}}{-1+i\sqrt{3}+2x}, x\right]}{\sqrt{3}}$$

Test complete!

IntegrationTest["0 Independent test suites\ Bronstein Problems"];

Testing Rubi on 14 integration problems...

Problem #6: Unable to integrate:

$$\left\{ \frac{x}{\sqrt{-71 - 96x + 10x^2 + x^4}}, x, -1, 0 \right\}$$

$$-\frac{1}{8} \operatorname{Log} \left[10001 + 3124x^2 - 1408x^3 + 54x^4 - 128x^5 + 20x^6 + x^8 + \sqrt{-71 - 96x + 10x^2 + x^4} (-781 + 528x - 27x^2 + 80x^3 - 15x^4 - x^6) \right]$$

$$\operatorname{Int} \left[\frac{x}{\sqrt{-71 - 96x + 10x^2 + x^4}}, x \right]$$

Problem #12: Unable to integrate:

$$\left\{ \frac{x^2 + 2x \operatorname{Log}[x] + \operatorname{Log}[x]^2 + (1+x) \sqrt{x + \operatorname{Log}[x]}}{x^3 + 2x^2 \operatorname{Log}[x] + x \operatorname{Log}[x]^2}, x, -3, 3 \right\}$$

$$\operatorname{Log}[x] - \frac{2}{\sqrt{x + \operatorname{Log}[x]}}$$

$$\operatorname{Log}[x] + \operatorname{Int} \left[\frac{1}{(x + \operatorname{Log}[x])^{3/2}}, x \right] - \operatorname{Int} \left[\frac{1}{\operatorname{Log}[x] (x + \operatorname{Log}[x])^{3/2}}, x \right] - \operatorname{Int} \left[\frac{1}{\operatorname{Log}[x]^2 \sqrt{x + \operatorname{Log}[x]}}, x \right] + \operatorname{Int} \left[\frac{\sqrt{x + \operatorname{Log}[x]}}{x \operatorname{Log}[x]^2}, x \right]$$

Test complete!

IntegrationTest["0 Independent test suites\\Bondarenko Problems"];

Testing Rubi on 35 integration problems...

Problem #21: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{(\operatorname{Cos}[x] + \operatorname{Cos}[3x])^5}, x, -57, 57 \right\}$$

$$-\frac{523}{256} \operatorname{ArcTanh}[\operatorname{Sin}[x]] + \frac{1483 \operatorname{ArcTanh}[\sqrt{2} \operatorname{Sin}[x]]}{512 \sqrt{2}} + \frac{\operatorname{Sin}[x]}{32 (1 - 2 \operatorname{Sin}[x]^2)^4} - \frac{17 \operatorname{Sin}[x]}{192 (1 - 2 \operatorname{Sin}[x]^2)^3} + \frac{203 \operatorname{Sin}[x]}{768 (1 - 2 \operatorname{Sin}[x]^2)^2} - \frac{437 \operatorname{Sin}[x]}{512 (1 - 2 \operatorname{Sin}[x]^2)} - \frac{43}{256} \operatorname{Sec}[x] \operatorname{Tan}[x] - \frac{1}{128} \operatorname{Sec}[x]^3 \operatorname{Tan}[x]$$

$$\begin{aligned}
 & \frac{523}{256} \operatorname{ArcTanh}[\operatorname{Sin}[x]] - \frac{1483 \operatorname{Log}\left[2 + \sqrt{2} + \operatorname{Cos}[x] + \sqrt{2} \operatorname{Cos}[x] - \operatorname{Sin}[x] - \sqrt{2} \operatorname{Sin}[x]\right]}{2048 \sqrt{2}} - \frac{1483 \operatorname{Log}\left[2 - \sqrt{2} + \operatorname{Cos}[x] - \sqrt{2} \operatorname{Cos}[x] + \operatorname{Sin}[x] - \sqrt{2} \operatorname{Sin}[x]\right]}{2048 \sqrt{2}} + \\
 & \frac{1483 \operatorname{Log}\left[2 - \sqrt{2} + \operatorname{Cos}[x] - \sqrt{2} \operatorname{Cos}[x] - \operatorname{Sin}[x] + \sqrt{2} \operatorname{Sin}[x]\right]}{2048 \sqrt{2}} + \frac{1483 \operatorname{Log}\left[2 + \sqrt{2} + \operatorname{Cos}[x] + \sqrt{2} \operatorname{Cos}[x] + \operatorname{Sin}[x] + \sqrt{2} \operatorname{Sin}[x]\right]}{2048 \sqrt{2}} - \frac{9372189581}{620780160 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)^4} - \\
 & \frac{115444608337}{931170240 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)^3} - \frac{125776127011}{196035840 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)^2} - \frac{273389009815}{106419456 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)} + \frac{9372189581}{620780160 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)^4} + \frac{115444608337}{931170240 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)^3} + \frac{125776127011}{196035840 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)^2} + \\
 & \frac{273389009815}{106419456 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)} - \frac{2174977520729 + 900895026797 \operatorname{Tan}\left[\frac{x}{2}\right]}{465585120 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^4} - \frac{900895026797 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{798145920 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^3} + \frac{38884413000239 + 18217810835525 \operatorname{Tan}\left[\frac{x}{2}\right]}{5587021440 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^3} + \\
 & \frac{94536076571 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{70946304 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2} - \frac{65580529660561 + 30915686684873 \operatorname{Tan}\left[\frac{x}{2}\right]}{7449361920 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2} - \frac{954063574769 \left(1 + \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{451476480 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)} + \frac{137658750217805 + 69771275635067 \operatorname{Tan}\left[\frac{x}{2}\right]}{14898723840 \left(1 - 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)} + \\
 & \frac{2174977520729 - 900895026797 \operatorname{Tan}\left[\frac{x}{2}\right]}{465585120 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^4} - \frac{38884413000239 - 18217810835525 \operatorname{Tan}\left[\frac{x}{2}\right]}{5587021440 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^3} + \frac{900895026797 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{798145920 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^3} + \frac{65580529660561 - 30915686684873 \operatorname{Tan}\left[\frac{x}{2}\right]}{7449361920 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2} + \\
 & \frac{94536076571 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{70946304 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2} - \frac{137658750217805 - 69771275635067 \operatorname{Tan}\left[\frac{x}{2}\right]}{14898723840 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)} + \frac{954063574769 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{451476480 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)} + \frac{1486633486121 \operatorname{Tan}\left[\frac{x}{2}\right]}{33256080 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \\
 & \frac{2833881904961 \operatorname{Tan}\left[\frac{x}{2}\right]^3}{232792560 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{801211946693 \operatorname{Tan}\left[\frac{x}{2}\right]^5}{232792560 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{1842220447 \operatorname{Tan}\left[\frac{x}{2}\right]^7}{1750320 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \\
 & \frac{11825071 \operatorname{Tan}\left[\frac{x}{2}\right]^9}{51480 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{17348075 \operatorname{Tan}\left[\frac{x}{2}\right]^{11}}{72072 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} - \frac{424843 \operatorname{Tan}\left[\frac{x}{2}\right]^{13}}{3960 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{34529 \operatorname{Tan}\left[\frac{x}{2}\right]^{15}}{360 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} - \\
 & \frac{48883 \operatorname{Tan}\left[\frac{x}{2}\right]^{17}}{1680 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{1813 \operatorname{Tan}\left[\frac{x}{2}\right]^{19}}{240 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} - \frac{35 \operatorname{Tan}\left[\frac{x}{2}\right]^{21}}{48 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4} + \frac{\operatorname{Tan}\left[\frac{x}{2}\right]^{23}}{16 \left(1 - 7 \operatorname{Tan}\left[\frac{x}{2}\right]^2 + 7 \operatorname{Tan}\left[\frac{x}{2}\right]^4 - \operatorname{Tan}\left[\frac{x}{2}\right]^6\right)^4}
 \end{aligned}$$

Problem #24: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Tanh}[x]}{\sqrt{e^x + e^{2x}}}, x, -13, 13 \right\}$$

$$2 e^{-x} \sqrt{e^x + e^{2x}} - \frac{\operatorname{ArcTan}\left[\frac{i - (1-2i) e^x}{2\sqrt{1+i} \sqrt{e^x + e^{2x}}}\right]}{\sqrt{1+i}} + \frac{\operatorname{ArcTan}\left[\frac{i + (1+2i) e^x}{2\sqrt{1-i} \sqrt{e^x + e^{2x}}}\right]}{\sqrt{1-i}}$$

$$\frac{2(1+e^x)}{\sqrt{e^x+e^{2x}}} - \frac{(1-i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\operatorname{ArcTanh}\left[\frac{\sqrt{1-i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}} - \frac{(1+i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\operatorname{ArcTanh}\left[\frac{\sqrt{1+i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}$$

Problem #26: Valid but suboptimal antiderivative:

$$\{\operatorname{Log}[x^2 + \sqrt{1-x^2}], x, -34, 34\}$$

$$-2x - \operatorname{ArcSin}[x] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] +$$

$$\sqrt{\frac{1}{2}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] - \sqrt{\frac{1}{2}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + x \operatorname{Log}[x^2 + \sqrt{1-x^2}]$$

$$-2x - \operatorname{ArcSin}[x] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] -$$

$$\sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] +$$

$$\sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] - 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] - \sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + x \operatorname{Log}[x^2 + \sqrt{1-x^2}]$$

Test complete!

1 Algebraic functions

■ 1 Linear products

```
IntegrationTest["1 Algebraic functions\\1 Linear products\\1.2 (a+b x)^m (c+d x)^n"];
```

Testing Rubi on 1568 integration problems...

Problem #336: Valid but suboptimal antiderivative:

$$\left\{ -\frac{bx^m}{2(a+bx)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx}}$$

$$\frac{x^m \left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, -m, \frac{1}{2}, 1 + \frac{bx}{a}\right]}{\sqrt{a+bx}} - \frac{2mx^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \text{Hypergeometric2F1}\left[\frac{1}{2}, 1-m, \frac{3}{2}, 1 + \frac{bx}{a}\right]}{a}$$

Test complete!

IntegrationTest["1 Algebraic functions\\1 Linear products\\1.3 (a+b x)^m (c+d x)^n (e+f x)^p"];

Testing Rubi on 3129 integration problems...

Problem #856: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{-1 + \frac{1}{x}} \sqrt{\frac{1}{x}} \sqrt{x}}{\sqrt{1+x}}, x, -6, 6 \right\}$$

$$\frac{2\sqrt{-x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{-x}\right], -1\right]}{\sqrt{x}}$$

$$\frac{2\sqrt{-1 + \frac{1}{x}} \left(1 + \frac{1}{x}\right) \sqrt{\frac{1}{x}} x^{3/2}}{\sqrt{1+x}} - \frac{2\sqrt{1 - \frac{1}{x}} \sqrt{1 + \frac{1}{x}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right], -1\right]}{\sqrt{-1 + \frac{1}{x}} \sqrt{\frac{1}{x}} \sqrt{1+x}}$$

Problem #986: Valid but suboptimal antiderivative:

$$\left\{ \frac{(1-ax)^{1-n} (1+ax)^{1+n}}{x^2}, x, -1, 1 \right\}$$

$$\frac{(1-ax)^{-n} (1+ax)^n (1+a^2x^2)}{x} - \frac{2an(1-ax)^{1-n} (1+ax)^{-1+n} \text{Hypergeometric2F1}\left[1, 1-n, 2-n, \frac{1-ax}{1+ax}\right]}{1-n} + \frac{2^{-n}an(1+ax)^{1+n} \text{Hypergeometric2F1}\left[1+n, 1+n, 2+n, \frac{1}{2}(1+ax)\right]}{1+n}$$

$$\frac{2^{1-n}a(1+ax)^{2+n} \text{AppellF1}\left[2+n, -1+n, 2, 3+n, \frac{1}{2}(1+ax), 1+ax\right]}{2+n}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1 Linear products\\1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q"];
```

Testing Rubi on 81 integration problems...

Test complete!

■ 2 Quadratic products

```
IntegrationTest["1 Algebraic functions\\2 Quadratic products\\2.2 (d+e x)^m (a+b x+c x^2)^p"];
```

Testing Rubi on 2998 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\2 Quadratic products\\2.3 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p"];
```

Testing Rubi on 3199 integration problems...

Problem #3147: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2}, x, -9, 9 \right\}$$

$$-\text{ArcCosh}[x] + \sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right]$$

$$\frac{\sqrt{\frac{1}{10}(-1+\sqrt{5})} \sqrt{-1+x} \sqrt{1+x} \text{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] - \sqrt{-1+x} \sqrt{1+x} \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \sqrt{-1+x} \sqrt{1+x} \text{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2} - \sqrt{-1+x^2} - \sqrt{-1+x^2}}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\2 Quadratic products\\2.4 (a+b x+c x^2)^p (d+e x+f x^2)^q"];
```

Testing Rubi on 168 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\2 Quadratic products\\2.5 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q"];
```

Testing Rubi on 227 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\2 Quadratic products\\2.6 (a+b x+c x^2)^p (d+e x+f x^2)^q (A+B x+C x^2)^r"];
```

Testing Rubi on 33 integration problems...

Test complete!

■ 3 Binomial products

IntegrationTest["1 Algebraic functions\\3 Binomial products\\3.2 (c x)^m (a+b x^n)^p"];

Testing Rubi on 3912 integration problems...

Problem #1217: Valid but suboptimal antiderivative:

$$\left\{ \frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}, x, -5, 5 \right\}$$

$$\frac{x^{2+m}\sqrt{a+bx^2}}{a x^{2+m}\sqrt{1+\frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]} + \frac{b(3+m)x^{4+m}\sqrt{1+\frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, -\frac{bx^2}{a}\right]}{(4+m)\sqrt{a+bx^2}}$$

Problem #1219: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{bx^{1+m}}{(a+bx^2)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^2}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx^2}} - \frac{bx^{2+m}}{a\sqrt{a+bx^2}} + \frac{x^m\sqrt{a+bx^2}}{a}$$

$$\frac{x^m\sqrt{1+\frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, -\frac{bx^2}{a}\right]}{\sqrt{a+bx^2}} - \frac{bx^{2+m}\sqrt{1+\frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]}{a(2+m)\sqrt{a+bx^2}}$$

Problem #3686: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^n}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx^n}} - \frac{bx^{m+n}}{a\sqrt{a+bx^n}} + \frac{x^m\sqrt{a+bx^n}}{a}$$

$$\frac{x^m\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{bx^n}{a}\right]}{\sqrt{a+bx^n}} - \frac{bnx^{m+n}\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2+\frac{m}{n}, -\frac{bx^n}{a}\right]}{2a(m+n)\sqrt{a+bx^n}}$$

Problem #3697: Valid but not optimal or suboptimal antiderivative:

$$\left\{ \frac{6 a x^2}{b (4+m) \sqrt{a+b x^{-2+m}}} + \frac{x^m}{\sqrt{a+b x^{-2+m}}}, x, -5, 5 \right\}$$

$$\frac{2 x^3 \sqrt{a+b x^{-2+m}}}{b (4+m)}$$

$$\frac{2 x^{1+m} (b+a x^{2-m})}{b (4+m) \sqrt{a+b x^{-2+m}}}$$

$$\frac{2 a x^2 \sqrt{1+\frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{3}{2-m}, -\frac{1+m}{2-m}, -\frac{b x^{-2+m}}{a}\right] + x^{1+m} \sqrt{1+\frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1+m}{2-m}, \frac{1-2m}{2-m}, -\frac{b x^{-2+m}}{a}\right]}{b (4+m) \sqrt{a+b x^{-2+m}} + (1+m) \sqrt{a+b x^{-2+m}}}$$

Problem #3699: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{b n x^{-1+m+n}}{2 (a+b x^n)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a+b x^n}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+b x^n}}$$

$$-\frac{b x^{m+n}}{a \sqrt{a+b x^n}} + \frac{x^m \sqrt{a+b x^n}}{a}$$

$$\frac{x^m \sqrt{1+\frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{b x^n}{a}\right] - b n x^{m+n} \sqrt{1+\frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2+\frac{m}{n}, -\frac{b x^n}{a}\right]}{\sqrt{a+b x^n} - 2 a (m+n) \sqrt{a+b x^n}}$$

Test complete!

`IntegrationTest["1 Algebraic functions\3 Binomial products\3.3 (a+b x^n)^p (c+d x^n)^q"];`

Testing Rubi on 569 integration problems...

Problem #287: Valid but suboptimal antiderivative:

$$\left\{ \frac{(1-2x^2)^m}{\sqrt{1-x^2}}, x, -1, 1 \right\}$$

$$\frac{2^{-2-m} \sqrt{x^2} (2-4x^2)^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, (1-2x^2)^2\right]}{(1+m)x}$$

$$x \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right]$$

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q"];

Testing Rubi on 1795 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.5 (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r"];

Testing Rubi on 115 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r"];

Testing Rubi on 97 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.7 (c x)^m Pq(x) (a+b x^n)^p"];

Testing Rubi on 561 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.9 (c x)^m (a x^j+b x^n)^p"];

Testing Rubi on 454 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\3 Binomial products\3.10 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q"];

Testing Rubi on 295 integration problems...

Test complete!

■ 4 Trinomial products

IntegrationTest["1 Algebraic functions\4 Trinomial products\4.2 (d x)^m (a+b x^n+c x^(2 n))^p"];

Testing Rubi on 1762 integration problems...

Problem #1574: Valid but suboptimal antiderivative:

$$\left\{ \frac{(a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{x^2} - \frac{2 b^3 (1 - 2 p) (1 - p) p (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{3 a^3 x}, x, -7, 7 \right\}$$

$$- \frac{(a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a x} + \frac{b (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^2 x^{2/3}} - \frac{b^2 (1 - 2 p) (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^3 x^{1/3}}$$

$$+ \frac{1}{a^4 (1 + 2 p)} 2 b^3 (1 - 2 p) (1 - p) p (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[1, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right] +$$

$$- \frac{1}{a^4 (1 + 2 p)} 3 (a b^3 + b^4 x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[4, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right]$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\4 Trinomial products\\4.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p"];
```

Testing Rubi on 341 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\4 Trinomial products\\4.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p"];
```


Testing Rubi on 436 integration problems...

Problem #254: Valid but suboptimal antiderivative:

$$\left\{ \frac{(d + e x^2)^{3/2}}{x^2 (a + b x^2 + c x^4)}, x, -16, 16 \right\}$$

$$\frac{d \sqrt{d + e x^2}}{a x} - \frac{(2 c d - (b - \sqrt{b^2 - 4 a c}) e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right] + (2 c d - (b + \sqrt{b^2 - 4 a c}) e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{\sqrt{b^2 - 4 a c} (b - \sqrt{b^2 - 4 a c})^{3/2} + \sqrt{b^2 - 4 a c} (b + \sqrt{b^2 - 4 a c})^{3/2}}$$

$$- \frac{d \sqrt{d + e x^2}}{a x} - \frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \left(d + \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{2 a \sqrt{b - \sqrt{b^2 - 4 a c}}}$$

$$\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \left(d - \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right] + d \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right] - \sqrt{e} \left(d - \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right] - \sqrt{e} \left(d + \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{2 a \sqrt{b + \sqrt{b^2 - 4 a c}} + a - 2 a - 2 a}$$

Test complete!

`IntegrationTest["1 Algebraic functions\\4 Trinomial products\\4.5 (d x)^m Pq(x) (a+b x^n+c x^(2 n))^p"];`

Testing Rubi on 177 integration problems...

Test complete!

`IntegrationTest["1 Algebraic functions\\4 Trinomial products\\4.7 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p"];`

Testing Rubi on 122 integration problems...

Test complete!

■ 5 Miscellaneous

`IntegrationTest["1 Algebraic functions\\5 Miscellaneous\\5.1 Rational functions"];`

Testing Rubi on 269 integration problems...

Problem #154: Valid but suboptimal antiderivative:

$$\left\{ \frac{-20x + 4x^2}{9 - 10x^2 + x^4}, x, -10, 10 \right\}$$

$$\text{Log}[1 - x] - \frac{1}{2} \text{Log}[3 - x] + \frac{3}{2} \text{Log}[1 + x] - 2 \text{Log}[3 + x]$$

$$-\frac{3}{2} \text{ArcTanh}\left[\frac{x}{3}\right] + \frac{\text{ArcTanh}[x]}{2} + \frac{5}{4} \text{Log}[1 - x^2] - \frac{5}{4} \text{Log}[9 - x^2]$$

Problem #203: Unable to integrate:

$$\left\{ \frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2}, x, -7, 7 \right\}$$

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3 + x + x^4)^3}$$

$$-\frac{19}{4(3 + x + x^4)^3} + \frac{1}{(3 + x + x^4)^2} - \frac{621}{4} \text{Int}\left[\frac{1}{(3 + x + x^4)^4}, x\right] + 684 \text{Int}\left[\frac{x}{(3 + x + x^4)^4}, x\right] +$$

$$360 \text{Int}\left[\frac{x^2}{(3 + x + x^4)^4}, x\right] + 44 \text{Int}\left[\frac{1}{(3 + x + x^4)^3}, x\right] - 320 \text{Int}\left[\frac{x}{(3 + x + x^4)^3}, x\right] - 75 \text{Int}\left[\frac{x^2}{(3 + x + x^4)^3}, x\right] + 30 \text{Int}\left[\frac{x}{(3 + x + x^4)^2}, x\right]$$

Problem #204: Unable to integrate:

$$\left\{ \frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4}, x, -13, 13 \right\}$$

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3 + x + x^4)^3}$$

$$\frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{12x^2}{(3 + x + x^4)^3} - \frac{5x^3}{(3 + x + x^4)^3} + \frac{3x^4}{2(3 + x + x^4)^3} - \frac{10x^6}{(3 + x + x^4)^3} - \frac{1}{2(3 + x + x^4)^2} + \frac{5x^2}{(3 + x + x^4)^2} +$$

$$\frac{144}{11} \text{Int}\left[\frac{1}{(3 + x + x^4)^4}, x\right] + \frac{828}{11} \text{Int}\left[\frac{x}{(3 + x + x^4)^4}, x\right] + 18 \text{Int}\left[\frac{x^2}{(3 + x + x^4)^4}, x\right] - 4 \text{Int}\left[\frac{1}{(3 + x + x^4)^3}, x\right] - 20 \text{Int}\left[\frac{x}{(3 + x + x^4)^3}, x\right]$$

Test complete!

IntegrationTest["1 Algebraic functions\5 Miscellaneous\5.2 Algebraic functions"];

Testing Rubi on 717 integration problems...

Problem #380: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{a x^{2n}}}{\sqrt{1+x^n}} + \frac{2 x^{-n} \sqrt{a x^{2n}}}{(2+n) \sqrt{1+x^n}}, x, -5, 5 \right\}$$

$$\frac{2 x^{1-n} \sqrt{a x^{2n}} \sqrt{1+x^n}}{2+n}$$

$$\frac{x \sqrt{a x^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{1}{n}, 2 + \frac{1}{n}, -x^n\right]}{1+n} + \frac{2 x^{1-n} \sqrt{a x^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -x^n\right]}{2+n}$$

Problem #540: Unable to integrate:

$$\left\{ \frac{(d^3 + e^3 x^3)^p}{d + e x}, x, -1, 0 \right\}$$

$$\frac{1}{e^p} (d^3 + e^3 x^3)^p \left(1 + \frac{2(d+ex)}{(-3+i\sqrt{3})d} \right)^{-p} \left(1 - \frac{2(d+ex)}{(3+i\sqrt{3})d} \right)^{-p} \operatorname{AppellF1}\left[p, -p, -p, 1+p, -\frac{2(d+ex)}{(-3+i\sqrt{3})d}, \frac{2(d+ex)}{(3+i\sqrt{3})d}\right]$$

$$\operatorname{Int}\left[\frac{(d^3 + e^3 x^3)^p}{d + e x}, x\right]$$

Problem #636: Valid but suboptimal antiderivative:

$$\left\{ (1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6}, x, -3, 3 \right\}$$

$$-\frac{(1-x^6)^{2/3}}{5x^5} + \frac{1}{5} x (1-x^6)^{2/3}$$

$$-\frac{\operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, -\frac{2}{3}, \frac{1}{6}, x^6\right]}{5x^5} + x \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{6}, \frac{7}{6}, x^6\right]$$

Problem #694: Unable to integrate:

$$\left\{ \sqrt{1-x^2+x\sqrt{-1+x^2}}, x, -1, 0 \right\}$$

$$\frac{1}{4} \left(3x + \sqrt{-1+x^2} \right) \sqrt{1-x^2+x\sqrt{-1+x^2}} + \frac{3 \operatorname{ArcSin}\left[x - \sqrt{-1+x^2}\right]}{4\sqrt{2}}$$

$$\operatorname{Int}\left[\sqrt{1-x^2+x\sqrt{-1+x^2}}, x\right]$$

Problem #695: Unable to integrate:

$$\left\{ \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}}, x, -1, 1 \right\}$$

$$\frac{1}{2} (\sqrt{x} + 3\sqrt{1+x}) \sqrt{-x + \sqrt{x} \sqrt{1+x}} - \frac{3 \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2\sqrt{2}}$$

$$2 \operatorname{Subst} \left[\operatorname{Int} \left[\sqrt{1-x^2 + x\sqrt{-1+x^2}}, x \right], x, \sqrt{1+x} \right]$$

Problem #696: Valid but suboptimal antiderivative:

$$\left\{ -\frac{x + 2\sqrt{1+x^2}}{x + x^3 + \sqrt{1+x^2}}, x, -25, 25 \right\}$$

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan} \left[\sqrt{-2+\sqrt{5}} (x + \sqrt{1+x^2}) \right] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh} \left[\sqrt{2+\sqrt{5}} (x + \sqrt{1+x^2}) \right]$$

$$-2 \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTan} \left[\sqrt{\frac{2}{1+\sqrt{5}}} x \right] - \sqrt{\frac{1}{10(1+\sqrt{5})}} \operatorname{ArcTan} \left[\sqrt{\frac{2}{1+\sqrt{5}}} x \right] - \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan} \left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2} \right] - \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan} \left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2} \right] -$$

$$2 \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTanh} \left[\sqrt{\frac{2}{-1+\sqrt{5}}} x \right] + \sqrt{\frac{1}{10(-1+\sqrt{5})}} \operatorname{ArcTanh} \left[\sqrt{\frac{2}{-1+\sqrt{5}}} x \right] - \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh} \left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2} \right] + \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh} \left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2} \right]$$

Problem #716: Unable to integrate:

$$\left\{ \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}, x, -5, 5 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan} \left[\frac{1 - \frac{2 \cdot 2^{1/3} (1-x)}{(1-x^3)^{2/3}}}{\sqrt{3}} \right]}{2^{2/3}} - \frac{\operatorname{Log} [1 + 2(1-x)^3 - x^3]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log} [2^{1/3} (1-x) + (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

$$-x \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3 \right] - (1+i\sqrt{3}) \operatorname{Int} \left[\frac{1}{(-1-i\sqrt{3}+2x)(1-x^3)^{2/3}}, x \right] - (1-i\sqrt{3}) \operatorname{Int} \left[\frac{1}{(-1+i\sqrt{3}+2x)(1-x^3)^{2/3}}, x \right]$$

Problem #717: Valid but suboptimal antiderivative:

$$\left\{ \frac{x^2}{\sqrt{-1+x^4} (1+x^4)}, x, -17, 17 \right\}$$

$$-\frac{1}{4} \operatorname{ArcTan} \left[\frac{1+x^2}{x\sqrt{-1+x^4}} \right] - \frac{1}{4} \operatorname{ArcTanh} \left[\frac{1-x^2}{x\sqrt{-1+x^4}} \right]$$

$$\frac{i \sqrt{1-x^2} \sqrt{1+x^2} \operatorname{EllipticPi}[-i, \operatorname{ArcSin}[x], -1]}{2 \sqrt{-1+x^4}} - \frac{i \sqrt{1-x^2} \sqrt{1+x^2} \operatorname{EllipticPi}[i, \operatorname{ArcSin}[x], -1]}{2 \sqrt{-1+x^4}}$$

Test complete!

IntegrationTest["1 Algebraic functions\5 Miscellaneous\5.3 Expansion problems"];

Testing Rubi on 110 integration problems...

Problem #94: Boo; correct but **31** steps required!

$$\left\{ \frac{-1 + \sqrt{1-x^2}}{\sqrt{1-x^2} (2+x-2\sqrt{1-x^2})^2}, x, 0, \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} \right\}$$

Test complete!

IntegrationTest["1 Algebraic functions\5 Miscellaneous\5.4 Substitution problems"];

Testing Rubi on 364 integration problems...

Problem #54: Valid but suboptimal antiderivative:

$$\{b x^{1+p} (b x + c x^3)^p + 2 c x^{3+p} (b x + c x^3)^p, x, -7, 7\}$$

$$\frac{x^{1+p} (b x + c x^3)^{1+p}}{2 (1+p)}$$

$$\frac{b x^{2+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \operatorname{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{c x^2}{b}\right]}{2 (1+p)} + \frac{c x^{4+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \operatorname{Hypergeometric2F1}\left[-p, 2+p, 3+p, -\frac{c x^2}{b}\right]}{2+p}$$

Problem #59: Valid but suboptimal antiderivative:

$$\{(1+2x)(x+x^2)^3(-18+7(x+x^2)^3)^2, x, -3, 3\}$$

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Problem #60: Valid but suboptimal antiderivative:

$$\{x^3 (1+x)^3 (1+2x) (-18+7x^3(1+x)^3)^2, x, -2, 2\}$$

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Test complete!

IntegrationTest["1 Algebraic functions\5 Miscellaneous\5.5 Piecewise constant extraction"];

Testing Rubi on 66 integration problems...

Test complete!

IntegrationTest["1 Algebraic functions\5 Miscellaneous\5.6 Derivative integration problems"];

Testing Rubi on 97 integration problems...

Problem #24: Unable to integrate:

$$\{g[x] f'[x] + f[x] g'[x], x, -1, 1\}$$

$$f[x] g[x]$$

$$\text{Int}[g[x] f'[x], x] + \text{Int}[f[x] g'[x], x]$$

Problem #43: Unable to integrate:

$$\{\cos[x] g[e^x] f'[\sin[x]] + e^x f[\sin[x]] g'[e^x], x, -1, 1\}$$

$$f[\sin[x]] g[e^x]$$

$$\text{Int}[\cos[x] g[e^x] f'[\sin[x]], x] + \text{Int}[e^x f[\sin[x]] g'[e^x], x]$$

Test complete!

2 Exponentials

IntegrationTest["2 Exponentials\\2.1 u (F^(c (a+b x)))^n"];

Testing Rubi on 98 integration problems...

Test complete!

IntegrationTest["2 Exponentials\\2.2 (c+d x)^m (F^(g (e+f x)))^n (a+b (F^(g (e+f x))))^p"];

Testing Rubi on 88 integration problems...

Test complete!

IntegrationTest["2 Exponentials\\2.3 Exponential functions"];

Testing Rubi on 726 integration problems...

Problem #63: Unable to integrate:

$$\left\{ \frac{x}{(b f^{-x} + a f^x)^3}, x, -1, 1 \right\}$$

$$\frac{f^x}{8 a b (b + a f^{2x}) \text{Log}[f]^2} - \frac{f^x x}{4 a (b + a f^{2x})^2 \text{Log}[f]} + \frac{f^x x}{8 a b (b + a f^{2x}) \text{Log}[f]} + \frac{x \text{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]} - \frac{i \text{PolyLog}\left[2, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{16 a^{3/2} b^{3/2} \text{Log}[f]^2} + \frac{i \text{PolyLog}\left[2, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{16 a^{3/2} b^{3/2} \text{Log}[f]^2}$$

$$\text{Int}\left[\frac{f^{3x} x}{(b + a f^{2x})^3}, x\right]$$

Problem #64: Unable to integrate:

$$\left\{ \frac{x^2}{(b f^{-x} + a f^x)^3}, x, -1, 1 \right\}$$

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{4 a^{3/2} b^{3/2} \text{Log}[f]^3} + \frac{f^x x}{4 a b (b + a f^{2x}) \text{Log}[f]^2} - \frac{f^x x^2}{4 a (b + a f^{2x})^2 \text{Log}[f]} + \frac{f^x x^2}{8 a b (b + a f^{2x}) \text{Log}[f]} +$$

$$\frac{x^2 \text{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]} - \frac{i x \text{PolyLog}\left[2, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^2} + \frac{i x \text{PolyLog}\left[2, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^2} + \frac{i \text{PolyLog}\left[3, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^3} - \frac{i \text{PolyLog}\left[3, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^3}$$

$$\text{Int}\left[\frac{f^{3x} x^2}{(b + a f^{2x})^3}, x\right]$$

Problem #645: Unable to integrate:

$$\{e^{x^2} x^{2x} (1 + \text{Log}[x]), x, -3, 3\}$$

$$e^{x^2} (-1 + x^x)$$

$$\text{Int}\left[e^{x^2} x^{2x}, x\right] + \text{Log}[x] \text{Int}\left[e^{x^2} x^{2x}, x\right] - \text{Int}\left[\frac{\text{Int}\left[e^{x^2} x^{2x}, x\right]}{x}, x\right]$$

Problem #647: Unable to integrate:

$$\left\{ x^{-2-\frac{1}{x}} (1 - \text{Log}[x]), x, -3, 3 \right\}$$

$$-x^{-1/x}$$

$$\text{Int}\left[x^{-2-\frac{1}{x}}, x\right] - \text{Log}[x] \text{Int}\left[x^{-2-\frac{1}{x}}, x\right] + \text{Int}\left[\frac{\text{Int}\left[x^{-2-\frac{1}{x}}, x\right]}{x}, x\right]$$

Test complete!

3 Logarithms

```
IntegrationTest["#3 Logarithms\\3.1 u (a+b log(c (d (e+f x)^p)^q)^n"];
```

Testing Rubi on 280 integration problems...

Test complete!


```
IntegrationTest["3 Logarithms\\3.2 Logarithm functions"];
```

Testing Rubi on 806 integration problems...

Problem #199: Unable to integrate:

$$\left\{ \frac{(d + ex)^m \operatorname{Log}[cx]}{x}, x, -1, 0 \right\}$$

$$-\frac{\left(1 + \frac{d}{ex}\right)^{-m} (d + ex)^m \operatorname{HypergeometricPFQ}\left[\{-m, -m, -m\}, \{1 - m, 1 - m\}, -\frac{d}{ex}\right]}{m^2} + \frac{\left(1 + \frac{d}{ex}\right)^{-m} (d + ex)^m \operatorname{Hypergeometric2F1}\left[-m, -m, 1 - m, -\frac{d}{ex}\right] \operatorname{Log}[cx]}{m}$$

$$\operatorname{Int}\left[\frac{(d + ex)^m \operatorname{Log}[cx]}{x}, x\right]$$

Test complete!

4 Trig functions

■ 1a Sine

```
IntegrationTest["4 Trig functions\\1a Sine\\1.0 (a sin)^m (b trg)^n"];
```

Testing Rubi on 538 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\1a Sine\\1.1.1 (a+b sin)^n"];
```

Testing Rubi on 64 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\1a Sine\\1.1.2 (g cos)^p (a+b sin)^m"];
```

Testing Rubi on 653 integration problems...

Problem #648: Valid but suboptimal antiderivative:

{ (e Cos[c + d x])^{-3-m} (a + b Sin[c + d x])^m, x, -5, 5 }

$$\frac{1}{(a-b) d e^3 (2+m)} (e \operatorname{Cos}[c+d x])^{-m} \operatorname{Sec}[c+d x]^4 (-1+\operatorname{Sin}[c+d x]) (1+\operatorname{Sin}[c+d x]) (a+b \operatorname{Sin}[c+d x])^{1+m} +$$

$$\left((-2b+a(2+m)) (e \operatorname{Cos}[c+d x])^{-m} \operatorname{Sec}[c+d x]^4 (-1+\operatorname{Sin}[c+d x]) (1+\operatorname{Sin}[c+d x])^2 (a+b \operatorname{Sin}[c+d x])^{1+m} \right) / \left((a-b)^2 d e^3 m (2+m) \right) -$$

$$\left((-b^2+a^2(1+m)) (e \operatorname{Cos}[c+d x])^{-m} \operatorname{Hypergeometric2F1}\left[\frac{m}{2}, 1+m, 2+m, -\frac{2(a+b \operatorname{Sin}[c+d x])}{(a-b)(-1+\operatorname{Sin}[c+d x])}\right] \right.$$

$$\left. \operatorname{Sec}[c+d x]^4 (1+\operatorname{Sin}[c+d x])^3 \left(\frac{(a+b)(1+\operatorname{Sin}[c+d x])}{(a-b)(-1+\operatorname{Sin}[c+d x])} \right)^{\frac{1}{2}(-2+m)} (a+b \operatorname{Sin}[c+d x])^{1+m} \right) / \left((a-b)^3 d e^3 m (1+m) \right)$$

$$- \frac{(e \operatorname{Cos}[c+d x])^{-2-m} (a+b \operatorname{Sin}[c+d x])^{1+m}}{(a-b) d e (2+m)} -$$

$$\left(b (e \operatorname{Cos}[c+d x])^{-2-m} \operatorname{Hypergeometric2F1}\left[1+m, \frac{2+m}{2}, 2+m, \frac{2(a+b \operatorname{Sin}[c+d x])}{(a+b)(1+\operatorname{Sin}[c+d x])}\right] (1-\operatorname{Sin}[c+d x]) \left(-\frac{(a-b)(1-\operatorname{Sin}[c+d x])}{(a+b)(1+\operatorname{Sin}[c+d x])} \right)^{m/2} (a+b \operatorname{Sin}[c+d x])^{1+m} \right) /$$

$$\left((a^2-b^2) d e (1+m)(2+m) \right) + \frac{a (e \operatorname{Cos}[c+d x])^{-2-m} (1+\operatorname{Sin}[c+d x]) (a+b \operatorname{Sin}[c+d x])^{1+m}}{(a^2-b^2) d e (2+m)} +$$

$$\left(2^{-m/2} a (a+b+am) (e \operatorname{Cos}[c+d x])^{-2-m} \operatorname{Hypergeometric2F1}\left[-\frac{m}{2}, \frac{2+m}{2}, \frac{2-m}{2}, \frac{(a-b)(1-\operatorname{Sin}[c+d x])}{2(a+b \operatorname{Sin}[c+d x])}\right] (1-\operatorname{Sin}[c+d x]) \left(\frac{(a+b)(1+\operatorname{Sin}[c+d x])}{a+b \operatorname{Sin}[c+d x]} \right)^{\frac{2+m}{2}} (a+b \operatorname{Sin}[c+d x])^{1+m} \right) /$$

$$(a-b)(a+b)^2 d e m (2+m)$$

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.1.3 (g tan)^p (a+b sin)^m"];

Testing Rubi on 208 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.2.1 (a+b sin)^m (c+d sin)^n"];

Testing Rubi on 837 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n"];

Testing Rubi on 1562 integration problems...

Problem #1277: Valid but suboptimal antiderivative:

$$\left\{ \frac{(a + b \sin[e + f x])^2}{(g \cos[e + f x])^{5/2} \sqrt{d \sin[e + f x]}}, x, -11, 11 \right\}$$

$$\frac{2(a^2 + b^2) \sqrt{d \sin[e + f x]} + \frac{4ab (d \sin[e + f x])^{3/2}}{3d^2 f g (g \cos[e + f x])^{3/2}} + \frac{(2a^2 - b^2) \operatorname{EllipticF}\left[\frac{1}{4}(4e - \pi) + f x, 2\right] \sqrt{\sin[2e + 2fx]}}{3fg^2 \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}}{3dfg (g \cos[e + f x])^{3/2} + \frac{2a^2 \sqrt{d \sin[e + f x]}}{3dfg (g \cos[e + f x])^{3/2}} + \frac{2b^2 \sqrt{d \sin[e + f x]}}{3dfg (g \cos[e + f x])^{3/2}} + \frac{4ab (d \sin[e + f x])^{3/2}}{3d^2 f g (g \cos[e + f x])^{3/2}} + \frac{2a^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2e + 2fx]}}{3fg^2 \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}} - \frac{b^2 \operatorname{EllipticF}\left[e - \frac{\pi}{4} + f x, 2\right] \sqrt{\sin[2e + 2fx]}}{3fg^2 \sqrt{g \cos[e + f x]} \sqrt{d \sin[e + f x]}}$$

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n"];

Testing Rubi on 51 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin)"];

Testing Rubi on 356 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.4.1 (a+b sin)^m (A+B sin+C sin^2)"];

Testing Rubi on 19 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2)"];

Testing Rubi on 34 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.8 (a+b sin)^m (c+d trig)^n"];

Testing Rubi on 9 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.9 trig^m (a+b sin^n+c sin^(2n))^p"];

Testing Rubi on 239 integration problems...

Problem #168: Unable to integrate:

$$\left\{ \frac{\text{Sec}[c + dx]^2}{a + b \sin[c + dx]^3}, x, -1, 0 \right\}$$

$$\frac{2 (-1)^{2/3} b^{2/3} \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - (-1)^{2/3} b^{2/3})^{3/2} d} - \frac{2 b^{2/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - b^{2/3})^{3/2} d} + \frac{2 (-1)^{1/3} b^{2/3} \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} + (-1)^{1/3} b^{2/3})^{3/2} d} + \frac{\text{Sec}[c + dx] (b - a \sin[c + dx])}{(-a^2 + b^2) d}$$

$$\text{Int}\left[\frac{\text{Sec}[c + dx]^2}{a + b \sin[c + dx]^3}, x\right]$$

Problem #170: Unable to integrate:

$$\left\{ \frac{\text{Sec}[c + dx]^4}{a + b \sin[c + dx]^3}, x, -1, 0 \right\}$$

$$-\frac{2 (-1)^{2/3} a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^2 (2 a^2 + b^2) \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^2 (2 a^2 + b^2) \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d}$$

$$-\frac{2 b^{4/3} (a^2 + 2 b^2) \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 (-1)^{1/3} a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^2 (2 a^2 + b^2) \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d}$$

$$-\frac{2 b^{4/3} (a^2 + 2 b^2) \text{ArcTan}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \frac{2 b^{4/3} (a^2 + 2 b^2) \text{ArcTan}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\text{Cos}[c + dx]}{12 (a + b) d (1 - \sin[c + dx])^2} +$$

$$\frac{\text{Cos}[c + dx]}{12 (a + b) d (1 - \sin[c + dx])} + \frac{(a + 4 b) \text{Cos}[c + dx]}{4 (a + b)^2 d (1 - \sin[c + dx])} - \frac{\text{Cos}[c + dx]}{12 (a - b) d (1 + \sin[c + dx])^2} - \frac{(a - 4 b) \text{Cos}[c + dx]}{4 (a - b)^2 d (1 + \sin[c + dx])} - \frac{\text{Cos}[c + dx]}{12 (a - b) d (1 + \sin[c + dx])}$$

$$\text{Int}\left[\frac{\text{Sec}[c + dx]^4}{a + b \sin[c + dx]^3}, x\right]$$

Test complete!

`IntegrationTest["#4 Trig functions\\|a Sine\\|1.10 (c+d x)^m (a+b sin)^n"];`

Testing Rubi on 178 integration problems...

Test complete!

`IntegrationTest["#4 Trig functions\\|a Sine\\|1.11 (e x)^m (a+b x^n)^p sin"];`

Testing Rubi on 113 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.12 (e x)^m (a+b sin(c+d x^n))^p"];

Testing Rubi on 209 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1a Sine\\1.13 (d+e x)^m sin(a+b x+c x^2)^n"];

Testing Rubi on 36 integration problems...

Test complete!

■ 1b Cosine

IntegrationTest["4 Trig functions\\1b Cosine\\1.0 (a cos)^m (b trg)^n"];

Testing Rubi on 294 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.1.1 (a+b cos)^n"];

Testing Rubi on 62 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.1.2 (g sin)^p (a+b cos)^m"];

Testing Rubi on 88 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.1.3 (g tan)^p (a+b cos)^m"];

Testing Rubi on 22 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.2.1 (a+b cos)^m (c+d cos)^n"];

Testing Rubi on 936 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n"];

Testing Rubi on 4 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n"];

Testing Rubi on 1 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.3.1 (a+b cos)^m (c+d cos)^n (A+B cos)"];

Testing Rubi on 644 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.4.1 (a+b cos)^m (A+B cos+C cos^2)"];

Testing Rubi on 393 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2)"];

Testing Rubi on 1536 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.8 (a+b cos)^m (c+d trig)^n"];

Testing Rubi on 21 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.9 trig^m (a+b cos^n+c cos^(2 n))^p"];

Testing Rubi on 94 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.10 (c+d x)^m (a+b cos)^n"];

Testing Rubi on 189 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.12 (e x)^m (a+b cos(c+d x^n))^p"];

Testing Rubi on 93 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\1b Cosine\\1.13 (d+e x)^m cos(a+b x+c x^2)^n"];

Testing Rubi on 34 integration problems...

Test complete!

■ 2a Tangent

IntegrationTest["4 Trig functions\\2a Tangent\\2.0 (a trg)^m (b tan)^n"];

Testing Rubi on 352 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.1.2 (d sec)^m (a+b tan)^n"];

Testing Rubi on 700 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.1.3 (d sin)^m (a+b tan)^n"];

Testing Rubi on 93 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.2.1 (a+b tan)^m (c+d tan)^n"];

Testing Rubi on 1342 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.3.1 (a+b tan)^m (c+d tan)^n (A+B tan)"];

Testing Rubi on 859 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2)"];

Testing Rubi on 133 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.9 trig^m (a+b tan^n+c tan^(2 n))^p"];

Testing Rubi on 209 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2a Tangent\\2.10 (c+d x)^m (a+b tan)^n"];

Testing Rubi on 63 integration problems...

Problem #17: Unable to integrate:

$$\left\{ \frac{x^2}{\sqrt{\tan[a + b x^2]}} + \frac{\sqrt{\tan[a + b x^2]}}{b} + x^2 \tan[a + b x^2]^{3/2}, x, -1, 1 \right\}$$

$$\frac{x \sqrt{\tan[a + b x^2]}}{b}$$

$$\text{Int}\left[\frac{x^2}{\sqrt{\tan[a + b x^2]}} , x\right] + \frac{\text{Int}\left[\sqrt{\tan[a + b x^2]} , x\right]}{b} + \text{Int}\left[x^2 \tan[a + b x^2]^{3/2}, x\right]$$

Test complete!

IntegrationTest["4 Trig functions\2a Tangent\2.11 (e x)^m (a+b tan(c+d x^n))^p"];

Testing Rubi on 72 integration problems...

Test complete!

■ 2b Cotangent

IntegrationTest["4 Trig functions\2b Cotangent\2.0 (a trg)^m (b cot)^n"];

Testing Rubi on 52 integration problems...

Test complete!

IntegrationTest["4 Trig functions\2b Cotangent\2.1.2 (d csc)^m (a+b cot)^n"];

Testing Rubi on 23 integration problems...

Test complete!

IntegrationTest["4 Trig functions\2b Cotangent\2.1.3 (d cos)^m (a+b cot)^n"];

Testing Rubi on 19 integration problems...

Test complete!

IntegrationTest["4 Trig functions\2b Cotangent\2.2.1 (a+b cot)^m (c+d cot)^n"];

Testing Rubi on 106 integration problems...

Test complete!

IntegrationTest["4 Trig functions\2b Cotangent\2.9 trig^m (a+b cot^n+c cot^(2 n))^p"];

Testing Rubi on 94 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\2b Cotangent\\2.10 (c+d x)^m (a+b cot)^n"];

Testing Rubi on 61 integration problems...

Test complete!

■ 3a Secant

IntegrationTest["4 Trig functions\\3a Secant\\3.0 (a sec)^m (b trg)^n"];

Testing Rubi on 283 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\3a Secant\\3.1.2 (d sec)^n (a+b sec)^m"];

Testing Rubi on 885 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\3a Secant\\3.1.3 (d sin)^n (a+b sec)^m"];

Testing Rubi on 306 integration problems...

Test complete!

IntegrationTest["4 Trig functions\\3a Secant\\3.1.4 (d tan)^n (a+b sec)^m"];

Testing Rubi on 351 integration problems...

Problem #207: Valid but suboptimal antiderivative:

$$\left\{ \frac{\tan[e + f x]^2}{(a + a \operatorname{Sec}[e + f x])^{9/2}}, x, -7, 7 \right\}$$

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{32 \sqrt{2} a^{9/2} f} + \frac{\tan[e + f x]}{3 a f (a + a \operatorname{Sec}[e + f x])^{7/2}} + \frac{11 \tan[e + f x]}{24 a^2 f (a + a \operatorname{Sec}[e + f x])^{5/2}} + \frac{27 \tan[e + f x]}{32 a^3 f (a + a \operatorname{Sec}[e + f x])^{3/2}}$$

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{32 \sqrt{2} a^{9/2} f} + \frac{\tan[e + f x]}{3 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]}} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]} \right)^3 +$$

$$\frac{11 \tan[e + f x]}{24 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]}} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]} \right)^2 + \frac{27 \tan[e + f x]}{32 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]}} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]} \right)$$

Problem #329: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Cot}[c + d x]^3}{(a + b \operatorname{Sec}[c + d x])^{3/2}}, x, -18, 18 \right\}$$

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{(4 a-7 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{4(a-b)^{5/2} d} + \frac{(4 a+7 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{4(a+b)^{5/2} d} + \\
& \frac{2 b^4}{a(a-b)^2(a+b)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{\operatorname{Cot}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]}(a^2+b^2-2 a b \operatorname{Sec}[c+d x])}{2(a^2-b^2)^2 d} \\
& - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{a \sqrt{a-b} d} + \frac{(2 a-5 b) b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{4 a(a-b)^{5/2} d} - \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{4 a(a-b)^{3/2} d} + \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{4 a(a+b)^{3/2} d} + \\
& \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{a \sqrt{a+b} d} - \frac{b(2 a+5 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{4 a(a+b)^{5/2} d} + \frac{b^2(a^2+5 b^2)}{2 a(a^2-b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{4 a(a+b) d(1-\operatorname{Sec}[c+d x])} + \\
& \frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{4 a(a-b) d(1+\operatorname{Sec}[c+d x])} - \left(b^2(a^2+b^2-a(a+b \operatorname{Sec}[c+d x]))\right) / \left(2 a(a^2-b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}(a^2-b^2-2 a(a+b \operatorname{Sec}[c+d x])+(a+b \operatorname{Sec}[c+d x])^2)\right)
\end{aligned}$$

Problem #333: Unable to integrate:

$$\left\{\frac{(d \operatorname{Tan}[e+f x])^n}{a+b \operatorname{Sec}[e+f x]}, x, -1, 0\right\}$$

$$\begin{aligned}
& \frac{1}{a f(1-n)} d \operatorname{AppellF1}\left[1-n, \frac{1-n}{2}, \frac{1-n}{2}, 2-n, \frac{a+b}{a+b \operatorname{Sec}[e+f x]}, \frac{a-b}{a+b \operatorname{Sec}[e+f x]}\right] \left(-\frac{b(1-\operatorname{Sec}[e+f x])}{a+b \operatorname{Sec}[e+f x]}\right)^{\frac{1-n}{2}} \left(\frac{b(1+\operatorname{Sec}[e+f x])}{a+b \operatorname{Sec}[e+f x]}\right)^{\frac{1-n}{2}} (d \operatorname{Tan}[e+f x])^{-1+n} \left(-\operatorname{Tan}[e+f x]^2\right)^{\frac{1-n}{2}+\frac{1}{2}(-1+n)} - \\
& \frac{1}{a f(1+n)} d \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\operatorname{Tan}[e+f x]^2\right] (d \operatorname{Tan}[e+f x])^{-1+n} \left(-\operatorname{Tan}[e+f x]^2\right)^{\frac{1-n}{2}+\frac{1+n}{2}}
\end{aligned}$$

$$\operatorname{Int}\left[\frac{(d \operatorname{Tan}[e+f x])^n}{a+b \operatorname{Sec}[e+f x]}, x\right]$$

Test complete!

IntegrationTest["4 Trig functions\3a Secant\3.2.1 (a+b sec)^m (c+d sec)^n"];

Testing Rubi on 243 integration problems...

Test complete!

IntegrationTest["4 Trig functions\3a Secant\3.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n"];

Testing Rubi on 286 integration problems...

Test complete!

IntegrationTest["4 Trig functions\3a Secant\3.3.1 (a+b sec)^m (d sec)^n (A+B sec)^n"];

Testing Rubi on 645 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3a Secant\\3.4.1 (a+b sec)^m (A+B sec+C sec^2)"];
```

Testing Rubi on 70 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3a Secant\\3.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2)"];
```

Testing Rubi on 1373 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3a Secant\\3.7 trig^m (a+b sec^n+c sec^(2 n))^p"];
```

Testing Rubi on 34 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3a Secant\\3.10 (c+d x)^m (a+b sec)^n"];
```

Testing Rubi on 46 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3a Secant\\3.11 (e x)^m (a+b sec(c+d x^n))^p"];
```

Testing Rubi on 83 integration problems...

Test complete!

■ 3b Cosecant

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.0 (a csc)^m (b trg)^n"];
```

Testing Rubi on 70 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.1.2 (d csc)^n (a+b csc)^m"];
```

Testing Rubi on 59 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.1.3 (d cos)^n (a+b csc)^m"];
```

Testing Rubi on 16 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.1.4 (d cot)^n (a+b csc)^m"];
```

Testing Rubi on 23 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.3.1 (a+b csc)^m (d csc)^n (A+B csc)"];
```

Testing Rubi on 24 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2)"];
```

Testing Rubi on 1 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.7 trig^m (a+b csc^n+c csc^(2 n))^p"];
```

Testing Rubi on 27 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\3b Cosecant\\3.11 (e x)^m (a+b csc(c+d x^n))^p"];
```

Testing Rubi on 84 integration problems...

Test complete!

■ 4 Miscellaneous trig

```
IntegrationTest["4 Trig functions\\4 Miscellaneous\\4.1 (c trig)^m (d trig)^n"];
```

Testing Rubi on 250 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4 Miscellaneous\\4.2 trig^m (a trig+b trig)^n"];
```

Testing Rubi on 294 integration problems...

Problem #15: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]^3}{(a \cos[x] + b \sin[x])^2}, x, -19, 19 \right\}$$

$$\frac{6 a^2 b \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{5/2}} + \frac{3 a (a^2-b^2) + a (a^2+b^2) \cos[2 x] - b (a^2+b^2) \sin[2 x]}{2 (a^2+b^2)^2 (a \cos[x] + b \sin[x])}$$

$$- \frac{3 a^2 \operatorname{ArcTanh}\left[\frac{b \cos[x]-a \sin[x]}{\sqrt{a^2+b^2}}\right]}{b (a^2+b^2)^{3/2}} - \frac{2 a^2 b \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{5/2}} + \frac{2 a^2 (3 a^2+b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b (a^2+b^2)^{5/2}} - \frac{\cos[x]}{b^2} +$$

$$\frac{3 a^2 \cos[x]}{b^2 (a^2+b^2)} - \frac{2 a \sin[x]}{b^3} + \frac{3 a^3 \sin[x]}{b^3 (a^2+b^2)} - \frac{2 a^3 \cos\left[\frac{x}{2}\right]^2 (2 a b + (a^2-b^2) \operatorname{Tan}\left[\frac{x}{2}\right])}{b^3 (a^2+b^2)^2} + \frac{2 a^2 (a+b \operatorname{Tan}\left[\frac{x}{2}\right])}{(a^2+b^2)^2 (a+2 b \operatorname{Tan}\left[\frac{x}{2}\right] - a \operatorname{Tan}\left[\frac{x}{2}\right]^2)}$$

Problem #23: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]^2}{(a \cos[x] + b \sin[x])^3}, x, -16, 16 \right\}$$

$$-\frac{(a^2 - 2b^2) \operatorname{ArcTanh}\left[\frac{-b+a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2}} + \frac{a(3ab \cos[x] + (a^2 + 4b^2) \sin[x])}{2(a^2 + b^2)^2 (a \cos[x] + b \sin[x])^2}$$

$$\frac{2a^2 \operatorname{ArcTanh}\left[\frac{b \cos[x] - a \sin[x]}{\sqrt{a^2+b^2}}\right]}{b^2 (a^2 + b^2)^{3/2}} - \frac{\operatorname{ArcTanh}\left[\frac{b \cos[x] - a \sin[x]}{\sqrt{a^2+b^2}}\right]}{b^2 \sqrt{a^2 + b^2}} + \frac{(a^2 - 2b^2) \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2}} - \frac{2(a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b^2 (a^2 + b^2)^{3/2}} +$$

$$\frac{2a}{b(a^2 + b^2)(a \cos[x] + b \sin[x])} - \frac{a \tan\left[\frac{x}{2}\right]}{2b^2(a^2 + b^2)} - \frac{a^2(b - a \tan\left[\frac{x}{2}\right]) \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}{2b^2(a^2 + b^2) \left(a + 2b \tan\left[\frac{x}{2}\right] - a \tan\left[\frac{x}{2}\right]^2\right)^2} - \frac{a(2a^2 - b^2) + b(a^2 - 2b^2) \tan\left[\frac{x}{2}\right]}{b(a^2 + b^2)^2 \left(a + 2b \tan\left[\frac{x}{2}\right] - a \tan\left[\frac{x}{2}\right]^2\right)}$$

Problem #123: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[c + dx]^3}{(a \cos[c + dx] + b \sin[c + dx])^2}, x, -11, 11 \right\}$$

$$-\frac{3ab^2 \operatorname{ArcTanh}\left[\frac{b \cos[c+dx] - a \sin[c+dx]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2} d} + \frac{2ab \cos[c + dx]}{(a^2 + b^2)^2 d} + \frac{(a^2 - b^2) \sin[c + dx]}{(a^2 + b^2)^2 d} - \frac{b^3}{(a^2 + b^2)^2 d (a \cos[c + dx] + b \sin[c + dx])}$$

$$\frac{2b^4 \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2 + b^2)^{5/2} d} - \frac{2b^2(3a^2 + b^2) \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2 + b^2)^{5/2} d} + \frac{2(2ab + (a^2 - b^2) \tan\left[\frac{1}{2}(c + dx)\right])}{(a^2 + b^2)^2 d \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2\right)} - \frac{2b^3 \left(a + b \tan\left[\frac{1}{2}(c + dx)\right]\right)}{a(a^2 + b^2)^2 d \left(a + 2b \tan\left[\frac{1}{2}(c + dx)\right] - a \tan\left[\frac{1}{2}(c + dx)\right]^2\right)}$$

Problem #131: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[c + dx]^4}{(a \cos[c + dx] + b \sin[c + dx])^3}, x, -15, 15 \right\}$$

$$\frac{1}{2d} \left(\frac{6b^2(-4a^2 + b^2) \operatorname{ArcTanh}\left[\frac{-b+a \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{7/2}} - \frac{2b(-3a^2 + b^2) \cos[c + dx]}{(a^2 + b^2)^3} + \right.$$

$$\left. \frac{2a(a^2 - 3b^2) \sin[c + dx]}{(a^2 + b^2)^3} + \frac{b^4 \sin[c + dx]}{a(a - ib)^2(a + ib)^2(a \cos[c + dx] + b \sin[c + dx])^2} - \frac{b^3(8a^2 + b^2)}{a(a^2 + b^2)^3(a \cos[c + dx] + b \sin[c + dx])} \right)$$

$$\begin{aligned}
& - \frac{3 b^4 (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \frac{4 b^4 (3 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} - \frac{2 b^2 (6 a^4 + 3 a^2 b^2 + b^4) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \frac{2 (b (3 a^2 - b^2) + a (a^2 - 3 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{(a^2 + b^2)^3 d (1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2} + \\
& \frac{2 b^4 (a b + (a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^2 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2} - \frac{3 b^4 (a^2 + 2 b^2) (b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^3 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2} - \frac{4 b^3 (2 a^4 - b^4 + a b (3 a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^3 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2}
\end{aligned}$$

Problem #133: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c+d x]^2}{(a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3}, x, -9, 9 \right\} \\
& \frac{(2 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2} d} - \frac{b ((4 a^2 + b^2) \operatorname{Cos}[c+d x] + 3 a b \operatorname{Sin}[c+d x])}{2 (a^2 + b^2)^2 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^2} \\
& \frac{b^2 (a^2 - 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{5/2} d} - \frac{2 (a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d} - \frac{\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{2 a (a^2 + b^2) d} - \\
& \frac{(b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]) (1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2}{2 (a^2 + b^2) d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2} - \frac{b (a (2 a^2 - b^2) + b (a^2 - 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^2 (a^2 + b^2)^2 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])^2}
\end{aligned}$$

Problem #142: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c+d x]^3}{(a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4}, x, -13, 13 \right\} \\
& \frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{7/2} d} + \\
& \left(-3 (3 a^4 b - a^2 b^3 + b^5) \operatorname{Cos}[2(c+d x)] + \frac{1}{2} b (-9 a^2 + b^2) (2 (a^2 + b^2) + 3 a b \operatorname{Sin}[2(c+d x)]) \right) / \left((6 (a - i b)^3 (a + i b)^3 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3 \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{6 b^2 (a^2 - 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^3 (a^2+b^2)^{5/2} d} - \frac{2 (a^2 - 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^3 (a^2+b^2)^{3/2} d} - \frac{b^2 (3 a^4 - 8 b^4) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^3 (a^2+b^2)^{7/2} d} \\
& \frac{\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{3 a^2 (a^2+b^2) d} - \frac{(b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]) \left(1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right)^3}{3 (a^2+b^2) d (a+2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^3} + \frac{2 b^2 (a b (a^2 - 4 b^2) + (3 a^4 - 8 b^4) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{3 a^4 (a^2+b^2)^2 d (a+2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^2} \\
& \frac{b^2 (3 a^4 - 8 b^4) (b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^4 (a^2+b^2)^3 d (a+2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)} - \frac{2 b (6 a^4 - 13 a^2 b^2 + 8 b^4 + 9 a b (a^2 - 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{3 a^4 (a^2+b^2)^2 d (a+2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)}
\end{aligned}$$

Test complete!

IntegrationTest["4 Trig functions\4 Miscellaneous\4.3 (c+d x)^m trig^n trig^p"];

Testing Rubi on 397 integration problems...

Test complete!

IntegrationTest["4 Trig functions\4 Miscellaneous\4.4 x^m (a+b trig^n)^p"];

Testing Rubi on 9 integration problems...

Test complete!

IntegrationTest["4 Trig functions\4 Miscellaneous\4.5 x^m trig(a+b log(c x^n))^p"];

Testing Rubi on 268 integration problems...

Test complete!

IntegrationTest["4 Trig functions\4 Miscellaneous\4.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n"];

Testing Rubi on 132 integration problems...

Test complete!

IntegrationTest["4 Trig functions\4 Miscellaneous\4.7 Trig functions"];

Testing Rubi on 881 integration problems...

Problem #690: Valid but suboptimal antiderivative:

{Cos[x]¹² Sin[x]¹⁰ - Cos[x]¹⁰ Sin[x]¹², x, -25, 25}

$$\begin{aligned}
& \frac{1}{11} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^{11} \\
& \frac{3 \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]}{5632} - \frac{3 \operatorname{Cos}[x]^{13} \operatorname{Sin}[x]}{5632} + \frac{1}{512} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^3 - \frac{7 \operatorname{Cos}[x]^{13} \operatorname{Sin}[x]^3}{2816} + \frac{7 \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^5}{1280} \\
& - \frac{7}{880} \operatorname{Cos}[x]^{13} \operatorname{Sin}[x]^5 + \frac{1}{80} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^7 - \frac{9}{440} \operatorname{Cos}[x]^{13} \operatorname{Sin}[x]^7 + \frac{1}{40} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^9 - \frac{1}{22} \operatorname{Cos}[x]^{13} \operatorname{Sin}[x]^9 + \frac{1}{22} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^{11}
\end{aligned}$$

Problem #727: Unable to integrate:

$$\{e^{\sin[x]} \operatorname{Sec}[x]^2 (x \operatorname{Cos}[x]^3 - \operatorname{Sin}[x]), x, -2, 2\}$$

$$e^{\sin[x]} (-1 + x \operatorname{Cos}[x]) \operatorname{Sec}[x]$$

$$\operatorname{Int}[e^{\sin[x]} x \operatorname{Cos}[x], x] - \operatorname{Int}[e^{\sin[x]} \operatorname{Sec}[x] \operatorname{Tan}[x], x]$$

Problem #789: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{\operatorname{Cos}[x]^{3/2} \sqrt{3 \operatorname{Cos}[x] + \operatorname{Sin}[x]}}, x, -5, 5 \right\}$$

$$\frac{2 \sqrt{3 \operatorname{Cos}[x] + \operatorname{Sin}[x]}}{\sqrt{\operatorname{Cos}[x]}}$$

$$\frac{2 \operatorname{Cos}\left[\frac{x}{2}\right]^2 \left(3 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - 3 \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{\operatorname{Cos}\left[\frac{x}{2}\right]^2 \left(3 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - 3 \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)} \sqrt{\operatorname{Cos}\left[\frac{x}{2}\right]^2 \left(1 - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)}}$$

Problem #790: Unable to integrate:

$$\left\{ \frac{\operatorname{Csc}[x] \sqrt{\operatorname{Cos}[x] + \operatorname{Sin}[x]}}{\operatorname{Cos}[x]^{3/2}}, x, -1, 0 \right\}$$

$$-\operatorname{Log}[\operatorname{Sin}[x]] + 2 \operatorname{Log}\left[-\sqrt{\operatorname{Cos}[x]} + \sqrt{\operatorname{Cos}[x] + \operatorname{Sin}[x]}\right] + \frac{2 \sqrt{\operatorname{Cos}[x] + \operatorname{Sin}[x]}}{\sqrt{\operatorname{Cos}[x]}}$$

$$\operatorname{Int}\left[\frac{\operatorname{Csc}[x] \sqrt{\operatorname{Cos}[x] + \operatorname{Sin}[x]}}{\operatorname{Cos}[x]^{3/2}}, x\right]$$

Problem #791: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Cos}[x] + \operatorname{Sin}[x]}{\sqrt{1 + \operatorname{Sin}[2x]}}, x, -19, 19 \right\}$$

$$\frac{x \sqrt{1 + \operatorname{Sin}[2x]}}{\operatorname{Cos}[x] + \operatorname{Sin}[x]}$$

$$\frac{2 \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{x}{2}\right]\right] \operatorname{Cos}\left[\frac{x}{2}\right]^2 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{\operatorname{Cos}\left[\frac{x}{2}\right]^4 \left(1 + 2 \operatorname{Tan}\left[\frac{x}{2}\right] - \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2}}$$

Problem #843: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x]}}, x, -26, 26 \right\}$$

$$-\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

$$\frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}}$$

$$\frac{\operatorname{Log}\left[1 + \cot[x] - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \cot[x] + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\operatorname{Log}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}}$$

Problem #846: Unable to integrate:

$$\left\{ \cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x, -1, 0 \right\}$$

$$\frac{x}{2} - \frac{\cos[x]}{2} - \operatorname{Log}\left[\cos\left[\frac{\pi}{4} + \frac{x}{2}\right]\right]$$

$$\operatorname{Int}\left[\cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x\right]$$

Problem #862: Unable to integrate:

$$\left\{ \frac{x^4}{b \sqrt{x^3 + 3 \sin[a + bx]}} + \frac{x^2 \cos[a + bx]}{\sqrt{x^3 + 3 \sin[a + bx]}} + \frac{4x \sqrt{x^3 + 3 \sin[a + bx]}}{3b}, x, -1, 1 \right\}$$

$$\frac{2x^2 \sqrt{x^3 + 3 \sin[a + bx]}}{3b}$$

$$\frac{\operatorname{Int}\left[\frac{x^4}{\sqrt{x^3 + 3 \sin[a + bx]}}, x\right]}{b} + \operatorname{Int}\left[\frac{x^2 \cos[a + bx]}{\sqrt{x^3 + 3 \sin[a + bx]}}, x\right] + \frac{4 \operatorname{Int}\left[x \sqrt{x^3 + 3 \sin[a + bx]}, x\right]}{3b}$$

Problem #864: Unable to integrate:

$$\left\{ \frac{\cos[x] + \sin[x]}{e^{-x} + \sin[x]}, x, -5, 5 \right\}$$

$$\operatorname{Log}[1 + e^x \sin[x]]$$

$$x + \text{Log}[\text{Sin}[x]] - \text{Int}\left[\frac{1}{1 + e^x \text{Sin}[x]}, x\right] - \text{Int}\left[\frac{\text{Cot}[x]}{1 + e^x \text{Sin}[x]}, x\right]$$

Test complete!

5 Inverse trig functions

■ Inverse sine

```
IntegrationTest["5 Inverse trig functions\\1a Inverse sine\\1.2 (d x)^m (a+b arcsin(c x))^n"];
```

Testing Rubi on 227 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\1a Inverse sine\\1.4a (f x)^m (d-c^2 d x^2)^p (a+b arcsin(c x))^n"];
```

Testing Rubi on 502 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\1a Inverse sine\\1.4b (f x)^m (d+e x^2)^p (a+b arcsin(c x))^n"];
```

Testing Rubi on 109 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\1a Inverse sine\\1.5 Inverse sine functions"];
```

Testing Rubi on 495 integration problems...

Problem #493: Unable to integrate:

$$\left\{ \frac{x}{\text{ArcSin}[\text{Sin}[x]]}, x, -1, 0 \right\}$$

$$\text{ArcSin}[\text{Sin}[x]] + \text{Log}[\text{ArcSin}[\text{Sin}[x]]] \left(-\text{ArcSin}[\text{Sin}[x]] + x \sqrt{\text{Cos}[x]^2} \text{Sec}[x] \right)$$

$$\text{Int}\left[\frac{x}{\text{ArcSin}[\text{Sin}[x]]}, x\right]$$

Test complete!

■ Inverse cosine

```
IntegrationTest["5 Inverse trig functions\\1b Inverse cosine\\1.2 (d x)^m (a+b arccos(c x))^n"];
```

Testing Rubi on 227 integration problems...

Test complete!

```
IntegrationTest["5 Inverse Trig functions\\1b Inverse cosine\\1.5 Inverse cosine functions"];
```

Testing Rubi on 136 integration problems...

Test complete!

■ Inverse tangent

```
IntegrationTest["5 Inverse trig functions\\2a Inverse tangent\\2.1.1 (d x)^m (a+b arctan(c x))^n"];
```

Testing Rubi on 57 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\2a Inverse tangent\\2.1.2 (f x)^m (d-c^2 d x^2)^p (a+b arctan(c x))^n"];
```

Testing Rubi on 994 integration problems...

Problem #973: Unable to integrate:

$$\left\{ \frac{\text{ArcTan}[x] \text{Log}[1+x^2]}{x}, x, -1, 0 \right\}$$

$$\begin{aligned} & \frac{1}{2} \left(-\text{Log}[1-ix]^2 \text{Log}[x] + \text{Log}[1+ix]^2 \text{Log}[x] - 2 \text{Log}[1+ix] \text{Log}[x] \text{Log}[-i+x] - \text{Log}[-ix] \text{Log}[-i+x]^2 + \text{Log}[x] \text{Log}[-i+x]^2 + \right. \\ & \left. 2 \text{Log}[1-ix] \text{Log}[x] \text{Log}[i+x] + \text{Log}[ix] \text{Log}[i+x]^2 - \text{Log}[x] \text{Log}[i+x]^2 + 2 \text{Log}[i+x] \text{PolyLog}[2, 1-ix] - 2 \text{Log}[-i+x] \text{PolyLog}[2, 1+ix] - \right. \\ & \left. \text{Log}[1-ix] \text{PolyLog}[2, -ix] + \text{Log}[1+ix] \text{PolyLog}[2, -ix] - 2 \text{Log}[-i+x] \text{PolyLog}[2, -ix] + \text{Log}[1+x^2] \text{PolyLog}[2, -ix] - \text{Log}[1-ix] \text{PolyLog}[2, ix] + \right. \\ & \left. \text{Log}[1+ix] \text{PolyLog}[2, ix] + 2 \text{Log}[i+x] \text{PolyLog}[2, ix] - \text{Log}[1+x^2] \text{PolyLog}[2, ix] - 2 \text{PolyLog}[3, 1-ix] + 2 \text{PolyLog}[3, 1+ix] \right) \end{aligned}$$

$$\text{Int} \left[\frac{\text{ArcTan}[x] \text{Log}[1+x^2]}{x}, x \right]$$

Test complete!

```
IntegrationTest["5 Inverse trig functions\\2a Inverse tangent\\2.1.3 (f x)^m (d+e x^2)^p (a+b arctan(c x))^n"];
```

Testing Rubi on 132 integration problems...

Problem #24: Valid but suboptimal antiderivative:

$\{x^3 (d + e x^2)^3 (a + b \text{ArcTan}[c x]), x, -8, 8\}$

$$\frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x^3}{40 c^9} - \frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x^3}{120 c^7} - \frac{b e (20 c^4 d^2 - 15 c^2 d e + 4 e^2) x^5}{200 c^5} -$$

$$\frac{b (15 c^2 d - 4 e) e^2 x^7}{280 c^3} - \frac{b e^3 x^9}{90 c} + \frac{b (c^2 d - e)^4 (c^2 d + 4 e) \text{ArcTan}[c x]}{40 c^{10} e^2} - \frac{d (d + e x^2)^4 (a + b \text{ArcTan}[c x])}{8 e^2} + \frac{(d + e x^2)^5 (a + b \text{ArcTan}[c x])}{10 e^2}$$

$$\frac{b (325 c^8 d^4 + 1815 c^6 d^3 e - 4977 c^4 d^2 e^2 + 4305 c^2 d e^3 - 1260 e^4) x}{12600 c^9 e} + \frac{b (5 c^6 d^3 + 750 c^4 d^2 e - 1071 c^2 d e^2 + 420 e^3) x (d + e x^2)}{12600 c^7 e} - \frac{b (25 c^4 d^2 - 135 c^2 d e + 84 e^2) x (d + e x^2)^2}{4200 c^5 e}$$

$$\frac{b (23 c^2 d - 36 e) x (d + e x^2)^3}{2520 c^3 e} - \frac{b x (d + e x^2)^4}{90 c e} + \frac{b (c^2 d - e)^4 (c^2 d + 4 e) \text{ArcTan}[c x]}{40 c^{10} e^2} - \frac{d (d + e x^2)^4 (a + b \text{ArcTan}[c x])}{8 e^2} + \frac{(d + e x^2)^5 (a + b \text{ArcTan}[c x])}{10 e^2}$$

Test complete!

IntegrationTest["5 Inverse trig functions\2a Inverse tangent\2.1.4 Inverse tangent functions"];

Testing Rubi on 217 integration problems...

Test complete!

IntegrationTest["5 Inverse trig functions\2a Inverse tangent\2.2.1 x^m (c+a^2 c x^2)^p E^(n arctan(a x))"];

Testing Rubi on 142 integration problems...

Test complete!

IntegrationTest["5 Inverse trig functions\2a Inverse tangent\2.2.2 Exponentials of inverse tangent"];

Testing Rubi on 242 integration problems...

Test complete!

■ Inverse cotangent

IntegrationTest["5 Inverse trig functions\2b Inverse cotangent\2.1 Inverse cotangent functions"];

Testing Rubi on 228 integration problems...

Test complete!

IntegrationTest["5 Inverse trig functions\2b Inverse cotangent\2.2 Exponentials of inverse cotangent"];

Testing Rubi on 12 integration problems...

Test complete!

■ Inverse secant

IntegrationTest["5 Inverse trig functions\3a Inverse secant\3.1 Inverse secant functions"];

Testing Rubi on 166 integration problems...

Test complete!

■ Inverse cosecant

```
IntegrationTest["5 Inverse trig functions\\3b Inverse cosecant\\3.1 Inverse cosecant functions"];
```

Testing Rubi on 87 integration problems...

Test complete!

6 Hyperbolic functions

■ 1a Hyperbolic sine

```
IntegrationTest["6 Hyperbolic functions\\1a Hyperbolic sine\\1.1 (c+d x)^m (a+b sinh)^n"];
```

Testing Rubi on 186 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1a Hyperbolic sine\\1.3 (e x)^m (a+b sinh(c+d x^n))^p"];
```

Testing Rubi on 96 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1a Hyperbolic sine\\1.4 (d+e x)^m sinh(a+b x+c x^2)^n"];
```

Testing Rubi on 33 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1a Hyperbolic sine\\1.5 Hyperbolic sine functions"];
```

Testing Rubi on 694 integration problems...

Test complete!

■ 1b Hyperbolic cosine

```
IntegrationTest["6 Hyperbolic functions\\1b Hyperbolic cosine\\1.1 (c+d x)^m (a+b cosh)^n"];
```

Testing Rubi on 183 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1b Hyperbolic cosine\\1.2 (e x)^m (a+b x^n)^p cosh"];
```

Testing Rubi on 113 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1b Hyperbolic cosine\\1.3 (e x)^m (a+b cosh(c+d x^n))^p"];
```

Testing Rubi on 62 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1b Hyperbolic cosine\\1.4 (d+e x)^m cosh(a+b x+c x^2)^n"];
```

Testing Rubi on 33 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\1b Hyperbolic cosine\\1.5 Hyperbolic cosine functions"];
```

Testing Rubi on 407 integration problems...

Test complete!

■ 2a Hyperbolic tangent

```
IntegrationTest["6 Hyperbolic functions\\2a Hyperbolic tangent\\2.1 (c+d x)^m (a+b tanh)^n"];
```

Testing Rubi on 79 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\2a Hyperbolic tangent\\2.2 Hyperbolic tangent functions"];
```

Testing Rubi on 461 integration problems...

Test complete!

■ 2b Hyperbolic cotangent

```
IntegrationTest["6 Hyperbolic functions\\2b Hyperbolic cotangent\\2.1 (c+d x)^m (a+b coth)^n"];
```

Testing Rubi on 61 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\2b Hyperbolic cotangent\\2.2 Hyperbolic cotangent functions"];
```

Testing Rubi on 228 integration problems...

Test complete!

■ 3a Hyperbolic secant

```
IntegrationTest["6 Hyperbolic functions\\3a Hyperbolic secant\\3.1 (c+d x)^m (a+b sech)^n"];
```

Testing Rubi on 16 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\3a Hyperbolic secant\\3.2 (e x)^m (a+b sech(c+d x^n))^p"];
```

Testing Rubi on 84 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\3a Hyperbolic secant\\3.3 Hyperbolic secant functions"];
```

Testing Rubi on 394 integration problems...

Test complete!

■ 3b Hyperbolic cosecant

```
IntegrationTest["6 Hyperbolic functions\\3b Hyperbolic cosecant\\3.1 (c+d x)^m (a+b csch)^n"];
```

Testing Rubi on 16 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\3b Hyperbolic cosecant\\3.2 (e x)^m (a+b csch(c+d x^n))^p"];
```

Testing Rubi on 83 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\3b Hyperbolic cosecant\\3.3 Hyperbolic cosecant functions"];
```

Testing Rubi on 195 integration problems...

Test complete!

■ 4 Miscellaneous hyperbolic trig

```
IntegrationTest["6 Hyperbolic functions\\4 Miscellaneous\\4.1 Hyperbolic functions"];
```

Testing Rubi on 1055 integration problems...

Test complete!

7 Inverse hyperbolic functions

■ Inverse hyperbolic sine

```
IntegrationTest["7 Inverse hyperbolic functions\\1a Inverse hyperbolic sine\\1.2 (d x)^m (a+b arcsinh(c x))^n"];
```

Testing Rubi on 156 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1a Inverse hyperbolic sine\\1.4a (f x)^m (d+c^2 d x^2)^p (a+b arcsinh(c x))^n"];
```

Testing Rubi on 469 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1a Inverse hyperbolic sine\\1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n"];
```

Testing Rubi on 58 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1a Inverse hyperbolic sine\\1.5 Inverse hyperbolic sine functions"];
```

Testing Rubi on 371 integration problems...

Problem #369: Unable to integrate:

$$\left\{ \frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}, x, -1, 0 \right\}$$

$$\text{ArcSinh}[\text{Sinh}[x]] + \text{Log}[\text{ArcSinh}[\text{Sinh}[x]]] \left(-\text{ArcSinh}[\text{Sinh}[x]] + x \sqrt{\text{Cosh}[x]^2 \text{Sech}[x]} \right)$$

$$\text{Int} \left[\frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}, x \right]$$

Test complete!

■ Inverse hyperbolic cosine

```
IntegrationTest["7 Inverse hyperbolic functions\\1b Inverse hyperbolic cosine\\1.2 (d x)^m (a+b arccosh(c x))^n"];
```

Testing Rubi on 166 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1b Inverse hyperbolic cosine\\1.4a (f x)^m (d-c^2 d x^2)^p (a+b arccosh(c x))^n"];
```

Testing Rubi on 453 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1b Inverse hyperbolic cosine\\1.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n"];
```

Testing Rubi on 109 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\1b Inverse hyperbolic cosine\\1.5 Inverse hyperbolic cosine functions"];
```

Testing Rubi on 293 integration problems...

Test complete!

■ Inverse hyperbolic tangent

```
IntegrationTest["7 Inverse hyperbolic functions\\2a Inverse hyperbolic tangent\\2.1 Inverse hyperbolic tangent functions"];
```


Testing Rubi on 800 integration problems...

Problem #436: Unable to integrate:

$$\left\{ \frac{\text{ArcTanh}[a + b x]^2}{x}, x, -3, 3 \right\}$$

$$-\frac{2}{3} \text{ArcTanh}[a + b x]^3 - \text{ArcTanh}[a + b x]^2 \text{Log}\left[\frac{2}{1 + a + b x}\right] + \text{ArcTanh}[a + b x]^2 \text{Log}\left[1 - \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] +$$

$$\text{ArcTanh}[a + b x]^2 \text{Log}\left[1 + \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + 2 \text{ArcTanh}[a + b x] \text{PolyLog}\left[2, -\frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + 2 \text{ArcTanh}[a + b x] \text{PolyLog}\left[2, \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] +$$

$$\text{ArcTanh}[a + b x] \text{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right] - 2 \text{PolyLog}\left[3, -\frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] - 2 \text{PolyLog}\left[3, \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2}{1 + a + b x}\right]$$

$$\text{Subst}\left[\text{Int}\left[\frac{\text{ArcTanh}[x]^2}{-a + x}, x\right], x, a + b x\right]$$

Test complete!

IntegrationTest["7 Inverse hyperbolic functions\\2a Inverse hyperbolic tangent\\2.2.1 x^m (c-a^2 c x^2)^p E^(n arctanh(a x))"];

Testing Rubi on 496 integration problems...

Test complete!

IntegrationTest["7 Inverse hyperbolic functions\\2a Inverse hyperbolic tangent\\2.2.2 Exponentials of inverse hyperbolic tangent functions"];

Testing Rubi on 881 integration problems...

Test complete!

■ Inverse hyperbolic cotangent

IntegrationTest["7 Inverse hyperbolic functions\\2b Inverse hyperbolic cotangent\\2.1 Inverse hyperbolic cotangent functions"];

Testing Rubi on 253 integration problems...

Problem #72: Unable to integrate:

$$\left\{ \frac{\text{ArcCoth}[a + b x]^2}{x}, x, -3, 3 \right\}$$

$$-\frac{2}{3} \text{ArcCoth}[a + b x]^3 - \text{ArcCoth}[a + b x]^2 \text{Log}\left[\frac{2}{1 + a + b x}\right] + \text{ArcCoth}[a + b x]^2 \text{Log}\left[1 - \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] +$$

$$\text{ArcCoth}[a + b x]^2 \text{Log}\left[1 + \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + 2 \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, -\frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + 2 \text{ArcCoth}[a + b x] \text{PolyLog}\left[2, \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] +$$

$$\text{ArcCoth}[a + b x] \text{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right] - 2 \text{PolyLog}\left[3, -\frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] - 2 \text{PolyLog}\left[3, \frac{\sqrt{\frac{1-a}{b}}(1 + a + b x)}{\sqrt{\frac{1+a}{b}}\sqrt{1 - (a + b x)^2}}\right] + \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2}{1 + a + b x}\right]$$

$$\text{Subst}\left[\text{Int}\left[\frac{\text{ArcCoth}[x]^2}{-a + x}, x\right], x, a + b x\right]$$

Test complete!

IntegrationTest["7 Inverse hyperbolic functions\\2b Inverse hyperbolic cotangent\\2.2 Exponentials of inverse hyperbolic cotangent functions"];

Testing Rubi on 935 integration problems...

Test complete!

■ Inverse hyperbolic secant

IntegrationTest["7 Inverse hyperbolic functions\\3a Inverse hyperbolic secant\\3.1 Inverse hyperbolic secant functions"];

Testing Rubi on 120 integration problems...

Problem #117: Unable to integrate:

$$\left\{ \frac{x \left(-1 + a e^{\text{ArcSech}[a x] x} \right)}{1 - a^2 x^2}, x, -10, 10 \right\}$$

$$-\frac{e^{\text{ArcSech}[a x] x}}{a}$$

$$-\frac{e^{\text{ArcSech}[a x] x}}{a} + \frac{2 \text{ArcTanh}\left[\sqrt{\frac{1-a x}{1+a x}}\right]}{a^2} - \frac{\text{Log}[x]}{a^2} + \frac{\text{Log}[1 - a^2 x^2]}{2 a^2} + \frac{\text{Int}\left[\frac{e^{\text{ArcSech}[a x]}}{1-a x}, x\right]}{2 a} + \frac{\text{Int}\left[\frac{e^{\text{ArcSech}[a x]}}{1+a x}, x\right]}{2 a}$$

Test complete!

■ Inverse hyperbolic cosecant

```
IntegrationTest["7 Inverse hyperbolic functions\3b Inverse hyperbolic cosecant\3.1 Inverse hyperbolic cosecant functions"];
```

Testing Rubi on 63 integration problems...

Test complete!

8 Special functions

```
IntegrationTest["8 Special functions\8.1 Error functions"];
```

Testing Rubi on 115 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\8.2 Fresnel integral functions"];
```

Testing Rubi on 126 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\8.3 Exponential integral functions"];
```

Testing Rubi on 149 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\8.4 Trig integral functions"];
```

Testing Rubi on 122 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\8.5 Hyperbolic integral functions"];
```

Testing Rubi on 122 integration problems...

Test complete!

IntegrationTest["8 Special functions\\8.6 Gamma functions"];

Testing Rubi on 168 integration problems...

Test complete!

IntegrationTest["8 Special functions\\8.7 Zeta function"];

Testing Rubi on 14 integration problems...

Test complete!

IntegrationTest["8 Special functions\\8.8 Polylogarithm function"];

Testing Rubi on 27 integration problems...

Test complete!

IntegrationTest["8 Special functions\\8.9 Product logarithm function"];

Testing Rubi on 396 integration problems...

Test complete!

Test suite statistics

PrintTestStatistics[];

* * * Indefinite Integration Test Suite Results * * *

Integration function: Rubi 4's user-defined Int function

Time and date of test: 17:46 07 March 2010

Mathematica version: 11.0.1 for Microsoft Windows (64-bit) (September 20, 2016)

Integration rules defined: 5976

Integration rules applied: 370259

Maximum steps used: 547 steps Optimal steps: 547 steps Integrand: $x^3 (c + a^2 c x^2)^{5/2} \text{ArcTan}[a x]^3$

Largest result size: 3205 leaves Optimal size: 3205 leaves Integrand: $\frac{x^3}{(a + b \text{Csc}[c + d \sqrt{x}])^2}$

Longest compute time: 33.6026 seconds Result size: 1326 leaves Integrand:
$$\frac{(c+d \operatorname{Tan}[e+f x])^{5/2} (A+B \operatorname{Tan}[e+f x]+C \operatorname{Tan}[e+f x]^2)}{(a+b \operatorname{Tan}[e+f x])^{7/2}}$$

File	Optimal	Nonident	Unintegrable	Timeout	Invalid	Total	Int/sec	Time
0 Independent test suites\Apostol Problems	175	0	0	0	0	175	39.10	4.7
0 Independent test suites\Moses Problems	113	0	0	0	0	113	31.50	3.7
0 Independent test suites\Timofeev Problems	687	17	1	0	0	705	5.50	179.5
0 Independent test suites\Charlwood Problems	42	5	2	0	1	50	4.17	114.9
0 Independent test suites\Stewart Problems	376	0	0	0	0	376	33.80	11.4
0 Independent test suites\Hearn Problems	278	0	6	0	0	284	9.36	30.7
0 Independent test suites\Jeffrey Problems	4	3	2	0	0	9	0.79	11.8
0 Independent test suites\Hebisch Problems	3	0	4	0	0	7	2.80	2.7
0 Independent test suites\Wester Problems	8	0	0	0	0	8	32.10	0.3
0 Independent test suites\Welz Problems	47	8	8	0	0	63	1.03	74.8
0 Independent test suites\Bronstein Problems	12	0	2	0	0	14	5.37	2.7
0 Independent test suites\Bondarenko Problems	32	3	0	0	0	35	1.45	30.7
1 Algebraic functions\1 Linear products\1.2 (a+b x)^m (c+d x)^n	1567	1	0	0	0	1568	4.72	336.5
1 Algebraic functions\1 Linear products\1.3 (a+b x)^m (c+d x)^n (e+f x)^p	3127	2	0	0	0	3129	3.92	804.7
1 Algebraic functions\1 Linear products\1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q	81	0	0	0	0	81	1.45	56.4
1 Algebraic functions\2 Quadratic products\2.2 (d+e x)^m (a+b x+c x^2)^p	2998	0	0	0	0	2998	2.47	1228.8
1 Algebraic functions\2 Quadratic products\2.3 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p	3198	1	0	0	0	3199	1.98	1632.2
1 Algebraic functions\2 Quadratic products\2.4 (a+b x+c x^2)^p (d+e x+f x^2)^q	168	0	0	0	0	168	1.26	134.4
1 Algebraic functions\2 Quadratic products\2.5 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q	227	0	0	0	0	227	0.52	443.9
1 Algebraic functions\2 Quadratic products\2.6 (a+b x+c x^2)^p (d+e x+f x^2)^q (A+B x+C x^2)	33	0	0	0	0	33	2.68	12.5
1 Algebraic functions\3 Binomial products\3.2 (c x)^m (a+b x^n)^p	3907	5	0	0	0	3912	9.33	424.2
1 Algebraic functions\3 Binomial products\3.3 (a+b x^n)^p (c+d x^n)^q	568	1	0	0	0	569	2.50	230.2
1 Algebraic functions\3 Binomial products\3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q	1795	0	0	0	0	1795	2.21	819.4
1 Algebraic functions\3 Binomial products\3.5 (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r	115	0	0	0	0	115	1.37	84.5
1 Algebraic functions\3 Binomial products\3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r	97	0	0	0	0	97	1.07	90.8
1 Algebraic functions\3 Binomial products\3.7 (c x)^m Pq(x) (a+b x^n)^p	561	0	0	0	0	561	1.71	331.6
1 Algebraic functions\3 Binomial products\3.9 (c x)^m (a x^j+b x^n)^p	454	0	0	0	0	454	4.61	99.5
1 Algebraic functions\3 Binomial products\3.10 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q	295	0	0	0	0	295	3.38	87.7
1 Algebraic functions\4 Trinomial products\4.2 (d x)^m (a+b x^n+c x^(2 n))^p	1761	1	0	0	0	1762	3.11	575.9
1 Algebraic functions\4 Trinomial products\4.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p	341	0	0	0	0	341	1.45	238.8
1 Algebraic functions\4 Trinomial products\4.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p	435	1	0	0	0	436	0.92	479.5
1 Algebraic functions\4 Trinomial products\4.5 (d x)^m Pq(x) (a+b x^n+c x^(2 n))^p	177	0	0	0	0	177	0.66	272.7
1 Algebraic functions\4 Trinomial products\4.7 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p	122	0	0	0	0	122	2.47	50.5
1 Algebraic functions\5 Miscellaneous\5.1 Rational functions	266	1	2	0	0	269	6.37	42.9
1 Algebraic functions\5 Miscellaneous\5.2 Algebraic functions	709	4	4	0	0	717	4.46	162.3
1 Algebraic functions\5 Miscellaneous\5.3 Expansion problems	110	0	0	0	0	110	6.27	17.7
1 Algebraic functions\5 Miscellaneous\5.4 Substitution problems	361	3	0	0	0	364	3.59	103.0
1 Algebraic functions\5 Miscellaneous\5.5 Piecewise constant extraction	66	0	0	0	0	66	5.77	11.6
1 Algebraic functions\5 Miscellaneous\5.6 Derivative integration problems	95	0	2	0	0	97	15.40	6.4
2 Exponentials\2.1 u (F^c(a+b x))^n	98	0	0	0	0	98	3.61	27.4
2 Exponentials\2.2 (c+d x)^m (F^g(e+f x))^n (a+b (F^g(e+f x)))^p	88	0	0	0	0	88	3.22	27.4
2 Exponentials\2.3 Exponential functions	722	0	4	0	0	726	5.26	139.1
3 Logarithms\3.1 u (a+b log(c (d (e+f x)^p)^q))^n	280	0	0	0	0	280	1.41	200.2
3 Logarithms\3.2 Logarithm functions	805	0	1	0	0	806	4.57	178.2
4 Trig functions\1a Sine\1.0 (a sin)^m (b trg)^n	538	0	0	0	0	538	11.50	47.4
4 Trig functions\1a Sine\1.1.1 (a+b sin)^n	64	0	0	0	0	64	8.73	7.4
4 Trig functions\1a Sine\1.1.2 (a cos)^p (a+b sin)^m	652	1	0	0	0	653	2.48	277.0

4 Trig functions\la Sine\1.1.3 (g tan)^p (a+b sin)^m	208	0	0	0	0	208	2.92	71.9
4 Trig functions\la Sine\1.2.1 (a+b sin)^m (c+d sin)^n	837	0	0	0	0	837	1.91	440.7
4 Trig functions\la Sine\1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n	1561	1	0	0	0	1562	2.06	766.0
4 Trig functions\la Sine\1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n	51	0	0	0	0	51	2.47	20.8
4 Trig functions\la Sine\1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin)	356	0	0	0	0	356	1.79	200.3
4 Trig functions\la Sine\1.4.1 (a+b sin)^m (A+B sin+C sin^2)	19	0	0	0	0	19	5.54	3.5
4 Trig functions\la Sine\1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2)	34	0	0	0	0	34	0.95	36.2
4 Trig functions\la Sine\1.8 (a+b sin)^m (c+d trig)^n	9	0	0	0	0	9	4.89	1.9
4 Trig functions\la Sine\1.9 trig^m (a+b sin^n+c sin^(2 n))^p	237	0	2	0	0	239	2.68	90.7
4 Trig functions\la Sine\1.10 (c+d x)^m (a+b sin)^n	178	0	0	0	0	178	3.05	59.1
4 Trig functions\la Sine\1.11 (e x)^m (a+b x^n)^p sin	113	0	0	0	0	113	0.88	130.5
4 Trig functions\la Sine\1.12 (e x)^m (a+b sin(c+d x^n))^p	209	0	0	0	0	209	4.00	53.3
4 Trig functions\la Sine\1.13 (d+e x)^m sin(a+b x+c x^2)^n	36	0	0	0	0	36	4.69	7.8
4 Trig functions\lb Cosine\1.0 (a cos)^m (b trg)^n	294	0	0	0	0	294	17.50	17.0
4 Trig functions\lb Cosine\1.1.1 (a+b cos)^n	62	0	0	0	0	62	10.20	6.2
4 Trig functions\lb Cosine\1.1.2 (g sin)^p (a+b cos)^m	88	0	0	0	0	88	1.23	72.8
4 Trig functions\lb Cosine\1.1.3 (g tan)^p (a+b cos)^m	22	0	0	0	0	22	5.90	3.8
4 Trig functions\lb Cosine\1.2.1 (a+b cos)^m (c+d cos)^n	936	0	0	0	0	936	2.81	335.2
4 Trig functions\lb Cosine\1.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n	4	0	0	0	0	4	1.15	3.5
4 Trig functions\lb Cosine\1.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n	1	0	0	0	0	1	4.93	0.2
4 Trig functions\lb Cosine\1.3.1 (a+b cos)^m (c+d cos)^n (A+B cos)	644	0	0	0	0	644	1.32	492.0
4 Trig functions\lb Cosine\1.4.1 (a+b cos)^m (A+B cos+C cos^2)	393	0	0	0	0	393	5.25	75.7
4 Trig functions\lb Cosine\1.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2)	1536	0	0	0	0	1536	0.98	1569.6
4 Trig functions\lb Cosine\1.8 (a+b cos)^m (c+d trig)^n	21	0	0	0	0	21	2.19	10.0
4 Trig functions\lb Cosine\1.9 trig^m (a+b cos^n+c cos^(2 n))^p	94	0	0	0	0	94	1.50	63.8
4 Trig functions\lb Cosine\1.10 (c+d x)^m (a+b cos)^n	189	0	0	0	0	189	4.03	47.3
4 Trig functions\lb Cosine\1.12 (e x)^m (a+b cos(c+d x^n))^p	93	0	0	0	0	93	5.03	18.7
4 Trig functions\lb Cosine\1.13 (d+e x)^m cos(a+b x+c x^2)^n	34	0	0	0	0	34	4.65	7.4
4 Trig functions\2a Tangent\2.0 (a trg)^m (b tan)^n	352	0	0	0	0	352	6.40	55.5
4 Trig functions\2a Tangent\2.1.2 (d sec)^m (a+b tan)^n	700	0	0	0	0	700	3.99	176.8
4 Trig functions\2a Tangent\2.1.3 (d sin)^m (a+b tan)^n	93	0	0	0	0	93	1.67	56.8
4 Trig functions\2a Tangent\2.2.1 (a+b tan)^m (c+d tan)^n	1342	0	0	0	0	1342	1.52	890.2
4 Trig functions\2a Tangent\2.3.1 (a+b tan)^m (c+d tan)^n (A+B tan)	859	0	0	0	0	859	1.35	641.8
4 Trig functions\2a Tangent\2.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2)	133	0	0	0	0	133	0.19	716.3
4 Trig functions\2a Tangent\2.9 trig^m (a+b tan^n+c tan^(2 n))^p	209	0	0	0	0	209	0.58	368.5
4 Trig functions\2a Tangent\2.10 (c+d x)^m (a+b tan)^n	62	0	1	0	0	63	2.40	26.4
4 Trig functions\2a Tangent\2.11 (e x)^m (a+b tan(c+d x^n))^p	72	0	0	0	0	72	1.79	40.7
4 Trig functions\2b Cotangent\2.0 (a trg)^m (b cot)^n	52	0	0	0	0	52	6.56	8.0
4 Trig functions\2b Cotangent\2.1.2 (d csc)^m (a+b cot)^n	23	0	0	0	0	23	9.39	2.5
4 Trig functions\2b Cotangent\2.1.3 (d cos)^m (a+b cot)^n	19	0	0	0	0	19	6.51	2.9
4 Trig functions\2b Cotangent\2.2.1 (a+b cot)^m (c+d cot)^n	106	0	0	0	0	106	1.34	79.8
4 Trig functions\2b Cotangent\2.9 trig^m (a+b cot^n+c cot^(2 n))^p	94	0	0	0	0	94	0.45	209.9
4 Trig functions\2b Cotangent\2.10 (c+d x)^m (a+b cot)^n	61	0	0	0	0	61	2.38	25.8
4 Trig functions\3a Secant\3.0 (a sec)^m (b trg)^n	283	0	0	0	0	283	11.40	24.9
4 Trig functions\3a Secant\3.1.2 (d sec)^n (a+b sec)^m	885	0	0	0	0	885	2.54	349.8
4 Trig functions\3a Secant\3.1.3 (d sin)^n (a+b sec)^m	306	0	0	0	0	306	1.79	172.6
4 Trig functions\3a Secant\3.1.4 (d tan)^n (a+b sec)^m	348	2	1	0	0	351	2.99	122.4
4 Trig functions\3a Secant\3.2.1 (a+b sec)^m (c+d sec)^n	243	0	0	0	0	243	1.81	135.1
4 Trig functions\3a Secant\3.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n	286	0	0	0	0	286	2.66	108.5
4 Trig functions\3a Secant\3.3.1 (a+b sec)^m (d sec)^n (A+B sec)	645	0	0	0	0	645	1.09	593.2
4 Trig functions\3a Secant\3.4.1 (a+b sec)^m (A+B sec+C sec^2)	70	0	0	0	0	70	11.20	6.4
4 Trig functions\3a Secant\3.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2)	1373	0	0	0	0	1373	1.14	1205.6

4 Trig functions\3a Secant\3.7 trig ^m (a+b sec ⁿ +c sec ^(2 n)) ^p	34	0	0	0	0	34	6.94	4.9
4 Trig functions\3a Secant\3.10 (c+d x) ^m (a+b sec) ⁿ	46	0	0	0	0	46	2.11	21.9
4 Trig functions\3a Secant\3.11 (e x) ^m (a+b sec(c+d x ⁿ)) ^p	83	0	0	0	0	83	1.35	61.9
4 Trig functions\3b Cosecant\3.0 (a csc) ^m (b trg) ⁿ	70	0	0	0	0	70	25.10	2.9
4 Trig functions\3b Cosecant\3.1.2 (d csc) ⁿ (a+b csc) ^m	59	0	0	0	0	59	4.36	13.7
4 Trig functions\3b Cosecant\3.1.3 (d cos) ⁿ (a+b csc) ^m	16	0	0	0	0	16	6.14	2.7
4 Trig functions\3b Cosecant\3.1.4 (d cot) ⁿ (a+b csc) ^m	23	0	0	0	0	23	5.71	4.1
4 Trig functions\3b Cosecant\3.3.1 (a+b csc) ^m (d csc) ⁿ (A+B csc)	24	0	0	0	0	24	10.80	2.3
4 Trig functions\3b Cosecant\3.4.2 (a+b csc) ^m (d csc) ⁿ (A+B csc+C csc ²)	1	0	0	0	0	1	4.27	0.3
4 Trig functions\3b Cosecant\3.7 trig ^m (a+b csc ⁿ +c csc ^(2 n)) ^p	27	0	0	0	0	27	6.23	4.4
4 Trig functions\3b Cosecant\3.11 (e x) ^m (a+b csc(c+d x ⁿ)) ^p	84	0	0	0	0	84	1.36	62.1
4 Trig functions\4 Miscellaneous\4.1 (c trig) ^m (d trig) ⁿ	250	0	0	0	0	250	10.60	23.7
4 Trig functions\4 Miscellaneous\4.2 trig ^m (a trig+b trig) ⁿ	288	6	0	0	0	294	3.09	99.1
4 Trig functions\4 Miscellaneous\4.3 (c+d x) ^m trig ⁿ trig ^p	397	0	0	0	0	397	1.77	226.7
4 Trig functions\4 Miscellaneous\4.4 x ^m (a+b trig ⁿ) ^p	9	0	0	0	0	9	1.51	6.1
4 Trig functions\4 Miscellaneous\4.5 x ^m trig(a+b log(c x ⁿ)) ^p	268	0	0	0	0	268	9.69	28.2
4 Trig functions\4 Miscellaneous\4.6 f ^a (a+b x+c x ²) trig(d+e x+f x ²) ⁿ	132	0	0	0	0	132	2.77	48.2
4 Trig functions\4 Miscellaneous\4.7 Trig functions	872	4	5	0	0	881	5.07	176.3
5 Inverse trig functions\1a Inverse sine\1.2 (d x) ^m (a+b arcsin(c x)) ⁿ	227	0	0	0	0	227	4.60	49.7
5 Inverse trig functions\1a Inverse sine\1.4a (f x) ^m (d-c ² d x ²) ^p (a+b arcsin(c x)) ⁿ	502	0	0	0	0	502	2.36	213.4
5 Inverse trig functions\1a Inverse sine\1.4b (f x) ^m (d+e x ²) ^p (a+b arcsin(c x)) ⁿ	109	0	0	0	0	109	1.15	95.2
5 Inverse trig functions\1a Inverse sine\1.5 Inverse sine functions	494	0	1	0	0	495	1.34	372.8
5 Inverse trig functions\1b Inverse cosine\1.2 (d x) ^m (a+b arccos(c x)) ⁿ	227	0	0	0	0	227	4.60	49.7
5 Inverse Trig functions\1b Inverse cosine\1.5 Inverse cosine functions	136	0	0	0	0	136	2.92	46.8
5 Inverse trig functions\2a Inverse tangent\2.1.1 (d x) ^m (a+b arctan(c x)) ⁿ	57	0	0	0	0	57	7.04	8.2
5 Inverse trig functions\2a Inverse tangent\2.1.2 (f x) ^m (d-c ² d x ²) ^p (a+b arctan(c x)) ⁿ	993	0	1	0	0	994	2.06	482.9
5 Inverse trig functions\2a Inverse tangent\2.1.3 (f x) ^m (d+e x ²) ^p (a+b arctan(c x)) ⁿ	131	1	0	0	0	132	1.67	80.0
5 Inverse trig functions\2a Inverse tangent\2.1.4 Inverse tangent functions	217	0	0	0	0	217	4.18	52.6
5 Inverse trig functions\2a Inverse tangent\2.2.1 x ^m (c+a ² c x ²) ^p E ^{(n arctan(a x))}	142	0	0	0	0	142	7.36	19.4
5 Inverse trig functions\2a Inverse tangent\2.2.2 Exponentials of inverse tangent	242	0	0	0	0	242	3.06	79.8
5 Inverse trig functions\2b Inverse cotangent\2.1 Inverse cotangent functions	228	0	0	0	0	228	2.65	86.9
5 Inverse trig functions\2b Inverse cotangent\2.2 Exponentials of inverse cotangent	12	0	0	0	0	12	6.01	2.0
5 Inverse trig functions\3a Inverse secant\3.1 Inverse secant functions	166	0	0	0	0	166	2.07	80.8
5 Inverse trig functions\3b Inverse cosecant\3.1 Inverse cosecant functions	87	0	0	0	0	87	7.57	11.7
6 Hyperbolic functions\1a Hyperbolic sine\1.1 (c+d x) ^m (a+b sinh) ⁿ	186	0	0	0	0	186	2.72	69.1
6 Hyperbolic functions\1a Hyperbolic sine\1.3 (e x) ^m (a+b sinh(c+d x ⁿ)) ^p	96	0	0	0	0	96	5.64	17.3
6 Hyperbolic functions\1a Hyperbolic sine\1.4 (d+e x) ^m sinh(a+b x+c x ²) ⁿ	33	0	0	0	0	33	5.58	6.0
6 Hyperbolic functions\1a Hyperbolic sine\1.5 Hyperbolic sine functions	694	0	0	0	0	694	3.42	205.2
6 Hyperbolic functions\1b Hyperbolic cosine\1.1 (c+d x) ^m (a+b cosh) ⁿ	183	0	0	0	0	183	3.32	55.8
6 Hyperbolic functions\1b Hyperbolic cosine\1.2 (e x) ^m (a+b x ⁿ) ^p cosh	113	0	0	0	0	113	0.79	143.4
6 Hyperbolic functions\1b Hyperbolic cosine\1.3 (e x) ^m (a+b cosh(c+d x ⁿ)) ^p	62	0	0	0	0	62	5.47	11.6
6 Hyperbolic functions\1b Hyperbolic cosine\1.4 (d+e x) ^m cosh(a+b x+c x ²) ⁿ	33	0	0	0	0	33	5.54	6.0
6 Hyperbolic functions\1b Hyperbolic cosine\1.5 Hyperbolic cosine functions	407	0	0	0	0	407	4.64	89.3
6 Hyperbolic functions\2a Hyperbolic tangent\2.1 (c+d x) ^m (a+b tanh) ⁿ	79	0	0	0	0	79	2.75	28.9
6 Hyperbolic functions\2a Hyperbolic tangent\2.2 Hyperbolic tangent functions	461	0	0	0	0	461	3.61	128.7
6 Hyperbolic functions\2b Hyperbolic cotangent\2.1 (c+d x) ^m (a+b coth) ⁿ	61	0	0	0	0	61	2.31	26.6
6 Hyperbolic functions\2b Hyperbolic cotangent\2.2 Hyperbolic cotangent functions	228	0	0	0	0	228	6.24	36.9
6 Hyperbolic functions\3a Hyperbolic secant\3.1 (c+d x) ^m (a+b sech) ⁿ	16	0	0	0	0	16	7.33	2.2
6 Hyperbolic functions\3a Hyperbolic secant\3.2 (e x) ^m (a+b sech(c+d x ⁿ)) ^p	84	0	0	0	0	84	1.47	57.5
6 Hyperbolic functions\3a Hyperbolic secant\3.3 Hyperbolic secant functions	394	0	0	0	0	394	4.77	83.5
6 Hyperbolic functions\3b Hyperbolic cosecant\3.1 (c+d x) ^m (a+b csch) ⁿ	16	0	0	0	0	16	6.22	2.6
6 Hyperbolic functions\3b Hyperbolic cosecant\3.2 (e x) ^m (a+b csch(c+d x ⁿ)) ^p	83	0	0	0	0	83	1.53	54.6

6 Hyperbolic functions\3b Hyperbolic cosecant\3.3 Hyperbolic cosecant functions	195	0	0	0	0	195	6.99	28.3
6 Hyperbolic functions\4 Miscellaneous\4.1 Hyperbolic functions	1055	0	0	0	0	1055	4.38	243.3
7 Inverse hyperbolic functions\1a Inverse hyperbolic sine\1.2 (d x)^m (a+b arcsinh(c x))^n	156	0	0	0	0	156	3.60	43.6
7 Inverse hyperbolic functions\1a Inverse hyperbolic sine\1.4a (f x)^m (d+c^2 d x^2)^p (a+b arcsinh(c x))^n	469	0	0	0	0	469	2.37	198.8
7 Inverse hyperbolic functions\1a Inverse hyperbolic sine\1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n	58	0	0	0	0	58	1.95	30.0
7 Inverse hyperbolic functions\1a Inverse hyperbolic sine\1.5 Inverse hyperbolic sine functions	370	0	1	0	0	371	2.13	175.5
7 Inverse hyperbolic functions\1b Inverse hyperbolic cosine\1.2 (d x)^m (a+b arccosh(c x))^n	166	0	0	0	0	166	2.71	61.4
7 Inverse hyperbolic functions\1b Inverse hyperbolic cosine\1.4a (f x)^m (d-c^2 d x^2)^p (a+b arccosh(c x))^n	453	0	0	0	0	453	1.61	282.2
7 Inverse hyperbolic functions\1b Inverse hyperbolic cosine\1.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n	109	0	0	0	0	109	1.00	109.4
7 Inverse hyperbolic functions\1b Inverse hyperbolic cosine\1.5 Inverse hyperbolic cosine functions	293	0	0	0	0	293	1.69	173.9
7 Inverse hyperbolic functions\2a Inverse hyperbolic tangent\2.1 Inverse hyperbolic tangent functions	799	0	1	0	0	800	4.10	196.5
7 Inverse hyperbolic functions\2a Inverse hyperbolic tangent\2.2.1 x^m (c-a^2 c x^2)^p E^(n arctanh(a x))	496	0	0	0	0	496	4.56	109.3
7 Inverse hyperbolic functions\2a Inverse hyperbolic tangent\2.2.2 Exponentials of inverse hyperbolic tangent functions	881	0	0	0	0	881	3.89	227.5
7 Inverse hyperbolic functions\2b Inverse hyperbolic cotangent\2.1 Inverse hyperbolic cotangent functions	252	0	1	0	0	253	3.64	70.2
7 Inverse hyperbolic functions\2b Inverse hyperbolic cotangent\2.2 Exponentials of inverse hyperbolic cotangent functions	935	0	0	0	0	935	3.46	271.8
7 Inverse hyperbolic functions\3a Inverse hyperbolic secant\3.1 Inverse hyperbolic secant functions	119	0	1	0	0	120	3.46	35.1
7 Inverse hyperbolic functions\3b Inverse hyperbolic cosecant\3.1 Inverse hyperbolic cosecant functions	63	0	0	0	0	63	7.44	8.5
8 Special functions\8.1 Error functions	115	0	0	0	0	115	4.98	23.4
8 Special functions\8.2 Fresnel integral functions	126	0	0	0	0	126	5.05	25.3
8 Special functions\8.3 Exponential integral functions	149	0	0	0	0	149	5.70	26.6
8 Special functions\8.4 Trig integral functions	122	0	0	0	0	122	2.77	44.4
8 Special functions\8.5 Hyperbolic integral functions	122	0	0	0	0	122	2.81	44.0
8 Special functions\8.6 Gamma functions	168	0	0	0	0	168	8.20	20.8
8 Special functions\8.7 Zeta function	14	0	0	0	0	14	23.00	0.6
8 Special functions\8.8 Polylogarithm function	27	0	0	0	0	27	5.44	5.0
8 Special functions\8.9 Product logarithm function	396	0	0	0	0	396	6.43	61.9
Totals	63523	71	53	0	1	63648	2.41	26840.0
Percentages	99.80%	0.11%	0.08%	0.00%	0.00%	100.00%		