

# Rubi 4.14.2 IntegrationTest Results

This file shows the integration problems for which **Rubi** either failed to get an antiderivative *or* returned a nonoptimal one. Each entry consists of the following items:

1. a phrase indicating the type of deficiency;
2. a list of the integrand, integration variable, number of rule applications desired, and number required;
3. the optimal antiderivative as contained in the test suite; and
4. the antiderivative as generated by **Rubi**.

## 0 Independent test suites

```
IntegrationTest["0 Independent test suites\\Apostol Problems"];
```

Testing Rubi on 175 integration problems...

Test complete!

```
IntegrationTest["0 Independent test suites\\Moses Problems"];
```

Testing Rubi on 113 integration problems...

Test complete!

```
IntegrationTest["0 Independent test suites\\Timofeev Problems"];
```

Testing Rubi on 705 integration problems...

Problem #222: Unable to integrate:

$$\left\{ \frac{\sqrt{1-x} x (1+x)^{2/3}}{-(1-x)^{5/6} (1+x)^{1/3} + (1-x)^{2/3} \sqrt{1+x}}, x, -4, 4 \right\}$$
$$-\frac{1}{12} (1-3x) (1-x)^{2/3} (1+x)^{1/3} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4} (1-x) (3+x) + \frac{1}{12} (1-x)^{1/3} (1+x)^{2/3} (1+3x) + \frac{1}{12} (1-x)^{1/6} (1+x)^{5/6} (2+3x) -$$
$$\frac{1}{12} (1-x)^{5/6} (1+x)^{1/6} (10+3x) + \frac{1}{6} \text{ArcTan} \left[ \frac{(1+x)^{1/6}}{(1-x)^{1/6}} \right] - \frac{4 \text{ArcTan} \left[ \frac{(1-x)^{1/3} - 2(1+x)^{1/3}}{\sqrt{3} (1-x)^{1/3}} \right]}{3\sqrt{3}} - \frac{5}{6} \text{ArcTan} \left[ \frac{(1-x)^{1/3} - (1+x)^{1/3}}{(1-x)^{1/6} (1+x)^{1/6}} \right] + \frac{\text{ArcTanh} \left[ \frac{\sqrt{3} (1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} + (1+x)^{1/3}} \right]}{6\sqrt{3}}$$

$$3 \text{ Subst} \left[ \text{CannotIntegrate} \left[ \frac{x^3}{(2-x^3)^{1/3} - (-x^3(-2+x^3))^{1/6}}, x \right], x, (1+x)^{1/3} \right] - 3 \text{ Subst} \left[ \text{CannotIntegrate} \left[ \frac{x^6}{(2-x^3)^{1/3} - (-x^3(-2+x^3))^{1/6}}, x \right], x, (1+x)^{1/3} \right]$$

Problem #226: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{((-1+x)^2(1+x))^{1/3}}, x, -3, 3 \right\}$$

$$\sqrt{3} \text{ ArcTan} \left[ \frac{1 + \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}} \right] - \frac{1}{2} \text{ Log}[1+x] - \frac{3}{2} \text{ Log} \left[ 1 - \frac{-1+x}{((-1+x)^2(1+x))^{1/3}} \right]$$

$$- \frac{\sqrt{3}(-1+x)^{2/3}(1+x)^{1/3} \text{ ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2(1+x)^{1/3}}{\sqrt{3}(-1+x)^{1/3}} \right]}{(1-x-x^2+x^3)^{1/3}} - \frac{(-1+x)^{2/3}(1+x)^{1/3} \text{ Log}[-1+x]}{2(1-x-x^2+x^3)^{1/3}} - \frac{3(-1+x)^{2/3}(1+x)^{1/3} \text{ Log} \left[ -1 + \frac{(1+x)^{1/3}}{(-1+x)^{1/3}} \right]}{2(1-x-x^2+x^3)^{1/3}}$$

Problem #228: Valid but suboptimal antiderivative:

$$\left\{ \frac{((-1+x)^2(1+x))^{1/3}}{x^2}, x, -6, 6 \right\}$$

$$- \frac{((-1+x)^2(1+x))^{1/3}}{x} - \frac{\text{ArcTan} \left[ \frac{1 - \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3}} - \sqrt{3} \text{ ArcTan} \left[ \frac{1 + \frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}} \right] +$$

$$\frac{\text{Log}[x]}{6} - \frac{2}{3} \text{ Log}[1+x] - \frac{3}{2} \text{ Log} \left[ 1 - \frac{-1+x}{((-1+x)^2(1+x))^{1/3}} \right] - \frac{1}{2} \text{ Log} \left[ 1 + \frac{-1+x}{((-1+x)^2(1+x))^{1/3}} \right]$$

$$- \frac{(1-x-x^2+x^3)^{1/3}}{x} - \frac{(1-x-x^2+x^3)^{1/3} \text{ ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2(-1+x)^{1/3}}{\sqrt{3}(1+x)^{1/3}} \right]}{\sqrt{3}(-1+x)^{2/3}(1+x)^{1/3}} - \frac{\sqrt{3}(1-x-x^2+x^3)^{1/3} \text{ ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2(-1+x)^{1/3}}{\sqrt{3}(1+x)^{1/3}} \right]}{(-1+x)^{2/3}(1+x)^{1/3}} + \frac{(1-x-x^2+x^3)^{1/3} \text{ Log}[x]}{6(-1+x)^{2/3}(1+x)^{1/3}} -$$

$$\frac{(1-x-x^2+x^3)^{1/3} \text{ Log}[1+x]}{2(-1+x)^{2/3}(1+x)^{1/3}} - \frac{3(1-x-x^2+x^3)^{1/3} \text{ Log} \left[ -1 + \frac{(-1+x)^{1/3}}{(1+x)^{1/3}} \right]}{2(-1+x)^{2/3}(1+x)^{1/3}} - \frac{(1-x-x^2+x^3)^{1/3} \text{ Log}[-(-1+x)^{1/3} - (1+x)^{1/3}]}{2(-1+x)^{2/3}(1+x)^{1/3}}$$

Problem #232: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{(9+3x-5x^2+x^3)^{1/3}}, x, -2, 2 \right\}$$

$$\sqrt{3} \text{ ArcTan} \left[ \frac{1 + \frac{2(-3+x)}{(9+3x-5x^2+x^3)^{1/3}}}{\sqrt{3}} \right] - \frac{1}{2} \text{ Log}[1+x] - \frac{3}{2} \text{ Log} \left[ 1 - \frac{-3+x}{(9+3x-5x^2+x^3)^{1/3}} \right]$$

$$-\frac{\sqrt{3} (-3+x)^{2/3} (1+x)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1+x)^{1/3}}{\sqrt{3}(-3+x)^{1/3}}\right]}{(9+3x-5x^2+x^3)^{1/3}} - \frac{(-3+x)^{2/3} (1+x)^{1/3} \operatorname{Log}[-3+x]}{2(9+3x-5x^2+x^3)^{1/3}} - \frac{3(-3+x)^{2/3} (1+x)^{1/3} \operatorname{Log}\left[-1 + \frac{(1+x)^{1/3}}{(-3+x)^{1/3}}\right]}{2(9+3x-5x^2+x^3)^{1/3}}$$

Problem #288: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{x + \sqrt{1+x+x^2}}, x, -3, 3 \right\}$$

$$-x + \sqrt{1+x+x^2} - \frac{3}{2} \operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right] + 2 \operatorname{Log}\left[x + \sqrt{1+x+x^2}\right]$$

$$\frac{3}{2(1+2(x+\sqrt{1+x+x^2}))} + 2 \operatorname{Log}\left[x + \sqrt{1+x+x^2}\right] - \frac{3}{2} \operatorname{Log}\left[1+2(x+\sqrt{1+x+x^2})\right]$$

Problem #306: Valid but suboptimal antiderivative:

$$\left\{ (x(1-x^2))^{1/3}, x, -12, 12 \right\}$$

$$\frac{1}{2} x (x(1-x^2))^{1/3} + \frac{\operatorname{ArcTan}\left[\frac{2x-(x(1-x^2))^{1/3}}{\sqrt{3}(x(1-x^2))^{1/3}}\right]}{2\sqrt{3}} + \frac{\operatorname{Log}[x]}{12} - \frac{1}{4} \operatorname{Log}\left[x + (x(1-x^2))^{1/3}\right]$$

$$\frac{1}{2} x (x-x^3)^{1/3} - \frac{x^{2/3} (1-x^2)^{2/3} \operatorname{ArcTan}\left[\frac{1-\frac{2x^{2/3}}{(1-x^2)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}(x-x^3)^{2/3}} + \frac{x^{2/3} (1-x^2)^{2/3} \operatorname{Log}\left[1 + \frac{x^{4/3}}{(1-x^2)^{2/3}} - \frac{x^{2/3}}{(1-x^2)^{1/3}}\right]}{12(x-x^3)^{2/3}} - \frac{x^{2/3} (1-x^2)^{2/3} \operatorname{Log}\left[1 + \frac{x^{2/3}}{(1-x^2)^{1/3}}\right]}{6(x-x^3)^{2/3}}$$

Problem #401: Valid but suboptimal antiderivative:

$$\left\{ \frac{\operatorname{Tan}[x]}{(-1 + \sqrt{\operatorname{Tan}[x]})^2}, x, -19, 19 \right\}$$

$$-\frac{x}{2} + \frac{\operatorname{ArcTan}\left[\frac{1-\operatorname{Tan}[x]}{\sqrt{2}\sqrt{\operatorname{Tan}[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTanh}\left[\frac{1+\operatorname{Tan}[x]}{\sqrt{2}\sqrt{\operatorname{Tan}[x]}}\right]}{\sqrt{2}} + \frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x]] + \operatorname{Log}\left[1 - \sqrt{\operatorname{Tan}[x]}\right] + \frac{1}{1 - \sqrt{\operatorname{Tan}[x]}}$$

$$-\frac{x}{2} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{\operatorname{Tan}[x]}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{\operatorname{Tan}[x]}\right]}{\sqrt{2}} + \frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x]] +$$

$$\operatorname{Log}\left[1 - \sqrt{\operatorname{Tan}[x]}\right] - \frac{\operatorname{Log}\left[1 - \sqrt{2}\sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x]\right]}{2\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \sqrt{2}\sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x]\right]}{2\sqrt{2}} + \frac{1}{1 - \sqrt{\operatorname{Tan}[x]}}$$

Problem #411: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[x]^3 (\cos[2x] - 3 \tan[x])}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}}, x, -101, 101 \right\}$$

$$\frac{33}{32} \operatorname{ArcTanh}\left[\frac{1}{2} \sec[x] \sqrt{\sin[2x]}\right] - \frac{9 \cos[x]}{16 \sqrt{\sin[2x]}} - \frac{5 \cos[x] \cot[x]}{24 \sqrt{\sin[2x]}} + \frac{\cos[x] \cot[x]^2}{20 \sqrt{\sin[2x]}}$$

$$- \left( \left( 33 i \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{1}{2}(-1 - \sqrt{5}), i \operatorname{ArcSinh}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{1 - \tan\left[\frac{x}{2}\right]} \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 + \tan\left[\frac{x}{2}\right]} \right) / \right.$$

$$\left. \left( 32 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) \right) -$$

$$\left( 33 i \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), i \operatorname{ArcSinh}\left[\sqrt{\tan\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{1 - \tan\left[\frac{x}{2}\right]} \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 + \tan\left[\frac{x}{2}\right]} \right) /$$

$$\left( 32 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)} \right) + \frac{69 \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - \tan\left[\frac{x}{2}\right]}}{\sqrt{2}}\right], 2\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2}}{80 \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}} -$$

$$\frac{15(2 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - \tan\left[\frac{x}{2}\right]}}{\sqrt{2}}\right], 2\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2}}{8(3 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}} -$$

$$\frac{3(3 - \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - \tan\left[\frac{x}{2}\right]}}{\sqrt{2}}\right], 2\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2}}{32(1 - \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}} -$$

$$\frac{15(2 + \sqrt{5}) \cos\left[\frac{x}{2}\right]^2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - \tan\left[\frac{x}{2}\right]}}{\sqrt{2}}\right], 2\right] \sqrt{\tan\left[\frac{x}{2}\right]} \sqrt{1 - \tan\left[\frac{x}{2}\right]^2}}{8(3 + \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}} -$$

$$8(3 + \sqrt{5}) \sqrt{\cos\left[\frac{x}{2}\right]^3 \sin\left[\frac{x}{2}\right] \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}$$





$$\begin{aligned}
& -\sqrt{2} \operatorname{Log} \left[ \operatorname{Cos}[x] + \operatorname{Sin}[x] - \sqrt{2} \operatorname{Sec}[x] \sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]} \right] - \\
& \frac{\operatorname{ArcSin} \left[ \operatorname{Cos}[x] - \operatorname{Sin}[x] \right] \operatorname{Cos}[x] \sqrt{\operatorname{Sin}[2x]}}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} - \frac{\operatorname{ArcTanh} \left[ \operatorname{Sin}[x] \right] \operatorname{Cos}[x] \sqrt{\operatorname{Sin}[2x]}}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} - \frac{\operatorname{Sin}[2x]}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} \\
& - \frac{2 \operatorname{Cos}[x] \operatorname{Sin}[x]}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} - \frac{\sqrt{2} \operatorname{ArcSinh} \left[ \operatorname{Tan}[x] \right] \sqrt{\operatorname{Cos}[x] \operatorname{Sin}[x]}}{\sqrt{\operatorname{Sec}[x]^2} \sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} - \\
& \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 - \sqrt{2} \sqrt{\operatorname{Tan}[x]} \right] \operatorname{Cos}[x]^2 \sqrt{\operatorname{Tan}[x]}}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} + \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 + \sqrt{2} \sqrt{\operatorname{Tan}[x]} \right] \operatorname{Cos}[x]^2 \sqrt{\operatorname{Tan}[x]}}{\sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} - \\
& \frac{\operatorname{Cos}[x]^2 \operatorname{Log} \left[ 1 - \sqrt{2} \sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x] \right] \sqrt{\operatorname{Tan}[x]}}{\sqrt{2} \sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}} + \frac{\operatorname{Cos}[x]^2 \operatorname{Log} \left[ 1 + \sqrt{2} \sqrt{\operatorname{Tan}[x]} + \operatorname{Tan}[x] \right] \sqrt{\operatorname{Tan}[x]}}{\sqrt{2} \sqrt{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]}}
\end{aligned}$$

Problem #427: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[3x]}{-\sqrt{-1+8 \operatorname{Cos}[x]^2} + \sqrt{3 \operatorname{Cos}[x]^2 - \operatorname{Sin}[x]^2}}, x, -51, 51 \right\} \\
& \frac{5 \operatorname{ArcSin} \left[ 2 \sqrt{\frac{2}{7}} \operatorname{Sin}[x] \right]}{4 \sqrt{2}} + \frac{3}{4} \operatorname{ArcSin} \left[ \frac{2 \operatorname{Sin}[x]}{\sqrt{3}} \right] - \frac{3}{4} \operatorname{ArcTan} \left[ \frac{\operatorname{Sin}[x]}{\sqrt{-1+4 \operatorname{Cos}[x]^2}} \right] - \\
& \frac{3}{4} \operatorname{ArcTan} \left[ \frac{\operatorname{Sin}[x]}{\sqrt{-1+8 \operatorname{Cos}[x]^2}} \right] - \frac{1}{2} \sqrt{-1+4 \operatorname{Cos}[x]^2} \operatorname{Sin}[x] - \frac{1}{2} \sqrt{-1+8 \operatorname{Cos}[x]^2} \operatorname{Sin}[x] \\
& \frac{5 \operatorname{ArcSin} \left[ 2 \sqrt{\frac{2}{7}} \operatorname{Sin}[x] \right]}{4 \sqrt{2}} + \frac{3}{4} \operatorname{ArcSin} \left[ \frac{2 \operatorname{Sin}[x]}{\sqrt{3}} \right] + \frac{3}{8} \operatorname{ArcTan} \left[ \frac{7-8 \operatorname{Sin}[x]}{\sqrt{7-8 \operatorname{Sin}[x]^2}} \right] - \frac{3}{8} \operatorname{ArcTan} \left[ \frac{7+8 \operatorname{Sin}[x]}{\sqrt{7-8 \operatorname{Sin}[x]^2}} \right] + \\
& \frac{3}{8} \operatorname{ArcTan} \left[ \frac{3-4 \operatorname{Sin}[x]}{\sqrt{3-4 \operatorname{Sin}[x]^2}} \right] - \frac{3}{8} \operatorname{ArcTan} \left[ \frac{3+4 \operatorname{Sin}[x]}{\sqrt{3-4 \operatorname{Sin}[x]^2}} \right] - \frac{1}{2} \operatorname{Sin}[x] \sqrt{7-8 \operatorname{Sin}[x]^2} - \frac{1}{2} \operatorname{Sin}[x] \sqrt{3-4 \operatorname{Sin}[x]^2}
\end{aligned}$$

Problem #447: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Sec}[x]^2 (-\operatorname{Cos}[2x] + 2 \operatorname{Tan}[x]^2)}{(\operatorname{Tan}[x] \operatorname{Tan}[2x])^{3/2}}, x, -20, 20 \right\} \\
& 2 \operatorname{ArcTanh} \left[ \frac{\operatorname{Tan}[x]}{\sqrt{\operatorname{Tan}[x] \operatorname{Tan}[2x]}} \right] - \frac{11 \operatorname{ArcTanh} \left[ \frac{\sqrt{2} \operatorname{Tan}[x]}{\sqrt{\operatorname{Tan}[x] \operatorname{Tan}[2x]}} \right]}{4 \sqrt{2}} + \frac{\operatorname{Tan}[x]}{2 (\operatorname{Tan}[x] \operatorname{Tan}[2x])^{3/2}} + \frac{2 \operatorname{Tan}[x]^3}{3 (\operatorname{Tan}[x] \operatorname{Tan}[2x])^{3/2}} + \frac{3 \operatorname{Tan}[x]}{4 \sqrt{\operatorname{Tan}[x] \operatorname{Tan}[2x]}}
\end{aligned}$$

$$\frac{3 \tan [x]}{4 \sqrt{2} \sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}}} + \frac{\cot [x] (1-\tan [x]^2)}{4 \sqrt{2} \sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}}} + \frac{\tan [x] (1-\tan [x]^2)}{3 \sqrt{2} \sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}}} - \frac{11 \operatorname{ArcTan}\left[\sqrt{-1+\tan [x]^2}\right] \tan [x]}{4 \sqrt{2} \sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}} \sqrt{-1+\tan [x]^2}} + \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-1+\tan [x]^2}}{\sqrt{2}}\right] \tan [x]}{\sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}} \sqrt{-1+\tan [x]^2}}$$

Problem #455: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin [x]^6 \tan [x]}{\cos [2 x]^{3/4}}, x, -14, 14 \right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{\cos [2 x]}}{\sqrt{2} \cos [2 x]^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{1+\sqrt{\cos [2 x]}}{\sqrt{2} \cos [2 x]^{1/4}}\right]}{\sqrt{2}} + \frac{71}{45} \cos [2 x]^{1/4} + \frac{13}{45} \cos [2 x]^{1/4} \sin [x]^2 + \frac{1}{9} \cos [2 x]^{1/4} \sin [x]^4$$

$$\frac{\operatorname{ArcTan}\left[1-\sqrt{2}(-1+2 \cos [x]^2)^{1/4}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1+\sqrt{2}(-1+2 \cos [x]^2)^{1/4}\right]}{\sqrt{2}} + \frac{7}{4}(-1+2 \cos [x]^2)^{1/4} - \frac{1}{5}(-1+2 \cos [x]^2)^{5/4} +$$

$$\frac{1}{36}(-1+2 \cos [x]^2)^{9/4} + \frac{\operatorname{Log}\left[1-\sqrt{2}(-1+2 \cos [x]^2)^{1/4}+\sqrt{-1+2 \cos [x]^2}\right]}{2 \sqrt{2}} - \frac{\operatorname{Log}\left[1+\sqrt{2}(-1+2 \cos [x]^2)^{1/4}+\sqrt{-1+2 \cos [x]^2}\right]}{2 \sqrt{2}}$$

Problem #456: Valid but suboptimal antiderivative:

$$\left\{ \sqrt{\tan [x] \tan [2 x]}, x, -6, 6 \right\}$$

$$-\operatorname{ArcTanh}\left[\frac{\tan [x]}{\sqrt{\tan [x] \tan [2 x]}}\right]$$

$$\operatorname{ArcTan}\left[\frac{\sqrt{-1+\tan [x]^2}}{\sqrt{2}}\right] \cot [x] \sqrt{\frac{\tan [x]^2}{1-\tan [x]^2}} \sqrt{-1+\tan [x]^2}$$

Problem #493: Unable to integrate:

$$\left\{ \frac{x^2}{(x \cos [x] - \sin [x])^2}, x, -1, 0 \right\}$$

$$\frac{\cos [x] + x \sin [x]}{x \cos [x] - \sin [x]}$$

$$x \cos [x] - \sin [x]$$

$$\text{CannotIntegrate}\left[\frac{x^2}{(x \cos [x] - \sin [x])^2}, x\right]$$



Problem #567: Valid but suboptimal antiderivative:

$$\{e^{x/2} x^2 \cos[x]^3, x, -31, 31\}$$

$$\begin{aligned} & -\frac{132}{125} e^{x/2} \cos[x] + \frac{18}{25} e^{x/2} x \cos[x] + \frac{48}{185} e^{x/2} x^2 \cos[x] + \frac{2}{37} e^{x/2} x^2 \cos[x]^3 - \frac{428 e^{x/2} \cos[3x]}{50653} + \frac{70 e^{x/2} x \cos[3x]}{1369} - \\ & \frac{24}{125} e^{x/2} \sin[x] - \frac{24}{25} e^{x/2} x \sin[x] + \frac{96}{185} e^{x/2} x^2 \sin[x] + \frac{12}{37} e^{x/2} x^2 \cos[x]^2 \sin[x] - \frac{792 e^{x/2} \sin[3x]}{50653} - \frac{24 e^{x/2} x \sin[3x]}{1369} \\ & - \frac{6687696 e^{x/2} \cos[x]}{6331625} + \frac{24792 e^{x/2} x \cos[x]}{34225} + \frac{48}{185} e^{x/2} x^2 \cos[x] + \frac{16 e^{x/2} \cos[x]^3}{50653} - \frac{8 e^{x/2} x \cos[x]^3}{1369} + \\ & \frac{2}{37} e^{x/2} x^2 \cos[x]^3 - \frac{432 e^{x/2} \cos[3x]}{50653} + \frac{72 e^{x/2} x \cos[3x]}{1369} - \frac{1218672 e^{x/2} \sin[x]}{6331625} - \frac{32556 e^{x/2} x \sin[x]}{34225} + \frac{96}{185} e^{x/2} x^2 \sin[x] + \\ & \frac{96 e^{x/2} \cos[x]^2 \sin[x]}{50653} - \frac{48 e^{x/2} x \cos[x]^2 \sin[x]}{1369} + \frac{12}{37} e^{x/2} x^2 \cos[x]^2 \sin[x] - \frac{816 e^{x/2} \sin[3x]}{50653} - \frac{12 e^{x/2} x \sin[3x]}{1369} \end{aligned}$$

Problem #592: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cosh[x] (-\cosh[2x] + \tanh[x])}{\sqrt{\sinh[2x]} (\sinh[x]^2 + \sinh[2x])}, x, -52, 52 \right\}$$

$$\begin{aligned} & \sqrt{2} \operatorname{ArcTan}\left[\operatorname{Sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{1}{6} \operatorname{ArcTan}\left[\frac{\sinh[x]}{\sqrt{\sinh[2x]}}\right] - \frac{1}{3} \sqrt{2} \operatorname{ArcTanh}\left[\operatorname{Sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{\cosh[x]}{\sqrt{\sinh[2x]}} \\ & \left( (-1)^{1/4} \cosh\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{2}{i-\sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{\tanh\left[\frac{x}{2}\right]} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \right) / \\ & \left( \sqrt{\sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) + \\ & \left( (-1)^{1/4} \cosh\left[\frac{x}{2}\right]^2 \operatorname{EllipticPi}\left[\frac{2}{i+\sqrt{3}}, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \sqrt{\tanh\left[\frac{x}{2}\right]} \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \right) / \\ & \left( \sqrt{\sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) - \frac{i \operatorname{Csch}\left[\frac{x}{2}\right] \operatorname{Sech}\left[\frac{x}{2}\right] \sqrt{i \sinh[2x]} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)}}{2 \sqrt{\sinh[2x]}} - \\ & \left( 4 (-1)^{3/4} \operatorname{EllipticPi}\left[-i, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \right. \\ & \left. \sqrt{-i\left(i + \tanh\left[\frac{x}{2}\right]\right)} \sqrt{i \cosh\left[\frac{x}{2}\right]^3 \sinh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) / \left( \sqrt{\sinh[2x]} \sqrt{\tanh\left[\frac{x}{2}\right] \left(1 + \tanh\left[\frac{x}{2}\right]^2\right)} \right) + \\ & \left( 4 (-1)^{3/4} \operatorname{EllipticPi}\left[i, -\operatorname{ArcSin}\left[(-1)^{3/4} \sqrt{\tanh\left[\frac{x}{2}\right]}\right], -1\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{i \sinh[2x]} \sqrt{-i\left(i - \tanh\left[\frac{x}{2}\right]\right)} \right. \end{aligned}$$



Problem #695: Valid but suboptimal antiderivative:

$$\left\{ \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right], x, -8, 8 \right\}$$

$$-\frac{\sqrt{2} a \sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}} + (a+x) \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

$$-\frac{2 a \sqrt{\frac{a}{a+x}} \sqrt{\frac{-a-x}{a+x}} \sqrt{a+x}}{\sqrt{\frac{a}{1+\frac{a-x}{a+x}}}\left(1+\frac{a-x}{a+x}\right)} + x \text{ArcSin}\left[\sqrt{\frac{-a-x}{a+x}}\right] - \frac{a \sqrt{\frac{a}{a+x}} \sqrt{a+x} \text{ArcTanh}\left[\frac{\sqrt{\frac{-a-x}{a+x}}}{\sqrt{1-\frac{a-x}{a+x}}}\right]}{\sqrt{-1-\frac{a-x}{a+x}} \sqrt{\frac{a}{1+\frac{a-x}{a+x}}}}$$

Problem #705: Valid but suboptimal antiderivative:

$$\left\{ e^x \text{ArcSin}[\text{Tanh}[x]], x, -5, 5 \right\}$$

$$e^x \text{ArcSin}[\text{Tanh}[x]] - \text{Cosh}[x] \text{Log}[1 + e^{2x}] \sqrt{\text{Sech}[x]^2}$$

$$-e^x \text{ArcSin}\left[\frac{1 - e^{2x}}{1 + e^{2x}}\right] - e^{-x} \sqrt{\frac{e^{2x}}{(1 + e^{2x})^2}} (1 + e^{2x}) \text{Log}[1 + e^{2x}]$$

Test complete!

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IntegrationTest["0 Independent test suites\\Charlwood Problems"];
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Testing Rubi on 50 integration problems...

Problem #3: Unable to integrate:

$$\left\{ -\text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right], x, -3, 3 \right\}$$

$$\frac{\left(\sqrt{x} + 3\sqrt{1+x}\right) \sqrt{-x + \sqrt{x} \sqrt{1+x}}}{4\sqrt{2}} - \left(\frac{3}{8} + x\right) \text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right]$$

$$-x \text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right] + \frac{\text{Subst}\left[\text{CannotIntegrate}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}, x\right], x, \sqrt{1+x}\right]}{\sqrt{2}}$$

Problem #4: Valid but suboptimal antiderivative:

$$\begin{aligned} & \left\{ \text{Log} \left[ 1 + x \sqrt{1 + x^2} \right], x, -32, 32 \right\} \\ & -2x + \sqrt{2(1+\sqrt{5})} \text{ArcTan} \left[ \sqrt{-2+\sqrt{5}} \left( x + \sqrt{1+x^2} \right) \right] - \sqrt{2(-1+\sqrt{5})} \text{ArcTanh} \left[ \sqrt{2+\sqrt{5}} \left( x + \sqrt{1+x^2} \right) \right] + x \text{Log} \left[ 1 + x \sqrt{1+x^2} \right] \\ & -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \text{ArcTan} \left[ \sqrt{\frac{2}{1+\sqrt{5}}} x \right] + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \text{ArcTan} \left[ \sqrt{\frac{2}{1+\sqrt{5}}} x \right] + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \text{ArcTan} \left[ \sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2} \right] + \\ & \sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan} \left[ \sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2} \right] + 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \text{ArcTanh} \left[ \sqrt{\frac{2}{-1+\sqrt{5}}} x \right] + \sqrt{\frac{1}{10}(-1+\sqrt{5})} \text{ArcTanh} \left[ \sqrt{\frac{2}{-1+\sqrt{5}}} x \right] + \\ & \sqrt{\frac{2}{5(1+\sqrt{5})}} \text{ArcTanh} \left[ \sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2} \right] - \sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTanh} \left[ \sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2} \right] + x \text{Log} \left[ 1 + x \sqrt{1+x^2} \right] \end{aligned}$$

Problem #5: NOT valid or unverifiable antiderivative:

$$\begin{aligned} & \left\{ \frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}, x, -4, 4 \right\} \\ & \frac{x}{3} + \frac{1}{3} \text{ArcTan} \left[ \frac{\text{Cos}[x] (1+\text{Cos}[x]^2) \text{Sin}[x]}{1+\text{Cos}[x]^2 \sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}} \right] \\ & \frac{\text{ArcTan} \left[ \frac{\text{Tan}[x]}{\sqrt{3+3 \text{Tan}[x]^2+\text{Tan}[x]^4}} \right] \text{Cos}[x]^2 \sqrt{3+3 \text{Tan}[x]^2+\text{Tan}[x]^4}}{2 \sqrt{\text{Cos}[x]^4 (3+3 \text{Tan}[x]^2+\text{Tan}[x]^4)}} - \\ & \frac{3^{1/4} \text{Cos}[x]^2 \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\text{Tan}[x]}{3^{1/4}} \right], \frac{1}{4} (2-\sqrt{3}) \right] (\sqrt{3} + \text{Tan}[x]^2) \sqrt{\frac{3+3 \text{Tan}[x]^2+\text{Tan}[x]^4}{(\sqrt{3} + \text{Tan}[x]^2)^2}}}{2 (3-\sqrt{3}) \sqrt{\text{Cos}[x]^4 (3+3 \text{Tan}[x]^2+\text{Tan}[x]^4)}} + \\ & \left( 3^{1/4} (1+\sqrt{3}) \text{Cos}[x]^2 \text{EllipticPi} \left[ \frac{1}{6} (3-2\sqrt{3}), 2 \text{ArcTan} \left[ \frac{\text{Tan}[x]}{3^{1/4}} \right], \frac{1}{4} (2-\sqrt{3}) \right] (\sqrt{3} + \text{Tan}[x]^2) \sqrt{\frac{3+3 \text{Tan}[x]^2+\text{Tan}[x]^4}{(\sqrt{3} + \text{Tan}[x]^2)^2}} \right) / \\ & \left( 4 (3-\sqrt{3}) \sqrt{\text{Cos}[x]^4 (3+3 \text{Tan}[x]^2+\text{Tan}[x]^4)} \right) \end{aligned}$$

Problem #12: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \text{ArcTan} \left[ x + \sqrt{1-x^2} \right], x, -43, 43 \right\} \\
& - \frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{-1 + \sqrt{3} x}{\sqrt{1-x^2}} \right] + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{1 + \sqrt{3} x}{\sqrt{1-x^2}} \right] - \\
& \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{-1 + 2x^2}{\sqrt{3}} \right] + x \text{ArcTan} \left[ x + \sqrt{1-x^2} \right] - \frac{1}{4} \text{ArcTanh} \left[ x \sqrt{1-x^2} \right] - \frac{1}{8} \text{Log} \left[ 1-x^2 + x^4 \right] \\
& - \frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{1-2x^2}{\sqrt{3}} \right] + \frac{\text{ArcTan} \left[ \frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} \sqrt{1-x^2}} \right]}{\sqrt{3}} + \frac{1}{12} (3i - \sqrt{3}) \text{ArcTan} \left[ \frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} \sqrt{1-x^2}} \right] + \\
& \frac{\text{ArcTan} \left[ \frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} x}{\sqrt{1-x^2}} \right]}{\sqrt{3}} - \frac{1}{12} (3i + \sqrt{3}) \text{ArcTan} \left[ \frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} x}{\sqrt{1-x^2}} \right] + x \text{ArcTan} \left[ x + \sqrt{1-x^2} \right] - \frac{1}{8} \text{Log} \left[ 1-x^2 + x^4 \right]
\end{aligned}$$

Problem #13: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{x \text{ArcTan} \left[ x + \sqrt{1-x^2} \right]}{\sqrt{1-x^2}}, x, -32, 32 \right\} \\
& - \frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{-1 + \sqrt{3} x}{\sqrt{1-x^2}} \right] + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{1 + \sqrt{3} x}{\sqrt{1-x^2}} \right] - \\
& \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{-1 + 2x^2}{\sqrt{3}} \right] - \sqrt{1-x^2} \text{ArcTan} \left[ x + \sqrt{1-x^2} \right] + \frac{1}{4} \text{ArcTanh} \left[ x \sqrt{1-x^2} \right] + \frac{1}{8} \text{Log} \left[ 1-x^2 + x^4 \right] \\
& - \frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan} \left[ \frac{1-2x^2}{\sqrt{3}} \right] + \frac{\text{ArcTan} \left[ \frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} \sqrt{1-x^2}} \right]}{2\sqrt{3}} - \frac{1}{12} (3i - \sqrt{3}) \text{ArcTan} \left[ \frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} \sqrt{1-x^2}} \right] + \\
& \frac{\text{ArcTan} \left[ \frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} x}{\sqrt{1-x^2}} \right]}{2\sqrt{3}} + \frac{1}{12} (3i + \sqrt{3}) \text{ArcTan} \left[ \frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}} x}{\sqrt{1-x^2}} \right] - \sqrt{1-x^2} \text{ArcTan} \left[ x + \sqrt{1-x^2} \right] + \frac{1}{8} \text{Log} \left[ 1-x^2 + x^4 \right]
\end{aligned}$$

Problem #42: Valid but suboptimal antiderivative:

$$\left\{ \frac{\text{Sec}[x]}{\sqrt{-1 + \text{Sec}[x]^4}}, x, -5, 5 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x] \text{Cot}[x] \sqrt{-1 + \text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \text{Sin}[x]}{\sqrt{2 \text{Sin}[x]^2 - \text{Sin}[x]^4}}\right] \sqrt{1 - \text{Cos}[x]^4} \text{Sec}[x]^2}{\sqrt{2} \sqrt{-1 + \text{Sec}[x]^4}}$$

Problem #45: Unable to integrate:

$$\left\{ \sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}, x, -1, 0 \right\}$$

$$\sqrt{2} \left( \sqrt{-1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{-2 + 2\sqrt{2}} \left(-\sqrt{2} - \sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}\right)}{2 \sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}}\right] - \right.$$

$$\left. \sqrt{1 + \sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2 + 2\sqrt{2}} \left(-\sqrt{2} - \sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}\right)}{2 \sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}}\right] - \sqrt{1 + \sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{-2 + 2\sqrt{2}} \sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}}{\sqrt{2} - \sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}\right] + \right.$$

$$\left. \sqrt{-1 + \sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{2 + 2\sqrt{2}} \sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}}{\sqrt{2} - \sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}\right] \right) \text{Cot}[x] \sqrt{-1 + \text{Sec}[x]} \sqrt{1 + \text{Sec}[x]}$$

$$\text{CannotIntegrate}\left[\sqrt{-\sqrt{-1 + \text{Sec}[x]} + \sqrt{1 + \text{Sec}[x]}}, x\right]$$

Test complete!

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IntegrationTest["0 Independent test suites\Stewart Problems"];
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Testing Rubi on 376 integration problems...

Test complete!

```
IntegrationTest["0 Independent test suites\Hearn Problems"];
```

Testing Rubi on 284 integration problems...

Problem #169: Unable to integrate:

$$\left\{ \frac{e^{1-e^{x^2} x+2 x^2} (x+2 x^3)}{(1-e^{x^2} x)^2}, x, -3, 3 \right\}$$

$$-\frac{e^{1-e^{x^2} x}}{-1+e^{x^2} x}$$

$$\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x}{(-1+e^{x^2} x)^2}, x\right] + 2 \text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x^3}{(-1+e^{x^2} x)^2}, x\right]$$

Problem #201: Unable to integrate:

$$\left\{ \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}, x, -4, 4 \right\}$$

$$-\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

$$-\sqrt{1+x^3} - \text{CannotIntegrate}\left[\frac{1}{\sqrt{1-2x+x^5}}, x\right] + \frac{5}{2} \text{CannotIntegrate}\left[\frac{x^4}{\sqrt{1-2x+x^5}}, x\right]$$

Problem #274: Unable to integrate:

$$\left\{ \frac{e^{x^2}}{x} + 2e^{x^2} x \text{Log}[x] + \frac{-2 + \text{Log}[x]}{(x + \text{Log}[x]^2)^2} + \frac{1 + \frac{1}{x} + \frac{2\text{Log}[x]}{x}}{x + \text{Log}[x]^2}, x, -9, 9 \right\}$$

$$e^{x^2} \text{Log}[x] - \frac{\text{Log}[x]}{x + \text{Log}[x]^2} + \text{Log}[x + \text{Log}[x]^2]$$

$$-2 \text{CannotIntegrate}\left[\frac{1}{(x + \text{Log}[x]^2)^2}, x\right] + \text{CannotIntegrate}\left[\frac{\text{Log}[x]}{(x + \text{Log}[x]^2)^2}, x\right] +$$

$$\text{CannotIntegrate}\left[\frac{1}{x + \text{Log}[x]^2}, x\right] + \text{CannotIntegrate}\left[\frac{1}{x(x + \text{Log}[x]^2)}, x\right] + 2 \text{CannotIntegrate}\left[\frac{\text{Log}[x]}{x(x + \text{Log}[x]^2)}, x\right] + e^{x^2} \text{Log}[x]$$

Problem #278: Unable to integrate:

$$\left\{ \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}}, x, -10, 10 \right\}$$

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)} - \text{ArcTanh}\left[\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3)\sqrt{1+2x^2+4x^3+x^4}}\right]$$

$$\begin{aligned} & \frac{9}{4} \text{CannotIntegrate} \left[ \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}, x \right] - \frac{13}{4} \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}-2x)^2 \sqrt{1+2x^2+4x^3+x^4}}, x \right] - \\ & \frac{13 \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}-2x) \sqrt{1+2x^2+4x^3+x^4}}, x \right]}{4\sqrt{2}} - \frac{1}{8} (2+15\sqrt{2}) \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}-2x) \sqrt{1+2x^2+4x^3+x^4}}, x \right] + \\ & \text{CannotIntegrate} \left[ \frac{x}{\sqrt{1+2x^2+4x^3+x^4}}, x \right] + \frac{1}{2} \text{CannotIntegrate} \left[ \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}, x \right] - \\ & \frac{13}{4} \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}+2x)^2 \sqrt{1+2x^2+4x^3+x^4}}, x \right] - \frac{13 \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}+2x) \sqrt{1+2x^2+4x^3+x^4}}, x \right]}{4\sqrt{2}} + \\ & \frac{1}{8} (2-15\sqrt{2}) \text{CannotIntegrate} \left[ \frac{1}{(\sqrt{2}+2x) \sqrt{1+2x^2+4x^3+x^4}}, x \right] - \frac{17}{2} \text{CannotIntegrate} \left[ \frac{x}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}}, x \right] \end{aligned}$$

Problem #279: Unable to integrate:

$$\begin{aligned} & \left\{ \frac{(1+2y) \sqrt{1-5y-5y^2}}{y(1+y)(2+y) \sqrt{1-y-y^2}}, y, -2, 2 \right\} \\ & - \frac{1}{4} \text{ArcTanh} \left[ \frac{(1-3y) \sqrt{1-5y-5y^2}}{(1-5y) \sqrt{1-y-y^2}} \right] - \frac{1}{2} \text{ArcTanh} \left[ \frac{(4+3y) \sqrt{1-5y-5y^2}}{(6+5y) \sqrt{1-y-y^2}} \right] + \frac{9}{4} \text{ArcTanh} \left[ \frac{(11+7y) \sqrt{1-5y-5y^2}}{3(7+5y) \sqrt{1-y-y^2}} \right] \\ & \frac{1}{2} \text{CannotIntegrate} \left[ \frac{\sqrt{1-5y-5y^2}}{y \sqrt{1-y-y^2}}, y \right] + \text{CannotIntegrate} \left[ \frac{\sqrt{1-5y-5y^2}}{(1+y) \sqrt{1-y-y^2}}, y \right] - \frac{3}{2} \text{CannotIntegrate} \left[ \frac{\sqrt{1-5y-5y^2}}{(2+y) \sqrt{1-y-y^2}}, y \right] \end{aligned}$$

Problem #284: Unable to integrate:

$$\begin{aligned} & \left\{ \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}, x, -2, 2 \right\} \\ & \frac{1}{2} \left( (1+\sqrt{2}) \text{Log} \left[ 1+x+\sqrt{2}x+\sqrt{2}x^2-x^7 \right] - (-1+\sqrt{2}) \text{Log} \left[ -1+(-1+\sqrt{2})x+\sqrt{2}x^2+x^7 \right] \right) \end{aligned}$$



$$\begin{aligned}
& 3 \text{ CannotIntegrate} \left[ \frac{1}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] + 3 \text{ CannotIntegrate} \left[ \frac{x}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] - \\
& 4 \text{ CannotIntegrate} \left[ \frac{x^2}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] - 4 \text{ CannotIntegrate} \left[ \frac{x^3}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] - \\
& 7 \text{ CannotIntegrate} \left[ \frac{x^6}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] + 4 \text{ CannotIntegrate} \left[ \frac{x^7}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] + \\
& 10 \text{ CannotIntegrate} \left[ \frac{x^8}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right] + 7 \text{ CannotIntegrate} \left[ \frac{x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}}, x \right]
\end{aligned}$$

Test complete!

```
IntegrationTest["0 Independent test suites\\Jeffrey Problems"];
```

Testing Rubi on 9 integration problems...

Problem #2: Valid but suboptimal antiderivative:

$$\left\{ \frac{1 + \cos[x] + 2 \sin[x]}{3 + \cos[x]^2 + 2 \sin[x] - 2 \cos[x] \sin[x]}, x, -43, 43 \right\}$$

$$-\text{ArcTan} \left[ \frac{2 \cos[x] - \sin[x]}{2 + \sin[x]} \right]$$

$$-\text{ArcTan} \left[ \frac{2 \cos[x] - \sin[x]}{2 + \sin[x]} \right] + \text{Cot} \left[ \frac{x}{2} \right] - \frac{\sin[x]}{1 - \cos[x]}$$

Problem #3: Valid but suboptimal antiderivative:

$$\left\{ \frac{2 + \cos[x] + 5 \sin[x]}{4 \cos[x] - 2 \sin[x] + \cos[x] \sin[x] - 2 \sin[x]^2}, x, -26, 26 \right\}$$

$$-\text{Log} [1 - 3 \cos[x] + \sin[x]] + \text{Log} [3 + \cos[x] + \sin[x]]$$

$$-\text{Log} \left[ 1 - 2 \tan \left[ \frac{x}{2} \right] \right] - \text{Log} \left[ 1 + \tan \left[ \frac{x}{2} \right] \right] + \text{Log} \left[ 2 + \tan \left[ \frac{x}{2} \right] + \tan \left[ \frac{x}{2} \right]^2 \right]$$

Problem #4: Valid but suboptimal antiderivative:

$$\left\{ \frac{3 + 7 \cos[x] + 2 \sin[x]}{1 + 4 \cos[x] + 3 \cos[x]^2 - 5 \sin[x] - \cos[x] \sin[x]}, x, -32, 32 \right\}$$

$$-\text{Log} [1 + \cos[x] - 2 \sin[x]] + \text{Log} [3 + \cos[x] + \sin[x]]$$

$$-\text{Log} \left[ 1 - 2 \tan \left[ \frac{x}{2} \right] \right] + \text{Log} \left[ 2 + \tan \left[ \frac{x}{2} \right] + \tan \left[ \frac{x}{2} \right]^2 \right]$$

Problem #5: Unable to integrate:

$$\left\{ \frac{-1 + 4 \cos[x] + 5 \cos[x]^2}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x, -2, 2 \right\}$$

$$x - 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{3 + \cos[x]}\right] - 2 \operatorname{ArcTan}\left[\frac{3 \sin[x] + 7 \cos[x] \sin[x]}{1 + 2 \cos[x] + 5 \cos[x]^2}\right]$$

$$\operatorname{CannotIntegrate}\left[\frac{1}{1 + 4 \cos[x] + 3 \cos[x]^2 - 4 \cos[x]^3}, x\right] +$$

$$4 \operatorname{CannotIntegrate}\left[\frac{\cos[x]}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x\right] + 5 \operatorname{CannotIntegrate}\left[\frac{\cos[x]^2}{-1 - 4 \cos[x] - 3 \cos[x]^2 + 4 \cos[x]^3}, x\right]$$

Problem #6: Unable to integrate:

$$\left\{ \frac{-5 + 2 \cos[x] + 7 \cos[x]^2}{-1 + 2 \cos[x] - 9 \cos[x]^2 + 4 \cos[x]^3}, x, -2, 2 \right\}$$

$$x - 2 \operatorname{ArcTan}\left[\frac{2 \cos[x] \sin[x]}{1 - \cos[x] + 2 \cos[x]^2}\right]$$

$$-5 \operatorname{CannotIntegrate}\left[\frac{1}{-1 + 2 \cos[x] - 9 \cos[x]^2 + 4 \cos[x]^3}, x\right] +$$

$$2 \operatorname{CannotIntegrate}\left[\frac{\cos[x]}{-1 + 2 \cos[x] - 9 \cos[x]^2 + 4 \cos[x]^3}, x\right] + 7 \operatorname{CannotIntegrate}\left[\frac{\cos[x]^2}{-1 + 2 \cos[x] - 9 \cos[x]^2 + 4 \cos[x]^3}, x\right]$$

Test complete!

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IntegrationTest["0 Independent test suites\\Hebisch Problems"];
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Testing Rubi on 7 integration problems...

Problem #2: Unable to integrate:

$$\left\{ \frac{e^{\frac{x}{2+x^2}} (2 - x^2)}{2x + x^3}, x, -5, 5 \right\}$$

$$\operatorname{ExpIntegralEi}\left[\frac{x}{2 + x^2}\right]$$

$$\operatorname{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2 - x}}, x\right] + \operatorname{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right] - \operatorname{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2 + x}}, x\right]$$

Problem #3: Unable to integrate:

$$\left\{ \frac{e^{\frac{x}{2+x^2}} (2 + 2x + 3x^2 - x^3 + 2x^4)}{2x + x^3}, x, -5, 5 \right\}$$

$$e^{\frac{x}{2+x^2}} (2 + x^2) + \text{ExpIntegralEi} \left[ \frac{x}{2+x^2} \right]$$

$$-\text{CannotIntegrate} \left[ e^{\frac{x}{2+x^2}}, x \right] + (1 + i\sqrt{2}) \text{CannotIntegrate} \left[ \frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2} - x}, x \right] +$$

$$\text{CannotIntegrate} \left[ \frac{e^{\frac{x}{2+x^2}}}{x}, x \right] + 2 \text{CannotIntegrate} \left[ e^{\frac{x}{2+x^2}} x, x \right] - (1 - i\sqrt{2}) \text{CannotIntegrate} \left[ \frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2} + x}, x \right]$$

Problem #5: Unable to integrate:

$$\left\{ \frac{e^{\frac{1}{-1+x^2}} (1 - 3x - x^2 + x^3)}{1 - x - x^2 + x^3}, x, -6, 6 \right\}$$

$$e^{\frac{1}{-1+x^2}} (1 + x)$$

$$\text{CannotIntegrate} \left[ e^{\frac{1}{-1+x^2}}, x \right] + \frac{1}{2} \text{CannotIntegrate} \left[ \frac{e^{\frac{1}{-1+x^2}}}{1-x}, x \right] - \text{CannotIntegrate} \left[ \frac{e^{\frac{1}{-1+x^2}}}{(-1+x)^2}, x \right] + \frac{1}{2} \text{CannotIntegrate} \left[ \frac{e^{\frac{1}{-1+x^2}}}{1+x}, x \right]$$

Problem #7: Unable to integrate:

$$\left\{ \frac{e^{x + \frac{1}{\text{Log}[x]}} (-1 + (1+x) \text{Log}[x]^2)}{\text{Log}[x]^2}, x, -2, 2 \right\}$$

$$e^{x + \frac{1}{\text{Log}[x]}} x$$

$$\text{CannotIntegrate} \left[ e^{x + \frac{1}{\text{Log}[x]}}, x \right] + \text{CannotIntegrate} \left[ e^{x + \frac{1}{\text{Log}[x]}} x, x \right] - \text{CannotIntegrate} \left[ \frac{e^{x + \frac{1}{\text{Log}[x]}}}{\text{Log}[x]^2}, x \right]$$

Test complete!

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IntegrationTest["0 Independent test suites\\Wester Problems"];
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Testing Rubi on 8 integration problems...

Test complete!

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IntegrationTest["0 Independent test suites\\Welz Problems"];
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Testing Rubi on 70 integration problems...

Problem #9: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{\sqrt{-1+x^2} (\sqrt{x} + \sqrt{-1+x^2})^2}, x, -18, 18 \right\}$$

$$\frac{2-4x}{5(\sqrt{x} + \sqrt{-1+x^2})} + \frac{1}{25} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right] - \frac{1}{50} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-(1-\sqrt{5})x}\right] -$$

$$\frac{1}{25} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right] - \frac{1}{50} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right]$$

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5} \sqrt{\frac{2}{5}(-11+5\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right] + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] -$$

$$\frac{2}{5} \sqrt{\frac{1}{5}(-2+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] - \frac{1}{5} \sqrt{\frac{2}{5}(11+5\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right] +$$

$$\sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right] - \frac{2}{5} \sqrt{\frac{1}{5}(2+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]$$

Problem #10: Valid but suboptimal antiderivative:

$$\left\{ \frac{(\sqrt{x} - \sqrt{-1+x^2})^2}{(1+x-x^2)^2 \sqrt{-1+x^2}}, x, -25, 25 \right\}$$

$$\frac{2-4x}{5(\sqrt{x} + \sqrt{-1+x^2})} + \frac{1}{25} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right] - \frac{1}{50} \sqrt{-110+50\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-(1-\sqrt{5})x}\right] -$$

$$\frac{1}{25} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right] - \frac{1}{50} \sqrt{110+50\sqrt{5}} \operatorname{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right]$$

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)} +$$

$$\frac{1}{5}\sqrt{\frac{2}{5}(-11+5\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right] - \frac{1}{5}\sqrt{\frac{1}{10}(-11+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] -$$

$$\frac{1}{5}\sqrt{\frac{1}{5}(-2+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] + \frac{1}{5}\sqrt{\frac{1}{5}(2+5\sqrt{5})} \operatorname{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right] -$$

$$\frac{1}{5}\sqrt{\frac{2}{5}(11+5\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right] - \frac{1}{5}\sqrt{\frac{1}{5}(-2+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right] -$$

$$\frac{1}{5}\sqrt{\frac{1}{5}(2+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right] + \frac{1}{5}\sqrt{\frac{1}{10}(11+5\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]$$

Problem #37: Unable to integrate:

$$\left\{\frac{1}{(1+x)(1-x^3)^{1/3}}, x, -1, 0\right\}$$

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{2(1-x)+2^{2/3}(1-x^3)^{1/3}}{2^{2/3}\sqrt{3}(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}} - \frac{\operatorname{Log}[1-x]}{4 \times 2^{1/3}} - \frac{\operatorname{Log}[1+x]}{2 \times 2^{1/3}} + \frac{3 \operatorname{Log}[-1+x+2^{2/3}(1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

CannotIntegrate $\left[\frac{1}{(1+x)(1-x^3)^{1/3}}, x\right]$

Problem #38: Unable to integrate:

$$\left\{\frac{x}{(1+x)(1-x^3)^{1/3}}, x, -9, 9\right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{2x-(1-x^3)^{1/3}}{\sqrt{3}(1-x^3)^{1/3}}\right]}{\sqrt{3}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{2(1-x)+2^{2/3}(1-x^3)^{1/3}}{2^{2/3}\sqrt{3}(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}} + \frac{\operatorname{Log}[1-x]}{4 \times 2^{1/3}} + \frac{\operatorname{Log}[1+x]}{2 \times 2^{1/3}} + \frac{1}{2} \operatorname{Log}[x+(1-x^3)^{1/3}] - \frac{3 \operatorname{Log}[-1+x+2^{2/3}(1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \text{CannotIntegrate}\left[\frac{1}{(-1-x)(1-x^3)^{1/3}}, x\right] - \frac{1}{6} \text{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{3} \text{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right]$$

Problem #39: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{x(2-3x+x^2)^{1/3}}, x, -2, 2\right\}$$

$$-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3}(2-x)}{\sqrt{3}(2-3x+x^2)^{1/3}}\right]}{2 \times 2^{1/3}} - \frac{\text{Log}[2-x]}{4 \times 2^{1/3}} - \frac{\text{Log}[x]}{2 \times 2^{1/3}} + \frac{3 \text{Log}[2-x-2^{2/3}(2-3x+x^2)^{1/3}]}{4 \times 2^{1/3}}$$

$$-\frac{\sqrt{3}(-2+x)^{1/3}(-1+x)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2^{1/3}(-2+x)^{2/3}}{\sqrt{3}(-1+x)^{1/3}}\right]}{2 \times 2^{1/3}(2-3x+x^2)^{1/3}} + \frac{3(-2+x)^{1/3}(-1+x)^{1/3} \text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}} - 2^{1/3}(-1+x)^{1/3}\right]}{4 \times 2^{1/3}(2-3x+x^2)^{1/3}} - \frac{(-2+x)^{1/3}(-1+x)^{1/3} \text{Log}[x]}{2 \times 2^{1/3}(2-3x+x^2)^{1/3}}$$

Problem #40: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{(-5+7x-3x^2+x^3)^{1/3}}, x, -11, 11\right\}$$

$$\frac{1}{2} \sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(-1+x)}{\sqrt{3}(-5+7x-3x^2+x^3)^{1/3}}\right] + \frac{1}{4} \text{Log}[1-x] - \frac{3}{4} \text{Log}\left[1-x+(-5+7x-3x^2+x^3)^{1/3}\right]$$

$$\frac{\sqrt{3}(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{4+(-1+x)^2}}{\sqrt{3}}\right]}{2(-5+7x-3x^2+x^3)^{1/3}} -$$

$$\frac{(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{Log}\left[1-\frac{(-1+x)^{2/3}}{4+(-1+x)^2}\right]}{2(-5+7x-3x^2+x^3)^{1/3}} + \frac{(-1+x)^{1/3}(5-2x+x^2)^{1/3} \text{Log}\left[1+\frac{(-1+x)^{2/3}}{4+(-1+x)^2} + \frac{(-1+x)^{4/3}}{4+(-1+x)^2}\right]}{4(-5+7x-3x^2+x^3)^{1/3}}$$

Problem #41: Valid but suboptimal antiderivative:

$$\left\{\frac{1}{(x(-q+x^2))^{1/3}}, x, -11, 11\right\}$$

$$\frac{1}{2} \sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2x}{\sqrt{3}(x(-q+x^2))^{1/3}}\right] + \frac{\text{Log}[x]}{4} - \frac{3}{4} \text{Log}\left[-x+(x(-q+x^2))^{1/3}\right]$$

$$\frac{\sqrt{3} x^{1/3} (-q + x^2)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2x^{2/3}}{(-q+x^2)^{1/3}}}{\sqrt{3}}\right]}{2 (-q x + x^3)^{1/3}} - \frac{x^{1/3} (-q + x^2)^{1/3} \operatorname{Log}\left[1 - \frac{x^{2/3}}{(-q+x^2)^{1/3}}\right]}{2 (-q x + x^3)^{1/3}} + \frac{x^{1/3} (-q + x^2)^{1/3} \operatorname{Log}\left[1 + \frac{x^{4/3}}{(-q+x^2)^{2/3}} + \frac{x^{2/3}}{(-q+x^2)^{1/3}}\right]}{4 (-q x + x^3)^{1/3}}$$

Problem #42: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{((-1+x)(q-2x+x^2))^{1/3}}, x, -12, 12 \right\}$$

$$\frac{1}{2} \sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(-1+x)}{\sqrt{3}((-1+x)(q-2x+x^2))^{1/3}}\right] + \frac{1}{4} \operatorname{Log}[1-x] - \frac{3}{4} \operatorname{Log}\left[1-x + ((-1+x)(q-2x+x^2))^{1/3}\right]$$

$$\frac{\sqrt{3}(-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}}}{\sqrt{3}}\right]}{2(-q+(2+q)x-3x^2+x^3)^{1/3}} -$$

$$\frac{(-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{Log}\left[1 - \frac{(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}}\right]}{2(-q+(2+q)x-3x^2+x^3)^{1/3}} + \frac{(-1+x)^{1/3}(q-2x+x^2)^{1/3} \operatorname{Log}\left[1 + \frac{(-1+x)^{2/3}}{(-1+q+(-1+x)^2)^{1/3}} + \frac{(-1+x)^{4/3}}{(-1+q+(-1+x)^2)^{2/3}}\right]}{4(-q+(2+q)x-3x^2+x^3)^{1/3}}$$

Problem #43: Unable to integrate:

$$\left\{ \frac{1}{x(-1+x)(q-2qx+x^2)^{1/3}}, x, -2, 2 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2q^{1/3}(-1+x)}{\sqrt{3}((-1+x)(q-2qx+x^2))^{1/3}}\right]}{2q^{1/3}} + \frac{\operatorname{Log}[1-x]}{4q^{1/3}} + \frac{\operatorname{Log}[x]}{2q^{1/3}} - \frac{3 \operatorname{Log}\left[-q^{1/3}(-1+x) + ((-1+x)(q-2qx+x^2))^{1/3}\right]}{4q^{1/3}}$$

$$\frac{(-1+x)^{1/3}(-q+2qx-x^2)^{1/3} \operatorname{Unintegrable}\left[\frac{1}{(-1+x)^{1/3}x(-q+2qx-x^2)^{1/3}}, x\right]}{(-q+3qx-(1+2q)x^2+x^3)^{1/3}}$$

Problem #44: Unable to integrate:

$$\left\{ \frac{2 - (1+k)x}{((1-x)x(1-kx))^{1/3}(1-(1+k)x)} , x, -6, 6 \right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2k^{1/3}x}{((1-x)x(1-kx))^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}} + \frac{\operatorname{Log}[x]}{2k^{1/3}} + \frac{\operatorname{Log}[1-(1+k)x]}{2k^{1/3}} - \frac{3 \operatorname{Log}\left[-k^{1/3}x + ((1-x)x(1-kx))^{1/3}\right]}{2k^{1/3}}$$

$$-\frac{3(1-x)^{1/3}x^{1/3}(kx)^{1/3}(1-kx)\left(1-\frac{1-kx}{1-k}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, 1-kx, \frac{1-kx}{1-k}\right]}{2k((1-x)x(1-kx))^{1/3}(x-x^2)^{1/3}} +$$

$$\frac{(1-x)^{1/3}x^{1/3}(1-kx)^{1/3}\text{Unintegrable}\left[\frac{1}{(1+(-1-k)x)(1-kx)^{1/3}(x-x^2)^{1/3}}, x\right]}{((1-x)x(1-kx))^{1/3}}$$

Problem #45: Unable to integrate:

$$\left\{\frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}, x, -1, 1\right\}$$

$$-\frac{\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2^{2/3}(1-kx)}{(1-k)^{1/3}((1-x)x(1-kx))^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}(1-k)^{1/3}} + \frac{\text{Log}[1-(2-k)x]}{2^{2/3}(1-k)^{1/3}} + \frac{\text{Log}[1-kx]}{2 \times 2^{2/3}(1-k)^{1/3}} - \frac{3\text{Log}[-1+kx+2^{2/3}(1-k)^{1/3}((1-x)x(1-kx))^{1/3}]}{2 \times 2^{2/3}(1-k)^{1/3}}$$

$$\frac{(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\text{CannotIntegrate}\left[\frac{(1-kx)^{1/3}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}, x\right]}{((1-x)x(1-kx))^{2/3}}$$

Problem #46: Unable to integrate:

$$\left\{\frac{a+bx+cx^2}{(1-x+x^2)(1-x^3)^{1/3}}, x, -11, 11\right\}$$

$$-\frac{1}{6}c\left(2\sqrt{3}\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] + \text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] - 2\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]\right) +$$

$$\frac{(a-b-2c)\left(-2\sqrt{3}\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right] - 3\text{Log}\left[2^{1/3} - (1-x^3)^{1/3}\right]\right)}{12 \times 2^{1/3}} +$$

$$\frac{(a+b)\left(2\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2 \cdot 2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] + \text{Log}\left[3-6x+6x^2-3x^3\right] - 3\text{Log}\left[-2^{1/3}(-1+x) + (1-x^3)^{1/3}\right]\right)}{4 \times 2^{1/3}} -$$

$$\frac{(a-b-2c)\left(2\sqrt{3}\text{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right] - 3\text{Log}\left[2^{1/3}x + (1-x^3)^{1/3}\right]\right)}{12 \times 2^{1/3}}$$



$$-\frac{c \operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{1}{3} \left(3b - \sqrt{3} \left(i(2a+b-c) - \sqrt{3}c\right)\right) \operatorname{CannotIntegrate}\left[\frac{1}{(-1-i\sqrt{3}+2x)(1-x^3)^{1/3}}, x\right] +$$

$$\frac{1}{3} \left(3b + \sqrt{3} \left(i(2a+b-c) + \sqrt{3}c\right)\right) \operatorname{CannotIntegrate}\left[\frac{1}{(-1+i\sqrt{3}+2x)(1-x^3)^{1/3}}, x\right] - \frac{1}{6} c \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{3} c \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right]$$

Problem #52: Valid but suboptimal antiderivative:

$$\left\{ \frac{-a - \sqrt{1+a^2} + x}{(-a + \sqrt{1+a^2} + x) \sqrt{(-a+x)(1+x^2)}}, x, -6, 6 \right\}$$

$$-\sqrt{2} \sqrt{a + \sqrt{1+a^2}} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{-a + \sqrt{1+a^2}} (-a+x)}{\sqrt{(-a+x)(1+x^2)}}\right]$$

$$\frac{2i \sqrt{\frac{a-x}{i+a}} \sqrt{1+x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-ix}}{\sqrt{2}}\right], \frac{2}{1-ia}\right] - 4 \sqrt{\frac{i-x}{i-a}} \sqrt{\frac{i+x}{i+a}} \sqrt{-a+x} \operatorname{EllipticPi}\left[-\frac{i-a}{\sqrt{1+a^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1}{i-a}} \sqrt{-a+x}\right], -\frac{i-a}{i+a}\right]}{\sqrt{-(a-x)(1+x^2)} \sqrt{\frac{1}{i-a} \sqrt{-(a-x)(1+x^2)}}}$$

Problem #55: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{x(4-6x+3x^2)^{1/3}}, x, -1, 1 \right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{-2+x-2^{2/3}(4-6x+3x^2)^{1/3}}{\sqrt{3}(-2+x)}\right]}{2^{2/3}\sqrt{3}} + \frac{\operatorname{Log}\left[\frac{-4+2x+2^{2/3}(4-6x+3x^2)^{1/3}}{x}\right]}{2 \times 2^{2/3}}$$

$$-\frac{\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{2/3}(2-x)}{\sqrt{3}(4-6x+3x^2)^{1/3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\operatorname{Log}[x]}{2 \times 2^{2/3}} + \frac{\operatorname{Log}[6-3x-3 \times 2^{1/3}(4-6x+3x^2)^{1/3}]}{2 \times 2^{2/3}}$$

Problem #58: Unable to integrate:

$$\left\{ \frac{(1-x^3)^{1/3}}{1+x}, x, -1, 0 \right\}$$

0

$$\text{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{1+x}, x\right]$$

Problem #59: Unable to integrate:

$$\left\{\frac{(1-x^3)^{1/3}}{1-x+x^2}, x, -2, 2\right\}$$

$$\begin{aligned} & \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2^{2/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2^{2/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\operatorname{Log}\left[-3(-1+x)(1-x+x^2)\right]}{2 \times 2^{2/3}} + \\ & \frac{\operatorname{Log}\left[2^{1/3} - (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log}\left[-2^{1/3}(-1+x) + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} + \frac{1}{2} \operatorname{Log}\left[x + (1-x^3)^{1/3}\right] - \frac{\operatorname{Log}\left[2^{1/3}x + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} \\ & \frac{2 \operatorname{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{1+i\sqrt{3}-2x}, x\right]}{\sqrt{3}} + \frac{2 \operatorname{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{-1+i\sqrt{3}+2x}, x\right]}{\sqrt{3}} \end{aligned}$$

Test complete!

```
IntegrationTest["0 Independent test suites\\Bronstein Problems"];
```

Testing Rubi on 14 integration problems...

Problem #6: Unable to integrate:

$$\left\{ \frac{x}{\sqrt{-71 - 96x + 10x^2 + x^4}}, x, -1, 0 \right\}$$

$$-\frac{1}{8} \operatorname{Log} \left[ 10001 + 3124x^2 - 1408x^3 + 54x^4 - 128x^5 + 20x^6 + x^8 + \sqrt{-71 - 96x + 10x^2 + x^4} (-781 + 528x - 27x^2 + 80x^3 - 15x^4 - x^6) \right]$$

$$\operatorname{CannotIntegrate} \left[ \frac{x}{\sqrt{-71 - 96x + 10x^2 + x^4}}, x \right]$$

Problem #12: Unable to integrate:

$$\left\{ \frac{x^2 + 2x \operatorname{Log}[x] + \operatorname{Log}[x]^2 + (1+x) \sqrt{x + \operatorname{Log}[x]}}{x^3 + 2x^2 \operatorname{Log}[x] + x \operatorname{Log}[x]^2}, x, -3, 3 \right\}$$

$$\operatorname{Log}[x] - \frac{2}{\sqrt{x + \operatorname{Log}[x]}}$$

$$\operatorname{CannotIntegrate} \left[ \frac{1}{(x + \operatorname{Log}[x])^{3/2}}, x \right] - \operatorname{CannotIntegrate} \left[ \frac{1}{\operatorname{Log}[x] (x + \operatorname{Log}[x])^{3/2}}, x \right] -$$

$$\operatorname{CannotIntegrate} \left[ \frac{1}{\operatorname{Log}[x]^2 \sqrt{x + \operatorname{Log}[x]}}, x \right] + \operatorname{CannotIntegrate} \left[ \frac{\sqrt{x + \operatorname{Log}[x]}}{x \operatorname{Log}[x]^2}, x \right] + \operatorname{Log}[x]$$

Test complete!

```
IntegrationTest["0 Independent test suites\\Bondarenko Problems"];
```

Testing Rubi on 35 integration problems...

Problem #7: Unable to integrate:

$$\left\{ \frac{\operatorname{Log}[1+x]}{x \sqrt{1 + \sqrt{1+x}}}, x, -1, 0 \right\}$$

$$\begin{aligned}
& -8 \operatorname{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right] - \frac{2 \operatorname{Log}[1+x]}{\sqrt{1+\sqrt{1+x}}} - \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right] \operatorname{Log}[1+x] + \\
& 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right] - 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right] + \sqrt{2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right] - \\
& \sqrt{2} \operatorname{PolyLog}\left[2, \frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right] - \sqrt{2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right] + \sqrt{2} \operatorname{PolyLog}\left[2, \frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right] \\
& \operatorname{Unintegrable}\left[\frac{\operatorname{Log}[1+x]}{x \sqrt{1+\sqrt{1+x}}}, x\right]
\end{aligned}$$

Problem #8: Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\sqrt{1+\sqrt{1+x}} \operatorname{Log}[1+x]}{x}, x, -1, 0 \right\} \\
& -16 \sqrt{1+\sqrt{1+x}} + 16 \operatorname{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right] + 4 \sqrt{1+\sqrt{1+x}} \operatorname{Log}[1+x] - 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right] \operatorname{Log}[1+x] + \\
& 4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right] - 4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right] + 2 \sqrt{2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right] - \\
& 2 \sqrt{2} \operatorname{PolyLog}\left[2, \frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right] - 2 \sqrt{2} \operatorname{PolyLog}\left[2, -\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right] + 2 \sqrt{2} \operatorname{PolyLog}\left[2, \frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right] \\
& \operatorname{Unintegrable}\left[\frac{\sqrt{1+\sqrt{1+x}} \operatorname{Log}[1+x]}{x}, x\right]
\end{aligned}$$

Problem #21: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{(\operatorname{Cos}[x] + \operatorname{Cos}[3x])^5}, x, -57, 57 \right\}$$

$$-\frac{523}{256} \operatorname{ArcTanh}[\sin[x]] + \frac{1483 \operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{512 \sqrt{2}} + \frac{\sin[x]}{32 (1 - 2 \sin[x]^2)^4} -$$
$$\frac{17 \sin[x]}{192 (1 - 2 \sin[x]^2)^3} + \frac{203 \sin[x]}{768 (1 - 2 \sin[x]^2)^2} - \frac{437 \sin[x]}{512 (1 - 2 \sin[x]^2)} - \frac{43}{256} \sec[x] \tan[x] - \frac{1}{128} \sec[x]^3 \tan[x]$$

$$\begin{aligned}
& -\frac{523}{256} \operatorname{ArcTanh}[\sin[x]] - \frac{1483 \operatorname{Log}\left[2 + \sqrt{2} + \cos[x] + \sqrt{2} \cos[x] - \sin[x] - \sqrt{2} \sin[x]\right]}{2048 \sqrt{2}} - \frac{1483 \operatorname{Log}\left[2 - \sqrt{2} + \cos[x] - \sqrt{2} \cos[x] + \sin[x] - \sqrt{2} \sin[x]\right]}{2048 \sqrt{2}} + \\
& \frac{1483 \operatorname{Log}\left[2 - \sqrt{2} + \cos[x] - \sqrt{2} \cos[x] - \sin[x] + \sqrt{2} \sin[x]\right]}{2048 \sqrt{2}} + \frac{1483 \operatorname{Log}\left[2 + \sqrt{2} + \cos[x] + \sqrt{2} \cos[x] + \sin[x] + \sqrt{2} \sin[x]\right]}{2048 \sqrt{2}} - \\
& \frac{9372189581}{620780160 \left(1 - \tan\left[\frac{x}{2}\right]\right)^4} - \frac{115444608337}{931170240 \left(1 - \tan\left[\frac{x}{2}\right]\right)^3} - \frac{125776127011}{196035840 \left(1 - \tan\left[\frac{x}{2}\right]\right)^2} - \frac{273389009815}{106419456 \left(1 - \tan\left[\frac{x}{2}\right]\right)} + \\
& \frac{9372189581}{620780160 \left(1 + \tan\left[\frac{x}{2}\right]\right)^4} + \frac{115444608337}{931170240 \left(1 + \tan\left[\frac{x}{2}\right]\right)^3} + \frac{125776127011}{196035840 \left(1 + \tan\left[\frac{x}{2}\right]\right)^2} + \frac{273389009815}{106419456 \left(1 + \tan\left[\frac{x}{2}\right]\right)} - \\
& \frac{2174977520729 + 900895026797 \tan\left[\frac{x}{2}\right]}{465585120 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^4} - \frac{900895026797 \left(1 + \tan\left[\frac{x}{2}\right]\right)}{798145920 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^3} + \frac{38884413000239 + 18217810835525 \tan\left[\frac{x}{2}\right]}{5587021440 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^3} + \\
& \frac{94536076571 \left(1 + \tan\left[\frac{x}{2}\right]\right)}{70946304 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2} - \frac{65580529660561 + 30915686684873 \tan\left[\frac{x}{2}\right]}{7449361920 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2} - \frac{954063574769 \left(1 + \tan\left[\frac{x}{2}\right]\right)}{451476480 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)} + \\
& \frac{137658750217805 + 69771275635067 \tan\left[\frac{x}{2}\right]}{14898723840 \left(1 - 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)} + \frac{2174977520729 - 900895026797 \tan\left[\frac{x}{2}\right]}{465585120 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^4} - \frac{38884413000239 - 18217810835525 \tan\left[\frac{x}{2}\right]}{5587021440 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^3} + \\
& \frac{900895026797 \left(1 - \tan\left[\frac{x}{2}\right]\right)}{798145920 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^3} + \frac{65580529660561 - 30915686684873 \tan\left[\frac{x}{2}\right]}{7449361920 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2} - \frac{94536076571 \left(1 - \tan\left[\frac{x}{2}\right]\right)}{70946304 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2} - \\
& \frac{137658750217805 - 69771275635067 \tan\left[\frac{x}{2}\right]}{14898723840 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)} + \frac{954063574769 \left(1 - \tan\left[\frac{x}{2}\right]\right)}{451476480 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)} + \frac{1486633486121 \tan\left[\frac{x}{2}\right]}{33256080 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \\
& \frac{2833881904961 \tan\left[\frac{x}{2}\right]^3}{232792560 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \frac{801211946693 \tan\left[\frac{x}{2}\right]^5}{232792560 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \\
& \frac{1842220447 \tan\left[\frac{x}{2}\right]^7}{1750320 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \frac{11825071 \tan\left[\frac{x}{2}\right]^9}{51480 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \frac{17348075 \tan\left[\frac{x}{2}\right]^{11}}{72072 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} - \\
& \frac{424843 \tan\left[\frac{x}{2}\right]^{13}}{3960 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \frac{34529 \tan\left[\frac{x}{2}\right]^{15}}{360 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} - \frac{48883 \tan\left[\frac{x}{2}\right]^{17}}{1680 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \\
& \frac{1813 \tan\left[\frac{x}{2}\right]^{19}}{240 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} - \frac{35 \tan\left[\frac{x}{2}\right]^{21}}{48 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4} + \frac{\tan\left[\frac{x}{2}\right]^{23}}{16 \left(1 - 7 \tan\left[\frac{x}{2}\right]^2 + 7 \tan\left[\frac{x}{2}\right]^4 - \tan\left[\frac{x}{2}\right]^6\right)^4}
\end{aligned}$$

Problem #24: Valid but suboptimal antiderivative:

$$\left\{ \frac{\text{Tanh}[x]}{\sqrt{e^x + e^{2x}}}, x, -13, 13 \right\}$$

$$2 e^{-x} \sqrt{e^x + e^{2x}} - \frac{\text{ArcTan}\left[\frac{i - (1-2i)e^x}{2\sqrt{1+i}\sqrt{e^x + e^{2x}}}\right]}{\sqrt{1+i}} + \frac{\text{ArcTan}\left[\frac{i + (1+2i)e^x}{2\sqrt{1-i}\sqrt{e^x + e^{2x}}}\right]}{\sqrt{1-i}}$$

$$\frac{2(1+e^x)}{\sqrt{e^x + e^{2x}}} - \frac{(1-i)^{3/2} \sqrt{e^x} \sqrt{1+e^x} \text{ArcTanh}\left[\frac{\sqrt{1-i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x + e^{2x}}} - \frac{(1+i)^{3/2} \sqrt{e^x} \sqrt{1+e^x} \text{ArcTanh}\left[\frac{\sqrt{1+i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x + e^{2x}}}$$

Problem #26: Valid but suboptimal antiderivative:

$$\left\{ \text{Log}\left[x^2 + \sqrt{1-x^2}\right], x, -34, 34 \right\}$$

$$-2x - \text{ArcSin}[x] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] +$$

$$\sqrt{\frac{1}{2}(-1+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] - \sqrt{\frac{1}{2}(-1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + x \text{Log}\left[x^2 + \sqrt{1-x^2}\right]$$

$$-2x - \text{ArcSin}[x] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] +$$

$$2\sqrt{\frac{1}{5}(2+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \sqrt{\frac{1}{10}(-1+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] -$$

$$2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] - \sqrt{\frac{1}{10}(-1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + x \text{Log}\left[x^2 + \sqrt{1-x^2}\right]$$

Test complete!

# 1 Algebraic functions

## 1.1 Binomial products

### 1.1.1 Linear

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.1 Linear\\1.1.1.2 (a+b x)^m (c+d x)^n"];
```

Testing Rubi on 1916 integration problems...

Problem #369: Valid but suboptimal antiderivative:

$$\left\{ -\frac{b x^m}{2 (a+b x)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a+b x}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+b x}} - \frac{x^m \left(-\frac{b x}{a}\right)^{-m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, -m, \frac{1}{2}, 1 + \frac{b x}{a}\right]}{\sqrt{a+b x}} - \frac{2 m x^m \left(-\frac{b x}{a}\right)^{-m} \sqrt{a+b x} \text{Hypergeometric2F1}\left[\frac{1}{2}, 1-m, \frac{3}{2}, 1 + \frac{b x}{a}\right]}{a}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.1 Linear\\1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p"];
```

Testing Rubi on 3132 integration problems...

Problem #856: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{-1 + \frac{1}{x}} \sqrt{\frac{1}{x}} \sqrt{x}}{\sqrt{1+x}}, x, -6, 6 \right\}$$

$$\frac{2 \sqrt{-x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{-x}\right], -1\right]}{\sqrt{x}} - \frac{2 \sqrt{-1 + \frac{1}{x}} \left(1 + \frac{1}{x}\right) \sqrt{\frac{1}{x}} x^{3/2}}{\sqrt{1+x}} - \frac{2 \sqrt{1 - \frac{1}{x}} \sqrt{1 + \frac{1}{x}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right], -1\right]}{\sqrt{-1 + \frac{1}{x}} \sqrt{\frac{1}{x}} \sqrt{1+x}}$$



Problem #943: Valid but suboptimal antiderivative:

$$\left\{ (e x)^m (a - b x)^{2+n} (a + b x)^n, x, -11, 11 \right\}$$

$$\begin{aligned} & - \frac{(e x)^{1+m} (a - b x)^{1+n} (a + b x)^{1+n}}{e (3 + m + 2 n)} + \frac{2 a^2 (2 + m + n) (e x)^{1+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e (1 + m) (3 + m + 2 n)} \\ & - \frac{2 a b (e x)^{2+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^2 (2 + m)} \\ & - \frac{a^2 (e x)^{1+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e (1 + m)} \\ & - \frac{2 a b (e x)^{2+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^2 (2 + m)} + \\ & - \frac{b^2 (e x)^{3+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{3+m}{2}, -n, \frac{5+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^3 (3 + m)} \end{aligned}$$

Problem #987: Valid but suboptimal antiderivative:

$$\left\{ \frac{(1 - a x)^{1-n} (1 + a x)^{1+n}}{x^2}, x, -1, 1 \right\}$$

$$\begin{aligned} & - \frac{(1 - a x)^{-n} (1 + a x)^n (1 + a^2 x^2)}{x} - \frac{2 a n (1 - a x)^{1-n} (1 + a x)^{-1+n} \text{Hypergeometric2F1}\left[1, 1 - n, 2 - n, \frac{1 - a x}{1 + a x}\right]}{1 - n} \\ & - \frac{2^{-n} a n (1 + a x)^{1+n} \text{Hypergeometric2F1}\left[1 + n, 1 + n, 2 + n, \frac{1}{2} (1 + a x)\right]}{1 + n} \\ & - \frac{2^{1-n} a (1 + a x)^{2+n} \text{AppellF1}\left[2 + n, -1 + n, 2, 3 + n, \frac{1}{2} (1 + a x), 1 + a x\right]}{2 + n} \end{aligned}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.1 Linear\\1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q"];
```

Testing Rubi on 83 integration problems...

Problem #73: Unable to integrate:

$$\left\{ \frac{1}{(a+bx)^{3/2} (c+dx)^{3/2} \sqrt{e+fx} \sqrt{g+hx}}, x, -1, 0 \right\}$$

$$-\frac{2d^3 \sqrt{a+bx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (de-cf) (dg-ch) \sqrt{c+dx}} - \frac{2b^3 \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (be-af) (bg-ah) \sqrt{a+bx}} +$$

$$\frac{2b(a^2 d^2 fh - ab d^2 (fg+eh) + b^2 (2d^2 eg + c^2 fh - cd (fg+eh))) \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (be-af) (de-cf) (bg-ah) (dg-ch) \sqrt{a+bx}} -$$

$$\left( 2\sqrt{fg-eh} (a^2 d^2 fh - ab d^2 (fg+eh) + b^2 (2d^2 eg + c^2 fh - cd (fg+eh))) \right.$$

$$\left. \sqrt{c+dx} \sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{bg-ah} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{a+bx}}\right], -\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right] \right) /$$

$$\left( (bc-ad)^2 (be-af) (de-cf) \sqrt{bg-ah} (dg-ch) \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}} \sqrt{g+hx} \right) -$$

$$\frac{4bd \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}} \sqrt{g+hx} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{bg-ah} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{a+bx}}\right], -\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right]}{(bc-ad)^2 \sqrt{bg-ah} \sqrt{fg-eh} \sqrt{c+dx} \sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a+bx)^{3/2} (c+dx)^{3/2} \sqrt{e+fx} \sqrt{g+hx}}, x\right]$$

Test complete!

## 1.1.2 Quadratic

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.2 (c x)^m (a+b x^2)^p"];
```

Testing Rubi on 1071 integration problems...

Problem #662: Valid but suboptimal antiderivative:

$$\left\{ \frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}, x, -5, 5 \right\}$$

$$x^{2+m} \sqrt{a+bx^2}$$

$$\frac{ax^{2+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{4+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, -\frac{bx^2}{a}\right]}{(4+m)\sqrt{a+bx^2}}$$

Problem #664: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{bx^{1+m}}{(a+bx^2)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^2}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx^2}}$$

$$-\frac{bx^{2+m}}{a\sqrt{a+bx^2}} + \frac{x^m \sqrt{a+bx^2}}{a}$$

$$\frac{x^m \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, -\frac{bx^2}{a}\right]}{\sqrt{a+bx^2}} - \frac{bx^{2+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]}{a(2+m)\sqrt{a+bx^2}}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.3 (a+b x^2)^p (c+d x^2)^q"];
```

Testing Rubi on 335 integration problems...

Problem #287: Valid but suboptimal antiderivative:

$$\left\{ \frac{(1-2x^2)^m}{\sqrt{1-x^2}}, x, -1, 1 \right\}$$

$$\frac{2^{-2-m} \sqrt{x^2} (2-4x^2)^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, (1-2x^2)^2\right]}{(1+m)x} \\ \times \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right]$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q"];
```

Testing Rubi on 1154 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r"];
```

Testing Rubi on 115 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r"];
```

Testing Rubi on 51 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.2 Quadratic\\1.1.2.8 (c x)^m Pq(x) (a+b x^2)^p"];
```

Testing Rubi on 174 integration problems...

Test complete!

### 1.1.3 General

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.3 General\\1.1.3.2 (c x)^m (a+b x^n)^p"];
```

Testing Rubi on 2912 integration problems...

Problem #2674: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^n}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx^n}}$$

$$-\frac{bx^{m+n}}{a\sqrt{a+bx^n}} + \frac{x^m\sqrt{a+bx^n}}{a}$$

$$\frac{x^m\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{bx^n}{a}\right]}{\sqrt{a+bx^n}} - \frac{bnx^{m+n}\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2+\frac{m}{n}, -\frac{bx^n}{a}\right]}{2a(m+n)\sqrt{a+bx^n}}$$

Problem #2685: Valid but not optimal or suboptimal antiderivative:

$$\left\{ \frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}} + \frac{x^m}{\sqrt{a+bx^{-2+m}}}, x, -5, 5 \right\}$$

$$\frac{2x^3\sqrt{a+bx^{-2+m}}}{b(4+m)}$$

$$\frac{2x^{1+m}(b+ax^{2-m})}{b(4+m)\sqrt{a+bx^{-2+m}}}$$

$$\frac{2ax^3\sqrt{1+\frac{bx^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{3}{2-m}, -\frac{1+m}{2-m}, -\frac{bx^{-2+m}}{a}\right]}{b(4+m)\sqrt{a+bx^{-2+m}}} + \frac{x^{1+m}\sqrt{1+\frac{bx^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1+m}{2-m}, \frac{1-2m}{2-m}, -\frac{bx^{-2+m}}{a}\right]}{(1+m)\sqrt{a+bx^{-2+m}}}$$

Problem #2687: Valid but not optimal or suboptimal antiderivative:

$$\left\{ -\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^n}}, x, -5, 5 \right\}$$

$$\frac{x^m}{\sqrt{a+bx^n}}$$

$$-\frac{bx^{m+n}}{a\sqrt{a+bx^n}} + \frac{x^m\sqrt{a+bx^n}}{a}$$

$$\frac{x^m\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{bx^n}{a}\right]}{\sqrt{a+bx^n}} - \frac{bnx^{m+n}\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2+\frac{m}{n}, -\frac{bx^n}{a}\right]}{2a(m+n)\sqrt{a+bx^n}}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.3 General\\1.1.3.3 (a+b x^n)^p (c+d x^n)^q"];
```

Testing Rubi on 285 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.3 General\\1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q"];
```

Testing Rubi on 728 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.3 General\\1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r"];
```

Testing Rubi on 46 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.3 General\\1.1.3.8 (c x)^m Pq(x) (a+b x^n)^p"];
```

Testing Rubi on 540 integration problems...

Problem #536: Unable to integrate:

$$\left\{ \frac{1+x^3}{(1-x^4)(1+x^4)^{1/4}}, x, -25, 25 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2^{1/4}x}{(1+x^4)^{1/4}}\right]}{2 \times 2^{1/4}} - \frac{\text{ArcTan}\left[\frac{(1+x^4)^{1/4}}{2^{1/4}}\right]}{2 \times 2^{1/4}} + \frac{\text{ArcTanh}\left[\frac{2^{1/4}x}{(1+x^4)^{1/4}}\right]}{2 \times 2^{1/4}} + \frac{\text{ArcTanh}\left[\frac{(1+x^4)^{1/4}}{2^{1/4}}\right]}{2 \times 2^{1/4}}$$

$$- \frac{1}{6} x^3 \text{AppellF1}\left[\frac{3}{4}, 1, \frac{1}{4}, \frac{7}{4}, x^4, -x^4\right] + \frac{\text{ArcTan}\left[\frac{2^{1/4}x}{(1+x^4)^{1/4}}\right]}{4 \times 2^{1/4}} - \frac{\text{ArcTan}\left[\frac{(1+x^4)^{1/4}}{2^{1/4}}\right]}{4 \times 2^{1/4}} + \frac{\text{ArcTanh}\left[\frac{2^{1/4}x}{(1+x^4)^{1/4}}\right]}{4 \times 2^{1/4}} + \frac{\text{ArcTanh}\left[\frac{(1+x^4)^{1/4}}{2^{1/4}}\right]}{4 \times 2^{1/4}} -$$

$$\frac{1}{2} \text{CannotIntegrate}\left[\frac{1}{(-1+x)(1+x^4)^{1/4}}, x\right] + \frac{\sqrt{-x^4} \text{EllipticPi}\left[-\frac{1}{\sqrt{2}}, \text{ArcSin}\left[(1+x^4)^{1/4}\right], -1\right]}{4 \sqrt{2} x^2} - \frac{\sqrt{-x^4} \text{EllipticPi}\left[\frac{1}{\sqrt{2}}, \text{ArcSin}\left[(1+x^4)^{1/4}\right], -1\right]}{4 \sqrt{2} x^2}$$

Test complete!

## 1.1.4 Improper

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.4 Improper\\1.1.4.2 (c x)^m (a x^j+b x^n)^p"];
```

Testing Rubi on 454 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.1 Binomial products\\1.1.4 Improper\\1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q"];
```

Testing Rubi on 298 integration problems...

Test complete!

## 1.2 Trinomial products

### 1.2.1 Quadratic

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.1 (a+b x+c x^2)^p"];
```

Testing Rubi on 143 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.2 (d+e x)^m (a+b x+c x^2)^p"];
```

Testing Rubi on 2584 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p"];
```

Testing Rubi on 2645 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p"];
```

Testing Rubi on 860 integration problems...

Problem #745: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2}, x, -9, 9 \right\}$$

$$-\text{ArcCosh}[x] + \sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right]$$

$$\frac{\sqrt{\frac{1}{10}(-1+\sqrt{5})}\sqrt{-1+x}\sqrt{1+x} \text{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}} -$$

$$\frac{\sqrt{-1+x}\sqrt{1+x} \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}} - \frac{\sqrt{\frac{1}{10}(1+\sqrt{5})}\sqrt{-1+x}\sqrt{1+x} \text{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q"];
```

Testing Rubi on 123 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q"];
```

Testing Rubi on 143 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.1 Quadratic\\1.2.1.9 (d+e x)^m Pq(x) (a+b x+c x^2)^p"];
```

Testing Rubi on 384 integration problems...

Test complete!

## 1.2.2 Quartic

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.2 Quartic\\1.2.2.2 (d x)^m (a+b x^2+c x^4)^p"];
```



Testing Rubi on 1126 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.2 Quartic\\1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p"];
```

Testing Rubi on 331 integration problems...

Problem #140: Unable to integrate:

$$\left\{ \frac{\sqrt{a + b x^2}}{\sqrt{1 - x^4}}, x, -1, 0 \right\}$$

$$\frac{a \sqrt{1 - x^2} \sqrt{\frac{a(1+x^2)}{a+b x^2}} \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} x}{\sqrt{a+b x^2}}\right], -\frac{a-b}{a+b}\right]}{\sqrt{a+b} \sqrt{1+x^2} \sqrt{\frac{a(1-x^2)}{a+b x^2}}}$$

$$\operatorname{Unintegrable}\left[\frac{\sqrt{a + b x^2}}{\sqrt{1 - x^4}}, x\right]$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.2 Quartic\\1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p"];
```

Testing Rubi on 374 integration problems...

Problem #336: Valid but suboptimal antiderivative:

$$\left\{ \frac{(d + e x^2)^{3/2}}{x^2 (a + b x^2 + c x^4)}, x, -16, 16 \right\}$$

$$\begin{aligned} & -\frac{d \sqrt{d + e x^2}}{a x} - \frac{(2 c d - (b - \sqrt{b^2 - 4 a c}) e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{\sqrt{b^2 - 4 a c} (b - \sqrt{b^2 - 4 a c})^{3/2}} + \frac{(2 c d - (b + \sqrt{b^2 - 4 a c}) e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{\sqrt{b^2 - 4 a c} (b + \sqrt{b^2 - 4 a c})^{3/2}} \\ & -\frac{d \sqrt{d + e x^2}}{a x} - \frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} \left(d + \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b - \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b - \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{2 a \sqrt{b - \sqrt{b^2 - 4 a c}}} \\ & \frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} \left(d - \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2 c d - (b + \sqrt{b^2 - 4 a c}) e} x}{\sqrt{b + \sqrt{b^2 - 4 a c}} \sqrt{d + e x^2}}\right]}{2 a \sqrt{b + \sqrt{b^2 - 4 a c}}} + \frac{d \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{a} \\ & \frac{\sqrt{e} \left(d - \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{2 a} - \frac{\sqrt{e} \left(d + \frac{b d - 2 a e}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{2 a} \end{aligned}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.2 Quartic\\1.2.2.5 Pq(x) (a+b x^2+c x^4)^p"];
```

Testing Rubi on 111 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.2 Quartic\\1.2.2.6 (d x)^m Pq(x) (a+b x^2+c x^4)^p"];
```

Testing Rubi on 145 integration problems...

Test complete!

## 1.2.3 General

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.3 General\\1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p"];
```

Testing Rubi on 664 integration problems...

Problem #24: Valid but suboptimal antiderivative:

$$\{x^8 (a^2 + 2 a b x^3 + b^2 x^6)^{3/2}, x, -4, 4\}$$

$$\frac{a^2 (a + b x^3)^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{12 b^3} - \frac{2 a (a + b x^3)^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{15 b^3} + \frac{(a + b x^3)^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{18 b^3}$$

$$\frac{a^3 x^9 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{9 (a + b x^3)} + \frac{a^2 b x^{12} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 (a + b x^3)} + \frac{a b^2 x^{15} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 (a + b x^3)} + \frac{b^3 x^{18} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{18 (a + b x^3)}$$

Problem #478: Valid but suboptimal antiderivative:

$$\left\{ \frac{(a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{x^2} - \frac{2 b^3 (1 - 2 p) (1 - p) p (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{3 a^3 x}, x, -7, 7 \right\}$$

$$- \frac{(a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a x} + \frac{b (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^2 x^{2/3}} - \frac{b^2 (1 - 2 p) (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^3 x^{1/3}}$$

$$\frac{1}{a^3 (1 + 2 p)} 2 b^3 (1 - 2 p) (1 - p) p \left( 1 + \frac{b x^{1/3}}{a} \right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[ 1, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right] +$$

$$\frac{3 b^3 \left( 1 + \frac{b x^{1/3}}{a} \right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[ 4, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right]}{a^3 (1 + 2 p)}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.3 General\\1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p"];
```

Testing Rubi on 96 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.3 General\\1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p"];
```

Testing Rubi on 131 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.3 General\\1.2.3.5 (d x)^m Pq(x) (a+b x^n+c x^(2 n))^p"];
```

Testing Rubi on 17 integration problems...

Test complete!

## 1.2.4 Improper

```
IntegrationTest["1 Algebraic functions\\1.2 Trinomial products\\1.2.4 Improper\\1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p"];
```

Testing Rubi on 140 integration problems...

Test complete!

## 1.3 Miscellaneous

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.1 Rational functions"];
```

Testing Rubi on 296 integration problems...

Problem #176: Valid but suboptimal antiderivative:

$$\left\{ \frac{-20x + 4x^2}{9 - 10x^2 + x^4}, x, -11, 11 \right\}$$

$$\text{Log}[1 - x] - \frac{1}{2} \text{Log}[3 - x] + \frac{3}{2} \text{Log}[1 + x] - 2 \text{Log}[3 + x]$$

$$-\frac{3}{2} \text{ArcTanh}\left[\frac{x}{3}\right] + \frac{\text{ArcTanh}[x]}{2} + \frac{5}{4} \text{Log}[1 - x^2] - \frac{5}{4} \text{Log}[9 - x^2]$$

Problem #225: Unable to integrate:

$$\left\{ \frac{(1 + x^2)^2}{a x^6 + b (1 + x^2)^3}, x, -5, 5 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} b^{5/6}}$$

$$\text{CannotIntegrate}\left[\frac{1}{a x^6 + b (1 + x^2)^3}, x\right] + 2 \text{CannotIntegrate}\left[\frac{x^2}{a x^6 + b (1 + x^2)^3}, x\right] + \text{CannotIntegrate}\left[\frac{x^4}{a x^6 + b (1 + x^2)^3}, x\right]$$

Problem #230: Unable to integrate:

$$\left\{ \frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3+x+x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2}, x, -7, 7 \right\}$$

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3+x+x^4)^3}$$

$$- \frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} - \frac{621}{4} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] +$$

$$684 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] + 360 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] + 44 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] -$$

$$320 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right] - 75 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^3}, x\right] + 30 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^2}, x\right]$$

Problem #231: Unable to integrate:

$$\left\{ \frac{-3 + 10x + 4x^3 - 30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}, x, -13, 13 \right\}$$

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3+x+x^4)^3}$$

$$\frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} -$$

$$\frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{144}{11} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] + \frac{828}{11} \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] +$$

$$18 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] - 4 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] - 20 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right]$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.2 Algebraic functions"];
```

Testing Rubi on 457 integration problems...

Problem #120: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}, x, -5, 5 \right\}$$

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\sqrt{1+x^n}}{2+n}$$

$$\frac{x \sqrt{a x^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{1}{n}, 2 + \frac{1}{n}, -x^n\right]}{1+n} + \frac{2 x^{1-n} \sqrt{a x^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -x^n\right]}{2+n}$$

Problem #280: Unable to integrate:

$$\left\{ \frac{(d^3 + e^3 x^3)^p}{d + e x}, x, -1, 0 \right\}$$

$$\frac{(d^3 + e^3 x^3)^p \left(1 + \frac{2(d+ex)}{(-3+i\sqrt{3})d}\right)^{-p} \left(1 - \frac{2(d+ex)}{(3+i\sqrt{3})d}\right)^{-p} \operatorname{AppellF1}\left[p, -p, -p, 1+p, -\frac{2(d+ex)}{(-3+i\sqrt{3})d}, \frac{2(d+ex)}{(3+i\sqrt{3})d}\right]}{e^p}$$

$$\operatorname{CannotIntegrate}\left[\frac{(d^3 + e^3 x^3)^p}{d + e x}, x\right]$$

Problem #375: Valid but suboptimal antiderivative:

$$\left\{ (1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6}, x, -3, 3 \right\}$$

$$-\frac{(1-x^6)^{2/3}}{5x^5} + \frac{1}{5}x(1-x^6)^{2/3}$$

$$-\frac{\operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, -\frac{2}{3}, \frac{1}{6}, x^6\right]}{5x^5} + x \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{6}, \frac{7}{6}, x^6\right]$$

Problem #433: Unable to integrate:

$$\left\{ \sqrt{1-x^2+x\sqrt{-1+x^2}}, x, -1, 0 \right\}$$

$$\frac{1}{4} \left( 3x + \sqrt{-1+x^2} \right) \sqrt{1-x^2+x\sqrt{-1+x^2}} + \frac{3 \operatorname{ArcSin}\left[x - \sqrt{-1+x^2}\right]}{4\sqrt{2}}$$

$$\operatorname{CannotIntegrate}\left[\sqrt{1-x^2+x\sqrt{-1+x^2}}, x\right]$$

Problem #434: Unable to integrate:

$$\left\{ \frac{\sqrt{-x+\sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}}, x, -1, 1 \right\}$$

$$\frac{1}{2} (\sqrt{x} + 3\sqrt{1+x}) \sqrt{-x + \sqrt{x} \sqrt{1+x}} - \frac{3 \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2\sqrt{2}}$$

$$2 \operatorname{Subst}\left[\operatorname{CannotIntegrate}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}, x\right], x, \sqrt{1+x}\right]$$

Problem #435: Valid but suboptimal antiderivative:

$$\left\{-\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}, x, -25, 25\right\}$$

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{-2+\sqrt{5}}(x+\sqrt{1+x^2})\right] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{2+\sqrt{5}}(x+\sqrt{1+x^2})\right]$$

$$-2\sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] -$$

$$\sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] - \sqrt{\frac{2}{5}(-1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] - 2\sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] +$$

$$\sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] - \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right]$$

Problem #455: Unable to integrate:

$$\left\{\frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}, x, -5, 5\right\}$$

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[1+2(1-x)^3-x^3\right]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log}\left[2^{1/3}(1-x)+(1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

$$-(1+i\sqrt{3}) \operatorname{CannotIntegrate}\left[\frac{1}{(-1-i\sqrt{3}+2x)(1-x^3)^{2/3}}, x\right] -$$

$$(1-i\sqrt{3}) \operatorname{CannotIntegrate}\left[\frac{1}{(-1+i\sqrt{3}+2x)(1-x^3)^{2/3}}, x\right] - x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right]$$

Problem #456: Valid but suboptimal antiderivative:

$$\left\{ \frac{x^2}{\sqrt{-1+x^4} (1+x^4)}, x, -17, 17 \right\}$$

$$-\frac{1}{4} \operatorname{ArcTan} \left[ \frac{1+x^2}{x \sqrt{-1+x^4}} \right] - \frac{1}{4} \operatorname{ArcTanh} \left[ \frac{1-x^2}{x \sqrt{-1+x^4}} \right]$$

$$\frac{i \sqrt{1-x^2} \sqrt{1+x^2} \operatorname{EllipticPi}[-i, \operatorname{ArcSin}[x], -1]}{2 \sqrt{-1+x^4}} - \frac{i \sqrt{1-x^2} \sqrt{1+x^2} \operatorname{EllipticPi}[i, \operatorname{ArcSin}[x], -1]}{2 \sqrt{-1+x^4}}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.3 Expansion problems"];
```

Testing Rubi on 110 integration problems...

Problem #94: Boo; correct but **31** steps required!

$$\left\{ \frac{-1 + \sqrt{1-x^2}}{\sqrt{1-x^2} (2+x-2\sqrt{1-x^2})^2}, x, 0, \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} \right\}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.4 Substitution problems"];
```



Testing Rubi on 364 integration problems...

Problem #54: Valid but suboptimal antiderivative:

$$\{b x^{1+p} (b x + c x^3)^p + 2 c x^{3+p} (b x + c x^3)^p, x, -7, 7\}$$

$$\frac{x^{1+p} (b x + c x^3)^{1+p}}{2 (1+p)}$$

$$\frac{b x^{2+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \text{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{c x^2}{b}\right]}{2 (1+p)} + \frac{c x^{4+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \text{Hypergeometric2F1}\left[-p, 2+p, 3+p, -\frac{c x^2}{b}\right]}{2+p}$$

Problem #59: Valid but suboptimal antiderivative:

$$\{(1+2x)(x+x^2)^3(-18+7(x+x^2)^3)^2, x, -3, 3\}$$

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Problem #60: Valid but suboptimal antiderivative:

$$\{x^3 (1+x)^3 (1+2x) (-18+7x^3(1+x)^3)^2, x, -2, 2\}$$

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.5 Piecewise constant extraction"];
```

Testing Rubi on 66 integration problems...

Test complete!

```
IntegrationTest["1 Algebraic functions\\1.3 Miscellaneous\\1.3.6 Derivative integration problems"];
```

Testing Rubi on 97 integration problems...

Problem #24: Unable to integrate:

$$\{g[x] f'[x] + f[x] g'[x], x, -1, 1\}$$

$$f[x] g[x]$$

CannotIntegrate[g[x] f'[x], x] + CannotIntegrate[f[x] g'[x], x]

Problem #43: Unable to integrate:

$$\{\cos[x] g[e^x] f'[\sin[x]] + e^x f[\sin[x]] g'[e^x], x, -1, 1\}$$

$$f[\sin[x]] g[e^x]$$

CannotIntegrate[Cos[x] g[e^x] f'[Sin[x]], x] + CannotIntegrate[e^x f[Sin[x]] g'[e^x], x]

Test complete!

## 2 Exponentials

```
IntegrationTest["2 Exponentials\\2.1 u (F^(c (a+b x)))^n"];
```

Testing Rubi on 98 integration problems...

Test complete!

```
IntegrationTest["2 Exponentials\\2.2 (c+d x)^m (F^(g (e+f x)))^n (a+b (F^(g (e+f x)))^n)^p"];
```

Testing Rubi on 88 integration problems...

Test complete!

```
IntegrationTest["2 Exponentials\\2.3 Exponential functions"];
```

Testing Rubi on 736 integration problems...

Problem #63: Unable to integrate:

$$\left\{ \frac{x}{(b f^{-x} + a f^x)^3}, x, -1, 1 \right\}$$

$$\frac{f^x}{8 a b (b + a f^{2x}) \operatorname{Log}[f]^2} - \frac{f^x x}{4 a (b + a f^{2x})^2 \operatorname{Log}[f]} + \frac{f^x x}{8 a b (b + a f^{2x}) \operatorname{Log}[f]} + \frac{x \operatorname{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \operatorname{Log}[f]} - \frac{i \operatorname{PolyLog}\left[2, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{16 a^{3/2} b^{3/2} \operatorname{Log}[f]^2} + \frac{i \operatorname{PolyLog}\left[2, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{16 a^{3/2} b^{3/2} \operatorname{Log}[f]^2}$$

$$\text{CannotIntegrate}\left[\frac{f^{3x} x}{(b + a f^{2x})^3}, x\right]$$

Problem #64: Unable to integrate:

$$\left\{\frac{x^2}{(b f^{-x} + a f^x)^3}, x, -1, 1\right\}$$

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{4 a^{3/2} b^{3/2} \text{Log}[f]^3} + \frac{f^x x}{4 a b (b + a f^{2x}) \text{Log}[f]^2} - \frac{f^x x^2}{4 a (b + a f^{2x})^2 \text{Log}[f]} + \frac{f^x x^2}{8 a b (b + a f^{2x}) \text{Log}[f]} + \\ & \frac{x^2 \text{ArcTan}\left[\frac{\sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]} - \frac{i x \text{PolyLog}\left[2, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^2} + \frac{i x \text{PolyLog}\left[2, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^2} + \frac{i \text{PolyLog}\left[3, -\frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^3} - \frac{i \text{PolyLog}\left[3, \frac{i \sqrt{a} f^x}{\sqrt{b}}\right]}{8 a^{3/2} b^{3/2} \text{Log}[f]^3} \end{aligned}$$

$$\text{CannotIntegrate}\left[\frac{f^{3x} x^2}{(b + a f^{2x})^3}, x\right]$$

Problem #654: Unable to integrate:

$$\{e^{x^x} x^{2x} (1 + \text{Log}[x]), x, -2, 2\}$$

$$e^{x^x} (-1 + x^x)$$

$$\text{CannotIntegrate}[e^{x^x} x^{2x}, x] + \text{CannotIntegrate}[e^{x^x} x^{2x} \text{Log}[x], x]$$

Problem #656: Unable to integrate:

$$\left\{x^{-2-\frac{1}{x}} (1 - \text{Log}[x]), x, -2, 2\right\}$$

$$-x^{-1/x}$$

$$\text{CannotIntegrate}\left[x^{-2-\frac{1}{x}}, x\right] - \text{CannotIntegrate}\left[x^{-2-\frac{1}{x}} \text{Log}[x], x\right]$$

Test complete!

## 3 Logarithms

```
IntegrationTest["3 Logarithms\\3.1 u (a+b log(c x^n))^p"];
```

Testing Rubi on 657 integration problems...

Test complete!

```
IntegrationTest["3 Logarithms\\3.2 u (a+b log(c (d+e x)^n))^p"];
```

Testing Rubi on 542 integration problems...

Problem #371: Unable to integrate:

$$\left\{ \frac{\text{Log}[f x^m] (a + b \text{Log}[c (d + e x)^n])^2}{x}, x, -1, 1 \right\}$$

$$\begin{aligned} & \frac{1}{2} m \text{Log}[x]^2 (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n])^2 + \text{Log}[x] (-m \text{Log}[x] + \text{Log}[f x^m]) (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n])^2 + \\ & 2 b n (-m \text{Log}[x] + \text{Log}[f x^m]) (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n]) \left( \text{Log}[x] \left( \text{Log}[d + e x] - \text{Log}\left[1 + \frac{e x}{d}\right]\right) - \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \\ & 2 b m n (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n]) \left( \frac{1}{2} \text{Log}[x]^2 \left( \text{Log}[d + e x] - \text{Log}\left[1 + \frac{e x}{d}\right]\right) - \text{Log}[x] \text{PolyLog}\left[2, -\frac{e x}{d}\right] + \text{PolyLog}\left[3, -\frac{e x}{d}\right] \right) - \\ & b^2 n^2 (m \text{Log}[x] - \text{Log}[f x^m]) \left( \text{Log}\left[-\frac{e x}{d}\right] \text{Log}[d + e x]^2 + 2 \text{Log}[d + e x] \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] - 2 \text{PolyLog}\left[3, 1 + \frac{e x}{d}\right] \right) + \\ & \frac{1}{12} b^2 m n^2 \left( \text{Log}\left[-\frac{e x}{d}\right]^4 + 6 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d + e x}\right]^2 - 4 \left( \text{Log}\left[-\frac{e x}{d}\right] + \text{Log}\left[\frac{d}{d + e x}\right] \right) \text{Log}\left[-\frac{e x}{d + e x}\right]^3 + \right. \\ & \quad \left. \text{Log}\left[-\frac{e x}{d + e x}\right]^4 + 6 \text{Log}[x]^2 \text{Log}[d + e x]^2 + 4 \left( 2 \text{Log}\left[-\frac{e x}{d}\right]^3 - 3 \text{Log}[x]^2 \text{Log}[d + e x] \right) \text{Log}\left[1 + \frac{e x}{d}\right] + \right. \\ & \quad \left. 6 \left( \text{Log}[x] - \text{Log}\left[-\frac{e x}{d}\right] \right) \left( \text{Log}[x] + 3 \text{Log}\left[-\frac{e x}{d}\right] \right) \text{Log}\left[1 + \frac{e x}{d}\right]^2 - 4 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d + e x}\right] \left( \text{Log}\left[-\frac{e x}{d}\right] + 3 \text{Log}\left[1 + \frac{e x}{d}\right] \right) + \right. \\ & \quad \left. 12 \left( \text{Log}\left[-\frac{e x}{d}\right]^2 - 2 \text{Log}\left[-\frac{e x}{d}\right] \left( \text{Log}\left[-\frac{e x}{d + e x}\right] + \text{Log}\left[1 + \frac{e x}{d}\right] \right) + 2 \text{Log}[x] \left( -\text{Log}[d + e x] + \text{Log}\left[1 + \frac{e x}{d}\right] \right) \right) \text{PolyLog}\left[2, -\frac{e x}{d}\right] - \right. \\ & \quad \left. 12 \text{Log}\left[-\frac{e x}{d + e x}\right]^2 \text{PolyLog}\left[2, \frac{e x}{d + e x}\right] + 12 \left( \text{Log}\left[-\frac{e x}{d}\right] - \text{Log}\left[-\frac{e x}{d + e x}\right] \right)^2 \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + 24 \left( \text{Log}[x] - \text{Log}\left[-\frac{e x}{d}\right] \right) \right. \\ & \quad \left. \text{Log}\left[1 + \frac{e x}{d}\right] \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + 24 \left( \text{Log}\left[-\frac{e x}{d + e x}\right] + \text{Log}[d + e x] \right) \text{PolyLog}\left[3, -\frac{e x}{d}\right] + 24 \text{Log}\left[-\frac{e x}{d + e x}\right] \text{PolyLog}\left[3, \frac{e x}{d + e x}\right] + \right. \\ & \quad \left. 24 \left( -\text{Log}[x] + \text{Log}\left[-\frac{e x}{d + e x}\right] \right) \text{PolyLog}\left[3, 1 + \frac{e x}{d}\right] - 24 \left( \text{PolyLog}\left[4, -\frac{e x}{d}\right] + \text{PolyLog}\left[4, \frac{e x}{d + e x}\right] - \text{PolyLog}\left[4, 1 + \frac{e x}{d}\right] \right) \right) \\ & \frac{\text{Log}[f x^m]^2 (a + b \text{Log}[c (d + e x)^n])^2}{2 m} - \frac{b e n \text{Unintegrable}\left[\frac{\text{Log}[f x^m]^2 (a + b \text{Log}[c (d + e x)^n])}{d + e x}, x\right]}{m} \end{aligned}$$

Problem #375: Unable to integrate:

$$\left\{ \frac{\text{Log}[x] \text{Log}[a + b x]^2}{x}, x, -1, 1 \right\}$$

$$\begin{aligned}
& \frac{1}{12} \left( \text{Log}\left[-\frac{bx}{a}\right]^4 + 6 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 - 4 \left( \text{Log}\left[-\frac{bx}{a}\right] + \text{Log}\left[\frac{a}{a+bx}\right] \right) \text{Log}\left[-\frac{bx}{a+bx}\right]^3 + \right. \\
& \quad \left. \text{Log}\left[-\frac{bx}{a+bx}\right]^4 + 6 \text{Log}[x]^2 \text{Log}[a+bx]^2 + 4 \left( 2 \text{Log}\left[-\frac{bx}{a}\right]^3 - 3 \text{Log}[x]^2 \text{Log}[a+bx] \right) \text{Log}\left[1 + \frac{bx}{a}\right] + \right. \\
& \quad 6 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \left( \text{Log}[x] + 3 \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right]^2 - 4 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right] \left( \text{Log}\left[-\frac{bx}{a}\right] + 3 \text{Log}\left[1 + \frac{bx}{a}\right] \right) + \\
& \quad 12 \left( \text{Log}\left[-\frac{bx}{a}\right]^2 - 2 \text{Log}\left[-\frac{bx}{a}\right] \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) + 2 \text{Log}[x] \left( -\text{Log}[a+bx] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) \right) \text{PolyLog}\left[2, -\frac{bx}{a}\right] - \\
& \quad 12 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 \text{PolyLog}\left[2, \frac{bx}{a+bx}\right] + 12 \left( \text{Log}\left[-\frac{bx}{a}\right] - \text{Log}\left[-\frac{bx}{a+bx}\right] \right)^2 \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + \\
& \quad 24 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right] \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + 24 \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}[a+bx] \right) \text{PolyLog}\left[3, -\frac{bx}{a}\right] + 24 \text{Log}\left[-\frac{bx}{a+bx}\right] \text{PolyLog}\left[3, \frac{bx}{a+bx}\right] + \\
& \quad 24 \left( -\text{Log}[x] + \text{Log}\left[-\frac{bx}{a+bx}\right] \right) \text{PolyLog}\left[3, 1 + \frac{bx}{a}\right] - 24 \left( \text{PolyLog}\left[4, -\frac{bx}{a}\right] + \text{PolyLog}\left[4, \frac{bx}{a+bx}\right] - \text{PolyLog}\left[4, 1 + \frac{bx}{a}\right] \right) \Big) \\
& \frac{1}{2} \text{Log}[x]^2 \text{Log}[a+bx]^2 - b \text{Unintegrable}\left[\frac{\text{Log}[x]^2 \text{Log}[a+bx]}{a+bx}, x\right]
\end{aligned}$$

Test complete!

```
IntegrationTest["3 Logarithms\\3.3 u (a+b log(c (d+e x^m)^n))^p"];
```

Testing Rubi on 436 integration problems...

Test complete!

```
IntegrationTest["3 Logarithms\\3.4 u log(e (f (a+b x)^p (c+d x)^q)^r)^s"];
```

Testing Rubi on 192 integration problems...

Problem #148: Unable to integrate:

$$\left\{ \frac{1}{(c+dx)(-a+c+(-b+d)x)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]} + \frac{\operatorname{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(a+bx)(c+dx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x, -3, 3 \right\}$$

$$- \frac{\operatorname{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(bc-ad)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}$$

$$\frac{b \operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(a+bx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} - \frac{d \operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(c+dx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} + \operatorname{Unintegrable}\left[\frac{1}{(c+dx)(-a+c+(-b+d)x)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}, x\right]$$

Problem #149: Unable to integrate:

$$\left\{ -\frac{1}{(a+bx)(a-c+(b-d)x)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]} + \frac{\operatorname{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(a+bx)(c+dx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x, -3, 3 \right\}$$

$$- \frac{\operatorname{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(bc-ad)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}$$

$$\frac{b \operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(a+bx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} - \frac{d \operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(c+dx)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} - \operatorname{Unintegrable}\left[\frac{1}{(a+bx)(a-c+(b-d)x)\operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}, x\right]$$

Test complete!

```
IntegrationTest["3 Logarithms\\3.5 Logarithm functions"];
```

Testing Rubi on 286 integration problems...

Test complete!

## 4 Trig functions

### 4.1a Sine

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.0 (a sin)^m (b trg)^n"];
```

Testing Rubi on 538 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.1.1 (a+b sin)^n"];
```

Testing Rubi on 72 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.1.2 (g cos)^p (a+b sin)^m"];
```

Testing Rubi on 653 integration problems...

Problem #648: Valid but suboptimal antiderivative:

{ (e Cos [c + d x])<sup>-3-m</sup> (a + b Sin [c + d x])<sup>m</sup>, x, -5, 5 }

$$\frac{(e \cos [c + d x])^{-m} \operatorname{Sec}[c + d x]^4 (-1 + \sin [c + d x]) (1 + \sin [c + d x]) (a + b \sin [c + d x])^{1+m}}{(a - b) d e^3 (2 + m)} + \frac{1}{(a - b)^2 d e^3 m (2 + m)}$$

$$\frac{(-2 b + a (2 + m)) (e \cos [c + d x])^{-m} \operatorname{Sec}[c + d x]^4 (-1 + \sin [c + d x]) (1 + \sin [c + d x])^2 (a + b \sin [c + d x])^{1+m}}{(a - b)^3 d e^3 m (1 + m)} - \frac{1}{(a - b)^3 d e^3 m (1 + m)} (-b^2 + a^2 (1 + m)) (e \cos [c + d x])^{-m} \operatorname{Hypergeometric2F1}\left[\frac{m}{2}, 1 + m, 2 + m, -\frac{2 (a + b \sin [c + d x])}{(a - b) (-1 + \sin [c + d x])}\right]$$

$$\operatorname{Sec}[c + d x]^4 (1 + \sin [c + d x])^3 \left(\frac{(a + b) (1 + \sin [c + d x])}{(a - b) (-1 + \sin [c + d x])}\right)^{\frac{1}{2} (-2+m)} (a + b \sin [c + d x])^{1+m}$$

$$- \frac{(e \cos [c + d x])^{-2-m} (a + b \sin [c + d x])^{1+m}}{(a - b) d e (2 + m)}$$

$$\left( b (e \cos [c + d x])^{-2-m} \operatorname{Hypergeometric2F1}\left[1 + m, \frac{2 + m}{2}, 2 + m, \frac{2 (a + b \sin [c + d x])}{(a + b) (1 + \sin [c + d x])}\right] (1 - \sin [c + d x]) \left(-\frac{(a - b) (1 - \sin [c + d x])}{(a + b) (1 + \sin [c + d x])}\right)^{m/2} \right. \\ \left. (a + b \sin [c + d x])^{1+m} \right) / \left( (a^2 - b^2) d e (1 + m) (2 + m) + \frac{a (e \cos [c + d x])^{-2-m} (1 + \sin [c + d x]) (a + b \sin [c + d x])^{1+m}}{(a^2 - b^2) d e (2 + m)} + \right.$$

$$\left. \left( 2^{-m/2} a (a + b + a m) (e \cos [c + d x])^{-2-m} \operatorname{Hypergeometric2F1}\left[-\frac{m}{2}, \frac{2 + m}{2}, \frac{2 - m}{2}, \frac{(a - b) (1 - \sin [c + d x])}{2 (a + b \sin [c + d x])}\right] \right. \right. \\ \left. \left. (1 - \sin [c + d x]) \left(\frac{(a + b) (1 + \sin [c + d x])}{a + b \sin [c + d x]}\right)^{\frac{2+m}{2}} (a + b \sin [c + d x])^{1+m} \right) / \left( (a - b) (a + b)^2 d e m (2 + m) \right)$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.1.3 (g tan)^p (a+b sin)^m"];
```

Testing Rubi on 208 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.2.1 (a+b sin)^m (c+d sin)^n"];
```

Testing Rubi on 837 integration problems...

Test complete!



IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n"];

Testing Rubi on 1563 integration problems...

Problem #1479: Unable to integrate:

$$\left\{ \frac{\text{Sec}[e + f x]^2 (a + b \text{Sin}[e + f x])^{3/2}}{\sqrt{d \text{Sin}[e + f x]}}, x, -1, 0 \right\}$$

$$\frac{\text{Sec}[e + f x] (b + a \text{Sin}[e + f x]) \sqrt{a + b \text{Sin}[e + f x]}}{f \sqrt{d \text{Sin}[e + f x]}}$$

$$\frac{(a + b)^{3/2} \sqrt{-\frac{a(-1 + \text{Csc}[e + f x])}{a + b}} \sqrt{\frac{a(1 + \text{Csc}[e + f x])}{a - b}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \text{Sin}[e + f x]}}{\sqrt{a + b} \sqrt{d \text{Sin}[e + f x]}}\right], -\frac{a + b}{a - b}\right] \text{Tan}[e + f x]}{\sqrt{d} f}$$

$$\left( b (a + b) \sqrt{-\frac{a(-1 + \text{Csc}[e + f x])}{a + b}} \sqrt{\frac{b + a \text{Csc}[e + f x]}{-a + b}} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{-\frac{b + a \text{Csc}[e + f x]}{a - b}}\right], \frac{-a + b}{a + b}\right] (1 + \text{Sin}[e + f x]) \text{Tan}[e + f x] \right) /$$

$$\left( f \sqrt{\frac{a(1 + \text{Csc}[e + f x])}{a - b}} \sqrt{d \text{Sin}[e + f x]} \sqrt{a + b \text{Sin}[e + f x]} \right)$$

$$\text{Unintegrable}\left[\frac{\text{Sec}[e + f x]^2 (a + b \text{Sin}[e + f x])^{3/2}}{\sqrt{d \text{Sin}[e + f x]}}, x\right]$$

Problem #1480: Unable to integrate:

$$\left\{ \frac{\text{Sec}[e + f x]^4 (a + b \text{Sin}[e + f x])^{5/2}}{\sqrt{d \text{Sin}[e + f x]}}, x, -1, 1 \right\}$$

$$\frac{5 a \operatorname{Sec}[e+f x] (b+a \operatorname{Sin}[e+f x]) \sqrt{a+b \operatorname{Sin}[e+f x]}}{6 f \sqrt{d \operatorname{Sin}[e+f x]}} + \frac{\operatorname{Sec}[e+f x]^3 \sqrt{d \operatorname{Sin}[e+f x]} (a+b \operatorname{Sin}[e+f x])^{5/2}}{3 d f} - \frac{1}{6 \sqrt{d} f}$$

$$5 a (a+b)^{3/2} \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{d \operatorname{Sin}[e+f x]}}\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}[e+f x] -$$

$$\left(5 a b (a+b) \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{b+a \operatorname{Csc}[e+f x]}{-a+b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \operatorname{Csc}[e+f x]}{a-b}}\right], \frac{-a+b}{a+b}\right] (1+\operatorname{Sin}[e+f x]) \operatorname{Tan}[e+f x]\right) /$$

$$\left(6 f \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \sqrt{d \operatorname{Sin}[e+f x]} \sqrt{a+b \operatorname{Sin}[e+f x]}\right)$$

$$\frac{\operatorname{Sec}[e+f x]^3 \sqrt{d \operatorname{Sin}[e+f x]} (a+b \operatorname{Sin}[e+f x])^{5/2}}{3 d f} + \frac{5}{6} a \operatorname{Unintegrable}\left[\frac{\operatorname{Sec}[e+f x]^2 (a+b \operatorname{Sin}[e+f x])^{3/2}}{\sqrt{d \operatorname{Sin}[e+f x]}}, x\right]$$

Problem #1515: Unable to integrate:

$$\left\{\frac{\operatorname{Sec}[e+f x]^6 (a+b \operatorname{Sin}[e+f x])^{9/2}}{\sqrt{d \operatorname{Sin}[e+f x]}}, x, -1, 1\right\}$$

$$-\frac{3 a b (-2 a^2 + b^2) \operatorname{Cos}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}}{5 f \sqrt{d \operatorname{Sin}[e+f x]}} + \frac{\operatorname{Sec}[e+f x]^5 \sqrt{d \operatorname{Sin}[e+f x]} (a+b \operatorname{Sin}[e+f x])^{9/2}}{5 d f} - \frac{1}{20 d f} 3 a \operatorname{Sec}[e+f x]^3 \sqrt{d \operatorname{Sin}[e+f x]}$$

$$\sqrt{a+b \operatorname{Sin}[e+f x]} (-a(7 a^2 + b^2) + 2 b(-7 a^2 + b^2) \operatorname{Sin}[e+f x] + 5 a(a^2 - b^2) \operatorname{Sin}[e+f x]^2 + (8 a^2 b - 4 b^3) \operatorname{Sin}[e+f x]^3) - \frac{1}{20 \sqrt{d} f}$$

$$3 a (a+b)^{3/2} (5 a^2 + 3 a b - 4 b^2) \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{d \operatorname{Sin}[e+f x]}}\right], -\frac{a+b}{a-b}\right] \operatorname{Tan}[e+f x] -$$

$$\frac{1}{5 d f \sqrt{a+b \operatorname{Sin}[e+f x]}} 3 b (2 a^4 - 3 a^2 b^2 + b^4) \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \operatorname{Csc}[e+f x]}{a-b}}\right], 1 - \frac{2 a}{a+b}\right]$$

$$\sqrt{d \operatorname{Sin}[e+f x]} \sqrt{-\frac{a \operatorname{Csc}[e+f x]^2 (1+\operatorname{Sin}[e+f x]) (a+b \operatorname{Sin}[e+f x])}{(a-b)^2}} \operatorname{Tan}[e+f x]$$

$$\frac{\text{Sec}[e + f x]^5 \sqrt{d \text{Sin}[e + f x]} (a + b \text{Sin}[e + f x])^{9/2}}{5 d f} + \frac{9}{10} a \text{Unintegrable}\left[\frac{\text{Sec}[e + f x]^4 (a + b \text{Sin}[e + f x])^{7/2}}{\sqrt{d \text{Sin}[e + f x]}}, x\right]$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n"];
```

Testing Rubi on 51 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin)"];
```

Testing Rubi on 356 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.4.1 (a+b sin)^m (A+B sin+C sin^2)"];
```

Testing Rubi on 19 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2)"];
```

Testing Rubi on 34 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.7 (d trig)^m (a+b (c sin)^n)^p"];
```

Testing Rubi on 592 integration problems...

Problem #391: Unable to integrate:

$$\left\{ \frac{\text{Sec}[c + d x]^2}{a + b \text{Sin}[c + d x]^3}, x, -1, 0 \right\}$$

$$\frac{2 (-1)^{2/3} b^{2/3} \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - (-1)^{2/3} b^{2/3})^{3/2} d} - \frac{2 b^{2/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - b^{2/3})^{3/2} d} +$$

$$\frac{2 (-1)^{1/3} b^{2/3} \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} + (-1)^{1/3} b^{2/3})^{3/2} d} + \frac{\text{Sec}[c + d x] (b - a \text{Sin}[c + d x])}{(-a^2 + b^2) d}$$

$$\text{Unintegrable}\left[\frac{\text{Sec}[c + d x]^2}{a + b \text{Sin}[c + d x]^3}, x\right]$$

Problem #392: Unable to integrate:

$$\left\{\frac{\text{Sec}[c + d x]^4}{a + b \text{Sin}[c + d x]^3}, x, -1, 0\right\}$$

$$\begin{aligned} & \frac{2(-1)^{2/3} a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2b^2 (2a^2 + b^2) \text{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \\ & \frac{2b^2 (2a^2 + b^2) \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2b^{4/3} (a^2 + 2b^2) \text{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \frac{2(-1)^{1/3} a^{2/3} b^{8/3} \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \\ & \frac{2b^2 (2a^2 + b^2) \text{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} - \frac{2b^{4/3} (a^2 + 2b^2) \text{ArcTan}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \\ & \frac{2b^{4/3} (a^2 + 2b^2) \text{ArcTan}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\text{Cos}[c + d x]}{12(a + b) d (1 - \text{Sin}[c + d x])^2} + \frac{\text{Cos}[c + d x]}{12(a + b) d (1 - \text{Sin}[c + d x])} + \\ & \frac{(a + 4b) \text{Cos}[c + d x]}{4(a + b)^2 d (1 - \text{Sin}[c + d x])} - \frac{\text{Cos}[c + d x]}{12(a - b) d (1 + \text{Sin}[c + d x])^2} - \frac{(a - 4b) \text{Cos}[c + d x]}{4(a - b)^2 d (1 + \text{Sin}[c + d x])} - \frac{\text{Cos}[c + d x]}{12(a - b) d (1 + \text{Sin}[c + d x])} \end{aligned}$$

$$\text{Unintegrable}\left[\frac{\text{Sec}[c + d x]^4}{a + b \text{Sin}[c + d x]^3}, x\right]$$

Problem #424: Unable to integrate:

$$\{\text{Sec}[e + f x]^3 (a + b \text{Sin}[e + f x]^4)^p, x, -9, 9\}$$

0

$$\begin{aligned} & \frac{\text{AppellF1}\left[\frac{1}{4}, 1, -p, \frac{5}{4}, \text{Sin}[e + f x]^4, -\frac{b \text{Sin}[e + f x]^4}{a}\right] \text{Sin}[e + f x] (a + b \text{Sin}[e + f x]^4)^p \left(1 + \frac{b \text{Sin}[e + f x]^4}{a}\right)^{-p}}{2f} + \frac{1}{6f} \\ & \text{AppellF1}\left[\frac{3}{4}, 1, -p, \frac{7}{4}, \text{Sin}[e + f x]^4, -\frac{b \text{Sin}[e + f x]^4}{a}\right] \text{Sin}[e + f x]^3 (a + b \text{Sin}[e + f x]^4)^p \left(1 + \frac{b \text{Sin}[e + f x]^4}{a}\right)^{-p} + \\ & \frac{\text{Subst}\left[\text{CannotIntegrate}\left[\frac{(a + b x^4)^p}{(-1 + x)^2}, x\right], x, \text{Sin}[e + f x]\right]}{4f} + \frac{\text{Subst}\left[\text{CannotIntegrate}\left[\frac{(a + b x^4)^p}{(1 + x)^2}, x\right], x, \text{Sin}[e + f x]\right]}{4f} \end{aligned}$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.8 (a+b sin)^m (c+d trig)^n"];
```

Testing Rubi on 9 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p"];
```

Testing Rubi on 19 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.10 (c+d x)^m (a+b sin)^n"];
```

Testing Rubi on 348 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.11 (e x)^m (a+b x^n)^p sin"];
```

Testing Rubi on 113 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.12 (e x)^m (a+b sin(c+d x^n))^p"];
```

Testing Rubi on 357 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1a Sine\\4.1.13 (d+e x)^m sin(a+b x+c x^2)^n"];
```

Testing Rubi on 36 integration problems...

Test complete!

## 4.1b Cosine

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.0 (a cos)^m (b trg)^n"];
```

Testing Rubi on 294 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.1.1 (a+b cos)^n"];
```

Testing Rubi on 62 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.1.2 (g sin)^p (a+b cos)^m"];
```

Testing Rubi on 88 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.1.3 (g tan)^p (a+b cos)^m"];
```

Testing Rubi on 22 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.2.1 (a+b cos)^m (c+d cos)^n"];
```

Testing Rubi on 932 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n"];
```

Testing Rubi on 4 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n"];
```

Testing Rubi on 1 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.3.1 (a+b cos)^m (c+d cos)^n (A+B cos)"];
```

Testing Rubi on 644 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.4.1 (a+b cos)^m (A+B cos+C cos^2)"];
```

Testing Rubi on 393 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2)"];
```

Testing Rubi on 1541 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.7 (d trig)^m (a+b (c cos)^n)^p"];
```

Testing Rubi on 98 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.8 (a+b cos)^m (c+d trig)^n"];
```

Testing Rubi on 21 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.9 trig^m (a+b cos^n+c cos^(2 n))^p"];
```

Testing Rubi on 20 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.10 (c+d x)^m (a+b cos)^n"];
```

Testing Rubi on 189 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.12 (e x)^m (a+b cos(c+d x^n))^p"];
```

Testing Rubi on 99 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.1b Cosine\\4.1.13 (d+e x)^m cos(a+b x+c x^2)^n"];
```

Testing Rubi on 34 integration problems...

Test complete!

## 4.2a Tangent

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.0 (a trg)^m (b tan)^n"];
```

Testing Rubi on 387 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.1.2 (d sec)^m (a+b tan)^n"];
```

Testing Rubi on 700 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.1.3 (d sin)^m (a+b tan)^n"];
```

Testing Rubi on 93 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.2.1 (a+b tan)^m (c+d tan)^n"];
```

Testing Rubi on 1342 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.3.1 (a+b tan)^m (c+d tan)^n (A+B tan)"];
```

Testing Rubi on 859 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2)"];
```

Testing Rubi on 127 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.7 (d trig)^m (a+b (c tan)^n)^p"];
```

Testing Rubi on 499 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.9 trig^m (a+b tan^n+c tan^(2 n))^p"];
```

Testing Rubi on 50 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.10 (c+d x)^m (a+b tan)^n"];
```



Testing Rubi on 63 integration problems...

Problem #17: Unable to integrate:

$$\left\{ \frac{x^2}{\sqrt{\tan[a + b x^2]}} + \frac{\sqrt{\tan[a + b x^2]}}{b} + x^2 \tan[a + b x^2]^{3/2}, x, -1, 1 \right\}$$

$$\frac{x \sqrt{\tan[a + b x^2]}}{b}$$

$$\text{Unintegrable}\left[\frac{x^2}{\sqrt{\tan[a + b x^2]}}, x\right] + \frac{\text{Unintegrable}\left[\sqrt{\tan[a + b x^2]}, x\right]}{b} + \text{Unintegrable}\left[x^2 \tan[a + b x^2]^{3/2}, x\right]$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.2a Tangent\\4.2.11 (e x)^m (a+b tan(c+d x^n))^p"];
```

Testing Rubi on 72 integration problems...

Test complete!

## 4.2b Cotangent

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.0 (a trg)^m (b cot)^n"];
```

Testing Rubi on 52 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.1.2 (d csc)^m (a+b cot)^n"];
```

Testing Rubi on 23 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.1.3 (d cos)^m (a+b cot)^n"];
```

Testing Rubi on 19 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.2.1 (a+b cot)^m (c+d cot)^n"];
```

Testing Rubi on 106 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.7 (d trig)^m (a+b (c cot)^n)^p"];
```

Testing Rubi on 64 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.9 trig^m (a+b cot^n+c cot^(2 n))^p"];
```

Testing Rubi on 32 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.2b Cotangent\\4.2.10 (c+d x)^m (a+b cot)^n"];
```

Testing Rubi on 61 integration problems...

Test complete!

## 4.3a Secant

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.0 (a sec)^m (b trg)^n"];
```

Testing Rubi on 299 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.1.2 (d sec)^n (a+b sec)^m"];
```

Testing Rubi on 879 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.1.3 (d sin)^n (a+b sec)^m"];
```

Testing Rubi on 306 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.1.4 (d tan)^n (a+b sec)^m"];
```

Testing Rubi on 365 integration problems...

Problem #207: Valid but suboptimal antiderivative:

$$\left\{ \frac{\tan[e + f x]^2}{(a + a \operatorname{Sec}[e + f x])^{9/2}}, x, -7, 7 \right\}$$

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{32 \sqrt{2} a^{9/2} f} + \frac{\tan[e + f x]}{3 a f (a + a \operatorname{Sec}[e + f x])^{7/2}} + \frac{11 \tan[e + f x]}{24 a^2 f (a + a \operatorname{Sec}[e + f x])^{5/2}} + \frac{27 \tan[e + f x]}{32 a^3 f (a + a \operatorname{Sec}[e + f x])^{3/2}}$$

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{32 \sqrt{2} a^{9/2} f} + \frac{\tan[e + f x]}{3 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]}\right)^3} +$$

$$\frac{11 \tan[e + f x]}{24 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]}\right)^2} + \frac{27 \tan[e + f x]}{32 a^4 f \sqrt{a + a \operatorname{Sec}[e + f x]} \left(2 + \frac{\tan[e + f x]^2}{1 + \operatorname{Sec}[e + f x]}\right)}$$

Problem #347: Unable to integrate:

$$\left\{ \frac{(d \tan[e + f x])^n}{a + b \operatorname{Sec}[e + f x]}, x, -1, 0 \right\}$$

$$\frac{1}{a f (1 - n)} d \operatorname{AppellF1}\left[1 - n, \frac{1 - n}{2}, \frac{1 - n}{2}, 2 - n, \frac{a + b}{a + b \operatorname{Sec}[e + f x]}, \frac{a - b}{a + b \operatorname{Sec}[e + f x]}\right] \left(\frac{b(1 - \operatorname{Sec}[e + f x])}{a + b \operatorname{Sec}[e + f x]}\right)^{\frac{1-n}{2}} \left(\frac{b(1 + \operatorname{Sec}[e + f x])}{a + b \operatorname{Sec}[e + f x]}\right)^{\frac{1-n}{2}}$$

$$(d \tan[e + f x])^{-1+n} (-\tan[e + f x]^2)^{\frac{1-n}{2} + \frac{1}{2}(-1+n)} - \frac{d \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan[e + f x]^2\right] (d \tan[e + f x])^{-1+n} (-\tan[e + f x]^2)^{\frac{1-n}{2} + \frac{1+n}{2}}}{a f (1 + n)}$$

$$\operatorname{Unintegrable}\left[\frac{(d \tan[e + f x])^n}{a + b \operatorname{Sec}[e + f x]}, x\right]$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.2.1 (a+b sec)^m (c+d sec)^n"];
```

Testing Rubi on 241 integration problems...

Problem #217: Unable to integrate:

$$\left\{ \frac{(c+d \operatorname{Sec}[e+fx])^{3/2}}{\sqrt{a+b \operatorname{Sec}[e+fx]}}, x, -1, 0 \right\}$$

$$- \left[ \left( 2c(c+d) \operatorname{Cot}[e+fx] \operatorname{EllipticPi} \left[ \frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin} \left[ \sqrt{\frac{(a+b)(c+d \operatorname{Sec}[e+fx])}{(c+d)(a+b \operatorname{Sec}[e+fx])}} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right] \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(c-d)(a+b \operatorname{Sec}[e+fx])}} \right. \right.$$

$$\left. \left. (a+b \operatorname{Sec}[e+fx])^{3/2} \sqrt{\frac{(a+b)(bc-ad)(-1+\operatorname{Sec}[e+fx])(c+d \operatorname{Sec}[e+fx])}{(c+d)^2(a+b \operatorname{Sec}[e+fx])^2}} \right) / \left( a(a+b) f \sqrt{c+d \operatorname{Sec}[e+fx]} \right) \right] +$$

$$\left[ 2d(c+d) \operatorname{Cot}[e+fx] \operatorname{EllipticPi} \left[ \frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin} \left[ \sqrt{\frac{(a+b)(c+d \operatorname{Sec}[e+fx])}{(c+d)(a+b \operatorname{Sec}[e+fx])}} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right] \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(c-d)(a+b \operatorname{Sec}[e+fx])}} \right.$$

$$\left. \left. (a+b \operatorname{Sec}[e+fx])^{3/2} \sqrt{-\frac{(a+b)(-bc+ad)(-1+\operatorname{Sec}[e+fx])(c+d \operatorname{Sec}[e+fx])}{(c+d)^2(a+b \operatorname{Sec}[e+fx])^2}} \right) / \left( b(a+b) f \sqrt{c+d \operatorname{Sec}[e+fx]} \right) \right] +$$

$$\frac{1}{abf \sqrt{\frac{(a+b)(c+d \operatorname{Sec}[e+fx])}{(c+d)(a+b \operatorname{Sec}[e+fx])}}} 2(bc-ad) \operatorname{Cot}[e+fx] \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(a+b)(c+d \operatorname{Sec}[e+fx])}{(c+d)(a+b \operatorname{Sec}[e+fx])}} \right], \frac{(a-b)(c+d)}{(a+b)(c-d)} \right]$$

$$\sqrt{\frac{(bc-ad)(-1+\operatorname{Sec}[e+fx])}{(c+d)(a+b \operatorname{Sec}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(c-d)(a+b \operatorname{Sec}[e+fx])}} \sqrt{a+b \operatorname{Sec}[e+fx]} \sqrt{c+d \operatorname{Sec}[e+fx]}$$

$$\operatorname{Unintegrable} \left[ \frac{(c+d \operatorname{Sec}[e+fx])^{3/2}}{\sqrt{a+b \operatorname{Sec}[e+fx]}}, x \right]$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n"];
```

Testing Rubi on 286 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.3.1 (a+b sec)^m (d sec)^n (A+B sec)"];
```

Testing Rubi on 633 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.4.1 (a+b sec)^m (A+B sec+C sec^2)"];
```

Testing Rubi on 70 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2)"];
```

Testing Rubi on 1373 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.7 (d trig)^m (a+b (c sec)^n)^p"];
```

Testing Rubi on 471 integration problems...

Problem #132: Unable to integrate:

$\{(a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m, x, -1, 0\}$

$$\frac{1}{f(1+m)} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right]$$

$$(\operatorname{Cos}[e+fx]^2)^{\frac{1}{2}+p} (a+b \operatorname{Sec}[e+fx]^2)^p (d \operatorname{Sin}[e+fx])^m \left(\frac{a+b-a \operatorname{Sin}[e+fx]^2}{a+b}\right)^{-p} \operatorname{Tan}[e+fx]$$

$\operatorname{Unintegrable}[(a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m, x]$

Problem #298: Unable to integrate:

$\{(d \operatorname{Sec}[e + f x])^m (a + b \operatorname{Sec}[e + f x]^2)^p, x, -1, 0\}$

$$\frac{1}{f^m} \operatorname{AppellF1}\left[\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \operatorname{Sec}[e+fx]^2, -\frac{b \operatorname{Sec}[e+fx]^2}{a}\right] \operatorname{Cot}[e+fx] (d \operatorname{Sec}[e+fx])^m (a+b \operatorname{Sec}[e+fx]^2)^p \left(1 + \frac{b \operatorname{Sec}[e+fx]^2}{a}\right)^{-p} \sqrt{-\operatorname{Tan}[e+fx]^2}$$

$\operatorname{Unintegrable}[(d \operatorname{Sec}[e + f x])^m (a + b \operatorname{Sec}[e + f x]^2)^p, x]$

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.10 (c+d x)^m (a+b sec)^n"];
```

Testing Rubi on 46 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3a Secant\\4.3.11 (e x)^m (a+b sec(c+d x^n))^p"];
```

Testing Rubi on 83 integration problems...

Test complete!

## 4.3b Cosecant

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.0 (a csc)^m (b trg)^n"];
```

Testing Rubi on 70 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.1.2 (d csc)^n (a+b csc)^m"];
```

Testing Rubi on 59 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.1.3 (d cos)^n (a+b csc)^m"];
```

Testing Rubi on 16 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.1.4 (d cot)^n (a+b csc)^m"];
```

Testing Rubi on 23 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.3.1 (a+b csc)^m (d csc)^n (A+B csc)"];
```

Testing Rubi on 24 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2)"];
```

Testing Rubi on 1 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.7 (d trig)^m (a+b (c csc)^n)^p"];
```

Testing Rubi on 27 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.3b Cosecant\\4.3.11 (e x)^m (a+b csc(c+d x^n))^p"];
```

Testing Rubi on 84 integration problems...

Test complete!

## 4.4 Miscellaneous trig

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.1 (c trig)^m (d trig)^n"];
```

Testing Rubi on 254 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.2 trig^m (a trig+b trig)^n"];
```

Testing Rubi on 294 integration problems...

Problem #15: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]^3}{(a \cos[x] + b \sin[x])^2}, x, -19, 19 \right\}$$

$$\frac{6 a^2 b \operatorname{ArcTanh}\left[\frac{-b+a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{5/2}} + \frac{3 a (a^2-b^2) + a (a^2+b^2) \cos[2 x] - b (a^2+b^2) \sin[2 x]}{2 (a^2+b^2)^2 (a \cos[x] + b \sin[x])}$$

$$- \frac{3 a^2 \operatorname{ArcTanh}\left[\frac{b \cos[x]-a \sin[x]}{\sqrt{a^2+b^2}}\right]}{b (a^2+b^2)^{3/2}} - \frac{2 a^2 b \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{5/2}} + \frac{2 a^2 (3 a^2+b^2) \operatorname{ArcTanh}\left[\frac{b-a \tan\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b (a^2+b^2)^{5/2}} - \frac{\cos[x]}{b^2} +$$

$$\frac{3 a^2 \cos[x]}{b^2 (a^2+b^2)} - \frac{2 a \sin[x]}{b^3} + \frac{3 a^3 \sin[x]}{b^3 (a^2+b^2)} - \frac{2 a^3 \cos\left[\frac{x}{2}\right]^2 (2 a b + (a^2-b^2) \tan\left[\frac{x}{2}\right])}{b^3 (a^2+b^2)^2} + \frac{2 a^2 (a+b \tan\left[\frac{x}{2}\right])}{(a^2+b^2)^2 (a+2 b \tan\left[\frac{x}{2}\right]-a \tan\left[\frac{x}{2}\right]^2)}$$

Problem #23: Valid but suboptimal antiderivative:

$$\left\{ \frac{\sin[x]^2}{(a \cos[x] + b \sin[x])^3}, x, -13, 13 \right\}$$

$$\begin{aligned}
& - \frac{(a^2 - 2b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2}} + \frac{a(3ab \operatorname{Cos}[x] + (a^2 + 4b^2) \operatorname{Sin}[x])}{2(a^2 + b^2)^2 (a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2} \\
& \frac{2a^2 \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[x] - a \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{b^2 (a^2 + b^2)^{3/2}} - \frac{\operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[x] - a \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{b^2 \sqrt{a^2 + b^2}} - \frac{a^2 (2a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b^2 (a^2 + b^2)^{5/2}} + \\
& \frac{2a}{b(a^2 + b^2)(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])} + \frac{2(ab + (a^2 + 2b^2) \operatorname{Tan}\left[\frac{x}{2}\right])}{a(a^2 + b^2)(a + 2b \operatorname{Tan}\left[\frac{x}{2}\right] - a \operatorname{Tan}\left[\frac{x}{2}\right]^2)} - \frac{4a^4 + 3a^2 b^2 + 2b^4 + ab(5a^2 + 2b^2) \operatorname{Tan}\left[\frac{x}{2}\right]}{ab(a^2 + b^2)^2 (a + 2b \operatorname{Tan}\left[\frac{x}{2}\right] - a \operatorname{Tan}\left[\frac{x}{2}\right]^2)}
\end{aligned}$$

Problem #123: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c + dx]^3}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}, x, -11, 11 \right\} \\
& - \frac{3ab^2 \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[c+dx] - a \operatorname{Sin}[c+dx]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2} d} + \frac{2ab \operatorname{Cos}[c + dx]}{(a^2 + b^2)^2 d} + \frac{(a^2 - b^2) \operatorname{Sin}[c + dx]}{(a^2 + b^2)^2 d} - \frac{b^3}{(a^2 + b^2)^2 d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])} \\
& \frac{2b^4 \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2 + b^2)^{5/2} d} - \frac{2b^2(3a^2 + b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{a(a^2 + b^2)^{5/2} d} + \\
& \frac{2(2ab + (a^2 - b^2) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right])}{(a^2 + b^2)^2 d (1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2)} - \frac{2b^3(a + b \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right])}{a(a^2 + b^2)^2 d (a + 2b \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2)}
\end{aligned}$$

Problem #131: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c + dx]^4}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3}, x, -15, 15 \right\} \\
& \frac{1}{2d} \left( - \frac{6b^2(-4a^2 + b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{7/2}} - \frac{2b(-3a^2 + b^2) \operatorname{Cos}[c + dx]}{(a^2 + b^2)^3} + \frac{2a(a^2 - 3b^2) \operatorname{Sin}[c + dx]}{(a^2 + b^2)^3} + \right. \\
& \left. \frac{b^4 \operatorname{Sin}[c + dx]}{a(a - ib)^2(a + ib)^2(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2} - \frac{b^3(8a^2 + b^2)}{a(a^2 + b^2)^3(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])} \right)
\end{aligned}$$



$$\begin{aligned}
& - \frac{3 b^4 (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \frac{4 b^4 (3 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} - \frac{2 b^2 (6 a^4 + 3 a^2 b^2 + b^4) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \\
& \frac{2 (b (3 a^2 - b^2) + a (a^2 - 3 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{(a^2 + b^2)^3 d (1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)} + \frac{2 b^4 (a b + (a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^2 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^2} - \\
& \frac{3 b^4 (a^2 + 2 b^2) (b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^3 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)} - \frac{4 b^3 (2 a^4 - b^4 + a b (3 a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^3 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)}
\end{aligned}$$

Problem #133: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c+d x]^2}{(a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3}, x, -6, 6 \right\} \\
& \frac{(2 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2} d} - \frac{b ((4 a^2 + b^2) \operatorname{Cos}[c+d x] + 3 a b \operatorname{Sin}[c+d x])}{2 (a^2 + b^2)^2 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^2} \\
& - \frac{(2 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{5/2} d} + \frac{2 b^2 (a b + (a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2) d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^2} - \\
& \frac{b (4 a^4 + 3 a^2 b^2 + 2 b^4 + a b (5 a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^3 (a^2 + b^2)^2 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)}
\end{aligned}$$

Problem #142: Valid but suboptimal antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{Cos}[c+d x]^3}{(a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4}, x, -7, 7 \right\} \\
& \frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{7/2} d} + \frac{-3 (3 a^4 b - a^2 b^3 + b^5) \operatorname{Cos}[2(c+d x)] + \frac{1}{2} b (-9 a^2 + b^2) (2 (a^2 + b^2) + 3 a b \operatorname{Sin}[2(c+d x)])}{6 (a - i b)^3 (a + i b)^3 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} \\
& - \frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{(a^2 + b^2)^{7/2} d} - \frac{8 b^3 (a (a^2 + 2 b^2) + b (3 a^2 + 4 b^2) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{3 a^5 (a^2 + b^2) d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^3} + \\
& \frac{2 b^2 (b (15 a^4 + 18 a^2 b^2 + 8 b^4) + a (9 a^4 + 30 a^2 b^2 + 16 b^4) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{3 a^5 (a^2 + b^2)^2 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^2} - \frac{b (6 a^6 + 9 a^4 b^2 + 12 a^2 b^4 + 4 b^6 + a b (9 a^4 + 6 a^2 b^2 + 2 b^4) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right])}{a^4 (a^2 + b^2)^3 d (a + 2 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2)^2}
\end{aligned}$$

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.3 (c+d x)^m trig^n trig^p"];
```

Testing Rubi on 397 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.4 x^m (a+b trig^n)^p"];
```

Testing Rubi on 9 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.5 x^m trig(a+b log(c x^n))^p"];
```

Testing Rubi on 268 integration problems...

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n"];
```

Testing Rubi on 142 integration problems...

Problem #28: Unable to integrate:

$$\{F^{c(a+bx)} (fx)^m \sin[d+ex], x, -1, 1\}$$

$$\frac{e^{-i d F^{ac}} (fx)^m \Gamma[1+m, x(i e - bc \log[F])] (x(i e - bc \log[F]))^{-m}}{2(e + i bc \log[F])} - \frac{e^{i d F^{ac}} (fx)^m \Gamma[1+m, -x(i e + bc \log[F])] (-x(i e + bc \log[F]))^{-m}}{2(e - i bc \log[F])}$$

CannotIntegrate[F<sup>a+c b c x</sup> (f x)<sup>m</sup> Sin[d+e x], x]

Problem #32: Unable to integrate:

$$\{f F^{c(a+bx)} (fx)^m (e x \cos[d+ex] + (1+m+bcx \log[F]) \sin[d+ex]), x, -6, 6\}$$

$$f F^{c(a+bx)} x (fx)^m \sin[d+ex]$$

e CannotIntegrate[F<sup>a+c b c x</sup> (f x)<sup>1+m</sup> Cos[d+e x], x] +

f (1+m) CannotIntegrate[F<sup>a+c b c x</sup> (f x)<sup>m</sup> Sin[d+e x], x] + b c CannotIntegrate[F<sup>a+c b c x</sup> (f x)<sup>1+m</sup> Sin[d+e x], x] Log[F]

Test complete!

```
IntegrationTest["4 Trig functions\\4.4 Miscellaneous\\4.4.7 Trig functions"];
```

Testing Rubi on 895 integration problems...

Problem #704: Valid but suboptimal antiderivative:

$$\{\cos[x]^{12} \sin[x]^{10} - \cos[x]^{10} \sin[x]^{12}, x, -25, 25\}$$

$$\frac{1}{11} \cos[x]^{11} \sin[x]^{11}$$

$$\frac{3 \cos[x]^{11} \sin[x]}{5632} - \frac{3 \cos[x]^{13} \sin[x]}{5632} + \frac{1}{512} \cos[x]^{11} \sin[x]^3 - \frac{7 \cos[x]^{13} \sin[x]^3}{2816} + \frac{7 \cos[x]^{11} \sin[x]^5}{1280} - \frac{7}{880} \cos[x]^{13} \sin[x]^5 + \frac{1}{80} \cos[x]^{11} \sin[x]^7 - \frac{9}{440} \cos[x]^{13} \sin[x]^7 + \frac{1}{40} \cos[x]^{11} \sin[x]^9 - \frac{1}{22} \cos[x]^{13} \sin[x]^9 + \frac{1}{22} \cos[x]^{11} \sin[x]^{11}$$

Problem #741: Unable to integrate:

$$\{e^{\sin[x]} \sec[x]^2 (x \cos[x]^3 - \sin[x]), x, -3, 3\}$$

$$e^{\sin[x]} (-1 + x \cos[x]) \sec[x]$$

$$\text{CannotIntegrate}[e^{\sin[x]} x \cos[x], x] - \text{CannotIntegrate}[e^{\sin[x]} \sec[x] \tan[x], x]$$

Problem #803: Valid but suboptimal antiderivative:

$$\left\{ \frac{1}{\cos[x]^{3/2} \sqrt{3 \cos[x] + \sin[x]}}, x, -5, 5 \right\}$$

$$\frac{2 \sqrt{3 \cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

$$\frac{2 \cos\left[\frac{x}{2}\right]^2 \left(3 + 2 \tan\left[\frac{x}{2}\right] - 3 \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{\cos\left[\frac{x}{2}\right]^2 \left(3 + 2 \tan\left[\frac{x}{2}\right] - 3 \tan\left[\frac{x}{2}\right]^2\right)} \sqrt{\cos\left[\frac{x}{2}\right]^2 \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}}$$

Problem #804: Unable to integrate:

$$\left\{ \frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}}, x, -1, 0 \right\}$$

$$-\log[\sin[x]] + 2 \log\left[-\sqrt{\cos[x]} + \sqrt{\cos[x] + \sin[x]}\right] + \frac{2 \sqrt{\cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

$$\text{CannotIntegrate}\left[\frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}}, x\right]$$

Problem #805: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[x] + \sin[x]}{\sqrt{1 + \sin[2x]}}, x, -17, 17 \right\}$$

$$x \sqrt{1 + \sin[2x]}$$

$$\cos[x] + \sin[x]$$

$$\frac{2 \operatorname{ArcTan}\left[\tan\left[\frac{x}{2}\right]\right] \cos\left[\frac{x}{2}\right]^2 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{\cos\left[\frac{x}{2}\right]^4 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2}}$$

Problem #857: Valid but suboptimal antiderivative:

$$\left\{ \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x]}}, x, -22, 22 \right\}$$

$$-\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

$$\frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}}$$

$$\frac{\log\left[1 + \cot[x] - \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\log\left[1 + \cot[x] + \frac{\sqrt{2} \sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\log\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}} - \frac{\log\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \tan[x]\right]}{2\sqrt{2}}$$

Problem #860: Unable to integrate:

$$\left\{ \cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x, -1, 0 \right\}$$

$$\frac{x}{2} - \frac{\cos[x]}{2} - \log\left[\cos\left[\frac{\pi}{4} + \frac{x}{2}\right]\right]$$

$$\text{CannotIntegrate}\left[\cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x\right]$$

Problem #876: Unable to integrate:

$$\left\{ \frac{x^4}{b \sqrt{x^3 + 3 \sin[a + b x]}} + \frac{x^2 \cos[a + b x]}{\sqrt{x^3 + 3 \sin[a + b x]}} + \frac{4 x \sqrt{x^3 + 3 \sin[a + b x]}}{3 b}, x, -1, 1 \right\}$$

$$\frac{2x^2 \sqrt{x^3 + 3 \sin[a + bx]}}{3b}$$

$$\frac{\text{CannotIntegrate}\left[\frac{x^4}{\sqrt{x^3 + 3 \sin[a + bx]}}, x\right]}{b} + \text{CannotIntegrate}\left[\frac{x^2 \cos[a + bx]}{\sqrt{x^3 + 3 \sin[a + bx]}}, x\right] + \frac{4 \text{CannotIntegrate}\left[x \sqrt{x^3 + 3 \sin[a + bx]}, x\right]}{3b}$$

Problem #878: Unable to integrate:

$$\left\{ \frac{\cos[x] + \sin[x]}{e^{-x} + \sin[x]}, x, -5, 5 \right\}$$

$$\text{Log}[1 + e^x \sin[x]]$$

$$x - \text{CannotIntegrate}\left[\frac{1}{1 + e^x \sin[x]}, x\right] - \text{CannotIntegrate}\left[\frac{\cot[x]}{1 + e^x \sin[x]}, x\right] + \text{Log}[\sin[x]]$$

Test complete!

## 5 Inverse trig functions

### 5.1a Inverse sine

```
IntegrationTest["5 Inverse trig functions\\5.1a Inverse sine\\5.1.2 (d x)^m (a+b arcsin(c x))^n"];
```

Testing Rubi on 227 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.1a Inverse sine\\5.1.4a (f x)^m (d-c^2 d x^2)^p (a+b arcsin(c x))^n"];
```

Testing Rubi on 502 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.1a Inverse sine\\5.1.4b (f x)^m (d+e x^2)^p (a+b arcsin(c x))^n"];
```

Testing Rubi on 108 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.1a Inverse sine\\5.1.5 Inverse sine functions"];
```

Testing Rubi on 492 integration problems...

Problem #490: Unable to integrate:

$$\left\{ \frac{x}{\text{ArcSin}[\text{Sin}[x]]}, x, -1, 0 \right\}$$

$$\text{ArcSin}[\text{Sin}[x]] + \text{Log}[\text{ArcSin}[\text{Sin}[x]]] \left( -\text{ArcSin}[\text{Sin}[x]] + x \sqrt{\text{Cos}[x]^2 \text{Sec}[x]} \right)$$

$$\text{CannotIntegrate}\left[\frac{x}{\text{ArcSin}[\text{Sin}[x]]}, x\right]$$

Test complete!

## 5.1b Inverse cosine

```
IntegrationTest["5 Inverse trig functions\\5.1b Inverse cosine\\5.1.2 (d x)^m (a+b arccos(c x))^n"];
```

Testing Rubi on 227 integration problems...

Test complete!

```
IntegrationTest["5 Inverse Trig functions\\5.1b Inverse cosine\\5.1.5 Inverse cosine functions"];
```

Testing Rubi on 136 integration problems...

Test complete!

## 5.2a Inverse tangent

```
IntegrationTest["5 Inverse trig functions\\5.2a Inverse tangent\\5.2.1 u (a+b arctan(c x^n))^p"];
```

Testing Rubi on 1391 integration problems...

Problem #1254: Valid but suboptimal antiderivative:

$\{x^3 (d + e x^2)^3 (a + b \text{ArcTan}[c x]), x, -8, 8\}$

$$\begin{aligned} & \frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x}{40 c^9} - \frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x^3}{120 c^7} - \frac{b e (20 c^4 d^2 - 15 c^2 d e + 4 e^2) x^5}{200 c^5} - \\ & \frac{b (15 c^2 d - 4 e) e^2 x^7}{280 c^3} - \frac{b e^3 x^9}{90 c} + \frac{b (c^2 d - e)^4 (c^2 d + 4 e) \text{ArcTan}[c x]}{40 c^{10} e^2} - \frac{d (d + e x^2)^4 (a + b \text{ArcTan}[c x])}{8 e^2} + \frac{(d + e x^2)^5 (a + b \text{ArcTan}[c x])}{10 e^2} \\ & b (325 c^8 d^4 + 1815 c^6 d^3 e - 4977 c^4 d^2 e^2 + 4305 c^2 d e^3 - 1260 e^4) x + \frac{b (5 c^6 d^3 + 750 c^4 d^2 e - 1071 c^2 d e^2 + 420 e^3) x (d + e x^2)}{12600 c^9 e} - \\ & \frac{b (25 c^4 d^2 - 135 c^2 d e + 84 e^2) x (d + e x^2)^2}{4200 c^5 e} - \frac{b (23 c^2 d - 36 e) x (d + e x^2)^3}{2520 c^3 e} - \frac{b x (d + e x^2)^4}{90 c e} + \\ & \frac{b (c^2 d - e)^4 (c^2 d + 4 e) \text{ArcTan}[c x]}{40 c^{10} e^2} - \frac{d (d + e x^2)^4 (a + b \text{ArcTan}[c x])}{8 e^2} + \frac{(d + e x^2)^5 (a + b \text{ArcTan}[c x])}{10 e^2} \end{aligned}$$

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.2a Inverse tangent\\5.2.2 u (a+b arctan(c+d x))^p"];
```

Testing Rubi on 70 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.2a Inverse tangent\\5.2.3 Inverse tangent functions"];
```

Testing Rubi on 119 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.2a Inverse tangent\\5.2.4 x^m (c+a^2 c x^2)^p E^(n arctan(a x))"];
```

Testing Rubi on 142 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.2a Inverse tangent\\5.2.5 Exponentials of inverse tangent"];
```

Testing Rubi on 243 integration problems...

Test complete!

## 5.2b Inverse cotangent

```
IntegrationTest["5 Inverse trig functions\\5.2b Inverse cotangent\\5.2.1 Inverse cotangent functions"];
```

Testing Rubi on 228 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.2b Inverse cotangent\\5.2.2 Exponentials of inverse cotangent"];
```

Testing Rubi on 12 integration problems...

Test complete!

## 5.3a Inverse secant

```
IntegrationTest["5 Inverse trig functions\\5.3a Inverse secant\\5.3.1 u (a+b arcsec(c x))^n"];
```

Testing Rubi on 174 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.3a Inverse secant\\5.3.2 Inverse secant functions"];
```

Testing Rubi on 50 integration problems...

Test complete!

## 5.3b Inverse cosecant

```
IntegrationTest["5 Inverse trig functions\\5.3b Inverse cosecant\\5.3.1 u (a+b arccsc(c x))^n"];
```

Testing Rubi on 159 integration problems...

Test complete!

```
IntegrationTest["5 Inverse trig functions\\5.3b Inverse cosecant\\5.3.2 Inverse cosecant functions"];
```

Testing Rubi on 49 integration problems...

Test complete!

---

# 6 Hyperbolic functions



## 6.1a Hyperbolic sine

```
IntegrationTest["6 Hyperbolic functions\\6.1a Hyperbolic sine\\6.1.1 (c+d x)^m (a+b sinh)^n"];
```

Testing Rubi on 492 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1a Hyperbolic sine\\6.1.3 (e x)^m (a+b sinh(c+d x^n))^p"];
```

Testing Rubi on 102 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1a Hyperbolic sine\\6.1.4 (d+e x)^m sinh(a+b x+c x^2)^n"];
```

Testing Rubi on 33 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1a Hyperbolic sine\\6.1.7 hyper^m (a+b sinh^n)^p"];
```

Testing Rubi on 525 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1a Hyperbolic sine\\6.1.5 Hyperbolic sine functions"];
```

Testing Rubi on 362 integration problems...

Test complete!

## 6.1b Hyperbolic cosine

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.1 (c+d x)^m (a+b cosh)^n"];
```

Testing Rubi on 183 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.2 (e x)^m (a+b x^n)^p cosh"];
```

Testing Rubi on 111 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.3 (e x)^m (a+b cosh(c+d x^n))^p"];
```

Testing Rubi on 68 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.4 (d+e x)^m cosh(a+b x+c x^2)^n"];
```

Testing Rubi on 33 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.7 hyper^m (a+b cosh^n)^p"];
```

Testing Rubi on 85 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.1b Hyperbolic cosine\\6.1.5 Hyperbolic cosine functions"];
```

Testing Rubi on 329 integration problems...

Test complete!

## 6.2a Hyperbolic tangent

```
IntegrationTest["6 Hyperbolic functions\\6.2a Hyperbolic tangent\\6.2.1 (c+d x)^m (a+b tanh)^n"];
```

Testing Rubi on 80 integration problems...

Problem #16: Unable to integrate:

$$\{x \sqrt{a \operatorname{Tanh}[b x]}, x, -1, 0\}$$

$$\frac{1}{8 b^2 \sqrt{\operatorname{Tanh}[b x]}}$$

$$\left( -8 b x \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[b x]}\right] + 4 i \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[b x]}\right]^2 - 4 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[b x]}\right] \operatorname{Log}\left[1 + \frac{(-i + \sqrt{\operatorname{Tanh}[b x]})^2}{(i + \sqrt{\operatorname{Tanh}[b x]})^2}\right] - 4 b x \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[b x]}\right] + \right.$$

$$\operatorname{Log}[4] \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[b x]}\right] - \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[b x]}\right]^2 + 2 \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[b x]}\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-i + \sqrt{\operatorname{Tanh}[b x]})\right] +$$

$$4 b x \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[b x]}\right] - \operatorname{Log}[4] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[b x]}\right] - 2 \operatorname{Log}\left[\frac{1}{2} \left((1 + i) - (1 - i) \sqrt{\operatorname{Tanh}[b x]}\right)\right] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[b x]}\right] -$$

$$2 \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) (i + \sqrt{\operatorname{Tanh}[b x]})\right] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[b x]}\right] + \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[b x]}\right]^2 +$$

$$2 \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[b x]}\right] \operatorname{Log}\left[\frac{1}{2} \left((1 + i) + (1 - i) \sqrt{\operatorname{Tanh}[b x]}\right)\right] - 2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 - \sqrt{\operatorname{Tanh}[b x]}\right)\right] +$$

$$2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[b x]})\right] + 2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[b x]})\right] + i \operatorname{PolyLog}\left[2, -\frac{(-i + \sqrt{\operatorname{Tanh}[b x]})^2}{(i + \sqrt{\operatorname{Tanh}[b x]})^2}\right] +$$

$$2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 + \sqrt{\operatorname{Tanh}[b x]}\right)\right] - 2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \sqrt{\operatorname{Tanh}[b x]}\right)\right] - 2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \sqrt{\operatorname{Tanh}[b x]}\right)\right] \right) \sqrt{a \operatorname{Tanh}[b x]}$$

Unintegrable  $\left[x \sqrt{a \operatorname{Tanh}[b x]}, x\right]$

Problem #18: Unable to integrate:

$\{(c + d x) \sqrt{b \operatorname{Tanh}[e + f x]}, x, -1, 0\}$

$$\frac{\sqrt{b} (d e - c f) \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} - \frac{\sqrt{b} (d e - c f) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} + \frac{1}{8 f^2 \sqrt{\operatorname{Tanh}[e+f x]}}$$

$$d \left( -8 (e+f x) \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[e+f x]}\right] + 4 i \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[e+f x]}\right]^2 - 4 \operatorname{ArcTan}\left[\sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[1 + \frac{(-i + \sqrt{\operatorname{Tanh}[e+f x]})^2}{(i + \sqrt{\operatorname{Tanh}[e+f x]})^2}\right] - \right.$$

$$4 (e+f x) \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[e+f x]}\right] + \operatorname{Log}[4] \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[e+f x]}\right] - \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[e+f x]}\right]^2 +$$

$$2 \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-i + \sqrt{\operatorname{Tanh}[e+f x]})\right] + 4 (e+f x) \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[e+f x]}\right] - \operatorname{Log}[4] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[e+f x]}\right] -$$

$$2 \operatorname{Log}\left[\frac{1}{2} \left((1+i) - (1-i) \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[e+f x]}\right] - 2 \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) (i + \sqrt{\operatorname{Tanh}[e+f x]})\right] \operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[e+f x]}\right] +$$

$$\operatorname{Log}\left[1 + \sqrt{\operatorname{Tanh}[e+f x]}\right]^2 + 2 \operatorname{Log}\left[1 - \sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[\frac{1}{2} \left((1+i) + (1-i) \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] -$$

$$2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 - \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] + 2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] +$$

$$2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] + i \operatorname{PolyLog}\left[2, -\frac{(-i + \sqrt{\operatorname{Tanh}[e+f x]})^2}{(i + \sqrt{\operatorname{Tanh}[e+f x]})^2}\right] + 2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 + \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] -$$

$$2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] - 2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \sqrt{\operatorname{Tanh}[e+f x]}\right)\right] \left. \right) \sqrt{b \operatorname{Tanh}[e+f x]}$$

$\operatorname{Unintegrable}\left[(c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}, x\right]$

Problem #27: Unable to integrate:

$$\left\{ \frac{c+d x}{\sqrt{b \operatorname{Tanh}[e+f x]}}, x, -1, 0 \right\}$$

$$\begin{aligned}
& \frac{c \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \frac{d x \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \frac{i d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]^2}{2 \sqrt{b} f^2} + \frac{c \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \frac{d x \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]^2}{2 \sqrt{b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{\sqrt{b} f^2} + \frac{d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+i \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{\sqrt{b} f^2} + \\
& \frac{d \operatorname{Log}\left[\frac{\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}-\sqrt{b}}\right] \operatorname{Log}\left[1-\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{Log}\left[\frac{\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}+\sqrt{b}}\right] \operatorname{Log}\left[1-\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \\
& \frac{i d \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right] \operatorname{Log}\left[1-\frac{i \sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \frac{i d \operatorname{Log}\left[\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right] \operatorname{Log}\left[1-\frac{i \sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \\
& \frac{i d \operatorname{Log}\left[\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right] \operatorname{Log}\left[1+\frac{i \sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \frac{i d \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right] \operatorname{Log}\left[1+\frac{i \sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \\
& \frac{d \operatorname{Log}\left[\frac{\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}+\sqrt{b}}\right] \operatorname{Log}\left[1+\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \frac{d \operatorname{Log}\left[\frac{\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}-\sqrt{b}}\right] \operatorname{Log}\left[1+\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}-\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \\
& \frac{d \operatorname{PolyLog}\left[2, \frac{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}+\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \frac{i d \operatorname{PolyLog}\left[2, \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{b}-i \sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} + \frac{i d \operatorname{PolyLog}\left[2, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{b}-i \sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \\
& \frac{i d \operatorname{PolyLog}\left[2, \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{b}+i \sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \frac{i d \operatorname{PolyLog}\left[2, \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{b}+i \sqrt{b \operatorname{Tanh}[e+fx]}\right)}{\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \frac{d \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}-\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \\
& \frac{d \operatorname{PolyLog}\left[2, \frac{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}+\sqrt{b}}\right]}{4 \sqrt{b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{2 \sqrt{b} f^2} + \frac{i d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+i \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{2 \sqrt{b} f^2} \\
& \operatorname{Unintegrable}\left[\frac{c+d x}{\sqrt{b \operatorname{Tanh}[e+fx]}}, x\right]
\end{aligned}$$

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.2a Hyperbolic tangent\\6.2.7 (d hyper)^m (a+b (c tanh)^n)^p"];
```

Testing Rubi on 263 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.2a Hyperbolic tangent\\6.2.2 Hyperbolic tangent functions"];
```

Testing Rubi on 198 integration problems...

Test complete!

## 6.2b Hyperbolic cotangent

```
IntegrationTest["6 Hyperbolic functions\\6.2b Hyperbolic cotangent\\6.2.1 (c+d x)^m (a+b coth)^n"];
```

Testing Rubi on 61 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.2b Hyperbolic cotangent\\6.2.7 (d hyper)^m (a+b (c coth)^n)^p"];
```

Testing Rubi on 53 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.2b Hyperbolic cotangent\\6.2.2 Hyperbolic cotangent functions"];
```

Testing Rubi on 175 integration problems...

Test complete!

## 6.3a Hyperbolic secant

```
IntegrationTest["6 Hyperbolic functions\\6.3a Hyperbolic secant\\6.3.1 (c+d x)^m (a+b sech)^n"];
```

Testing Rubi on 16 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3a Hyperbolic secant\\6.3.2 (e x)^m (a+b sech(c+d x^n))^p"];
```

Testing Rubi on 84 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3a Hyperbolic secant\\6.3.7 (d hyper)^m (a+b (c sech)^n)^p"];
```

Testing Rubi on 220 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3a Hyperbolic secant\\6.3.3 Hyperbolic secant functions"];
```

Testing Rubi on 194 integration problems...

Test complete!

## 6.3b Hyperbolic cosecant

```
IntegrationTest["6 Hyperbolic functions\\6.3b Hyperbolic cosecant\\6.3.1 (c+d x)^m (a+b csch)^n"];
```

Testing Rubi on 29 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3b Hyperbolic cosecant\\6.3.2 (e x)^m (a+b csch(c+d x^n))^p"];
```

Testing Rubi on 83 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3b Hyperbolic cosecant\\6.3.7 (d hyper)^m (a+b (c csch)^n)^p"];
```

Testing Rubi on 27 integration problems...

Test complete!

```
IntegrationTest["6 Hyperbolic functions\\6.3b Hyperbolic cosecant\\6.3.3 Hyperbolic cosecant functions"];
```

Testing Rubi on 168 integration problems...

Test complete!

## 6.4 Miscellaneous hyperbolic trig

```
IntegrationTest["6 Hyperbolic functions\\6.4 Miscellaneous\\6.4.1 Hyperbolic functions"];
```

Testing Rubi on 1059 integration problems...

Test complete!

---

# 7 Inverse hyperbolic functions

## 7.1a Inverse hyperbolic sine

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1a Inverse hyperbolic sine\\7.1.2 (d x)^m (a+b arcsinh(c x))^n"];
```

Testing Rubi on 156 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1a Inverse hyperbolic sine\\7.1.4a (f x)^m (d+c^2 d x^2)^p (a+b arcsinh(c x))^n"];
```

Testing Rubi on 469 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1a Inverse hyperbolic sine\\7.1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n"];
```

Testing Rubi on 58 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1a Inverse hyperbolic sine\\7.1.5 Inverse hyperbolic sine functions"];
```

Testing Rubi on 371 integration problems...

Problem #369: Unable to integrate:

$$\left\{ \frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}, x, -1, 0 \right\}$$

$$\text{ArcSinh}[\text{Sinh}[x]] + \text{Log}[\text{ArcSinh}[\text{Sinh}[x]]] \left( -\text{ArcSinh}[\text{Sinh}[x]] + x \sqrt{\text{Cosh}[x]^2} \text{Sech}[x] \right)$$

$$\text{CannotIntegrate}\left[ \frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}, x \right]$$

Test complete!

## 7.1b Inverse hyperbolic cosine

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1b Inverse hyperbolic cosine\\7.1.2 (d x)^m (a+b arccosh(c x))^n"];
```

Testing Rubi on 166 integration problems...

Test complete!



```
IntegrationTest["7 Inverse hyperbolic functions\\7.1b Inverse hyperbolic cosine\\7.1.4a (f x)^m (d-c^2 d x^2)^p (a+b arccosh(c x))^n"];
```

Testing Rubi on 453 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1b Inverse hyperbolic cosine\\7.1.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n"];
```

Testing Rubi on 109 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.1b Inverse hyperbolic cosine\\7.1.5 Inverse hyperbolic cosine functions"];
```

Testing Rubi on 293 integration problems...

Problem #61: Unable to integrate:

$$\left\{ \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{f + g x}, x, -30, 30 \right\}$$

$$\begin{aligned}
& - \frac{a d (c f - g) (c f + g) \sqrt{d - c^2 d x^2}}{g^3} + \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 d (c f - g) x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{a d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2}}{6 g} + \\
& \frac{b c d x (-12 - 9 c x + 4 c^2 x^2) \sqrt{d - c^2 d x^2}}{36 g \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^3} - \frac{a d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{2 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{6 g} - \frac{b d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]^2}{4 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c d (c f - g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^2} - \\
& \frac{d (c f - g) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{d (c f - g)^2 (c f + g)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \frac{d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^2 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \\
& \frac{2 a d (c f - g)^{3/2} (c f + g)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c f + g} \sqrt{1 + c x}}{\sqrt{c f - g} \sqrt{-1 + c x}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{a d (c f - g) (c f + g) \sqrt{d - c^2 d x^2}}{g^3} + \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 d (c f - g) x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^3} + \frac{c d (c f - g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^2} - \\
& \frac{d (c f - g) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{d (c f - g)^2 (c f + g)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \frac{d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^2 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \\
& \frac{2 a d (c f - g)^{3/2} (c f + g)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c f + g} \sqrt{1 + c x}}{\sqrt{c f - g} \sqrt{-1 + c x}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{c d \sqrt{d - c^2 d x^2} \operatorname{Unintegrable}\left[(-1 + c x)^{3/2} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x]), x\right]}{g \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Test complete!

## 7.2a Inverse hyperbolic tangent

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2a Inverse hyperbolic tangent\\7.2.1 u (a+b arctanh(c x^n))^p"];
```

Testing Rubi on 706 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2a Inverse hyperbolic tangent\\7.2.2 u (a+b arctanh(c+d x))^p"];
```

Testing Rubi on 62 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2a Inverse hyperbolic tangent\\7.2.3 Inverse hyperbolic tangent functions"];
```

Testing Rubi on 327 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2a Inverse hyperbolic tangent\\7.2.4  $x^m (c-a^2 c x^2)^p E^{(n \operatorname{arctanh}(a x))}$ "];
```

Testing Rubi on 496 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2a Inverse hyperbolic tangent\\7.2.5 Exponentials of inverse hyperbolic tangent functions"];
```

Testing Rubi on 881 integration problems...

Test complete!

## 7.2b Inverse hyperbolic cotangent

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2b Inverse hyperbolic cotangent\\7.2.1 Inverse hyperbolic cotangent functions"];
```

Testing Rubi on 293 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.2b Inverse hyperbolic cotangent\\7.2.2 Exponentials of inverse hyperbolic cotangent functions"];
```

Testing Rubi on 935 integration problems...

Test complete!

## 7.3a Inverse hyperbolic secant

```
IntegrationTest["7 Inverse hyperbolic functions\\7.3a Inverse hyperbolic secant\\7.3.1  $u (a+b \operatorname{arcsech}(c x))^n$ "];
```

Testing Rubi on 190 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.3a Inverse hyperbolic secant\\7.3.2 Inverse hyperbolic secant functions"];
```

Testing Rubi on 100 integration problems...

Test complete!

## 7.3b Inverse hyperbolic cosecant

```
IntegrationTest["7 Inverse hyperbolic functions\\7.3b Inverse hyperbolic cosecant\\7.3.1 u (a+b arccsch(c x))^n"];
```

Testing Rubi on 159 integration problems...

Test complete!

```
IntegrationTest["7 Inverse hyperbolic functions\\7.3b Inverse hyperbolic cosecant\\7.3.2 Inverse hyperbolic cosecant functions"];
```

Testing Rubi on 71 integration problems...

Test complete!

---

## 8 Special functions

```
IntegrationTest["8 Special functions\\8.1 Error functions"];
```

Testing Rubi on 290 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.2 Fresnel integral functions"];
```

Testing Rubi on 204 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.3 Exponential integral functions"];
```

Testing Rubi on 210 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.4 Trig integral functions"];
```

Testing Rubi on 122 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.5 Hyperbolic integral functions"];
```

Testing Rubi on 122 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.6 Gamma functions"];
```

Testing Rubi on 226 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.7 Zeta function"];
```

Testing Rubi on 14 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.8 Polylogarithm function"];
```

Testing Rubi on 53 integration problems...

Test complete!

```
IntegrationTest["8 Special functions\\8.9 Product logarithm function"];
```

Testing Rubi on 396 integration problems...

Test complete!

```
(* IntegrationTest["8 Special functions\\8.10 Bessel functions"]; *)
```

---

## Test suite statistics

```
PrintTestStatistics[];
```

```
* * * Indefinite Integration Test Suite Results * * *
```

```
Integration function: Rubi 4's user-defined Int function
```

```
Time and date of test: 00:46 19 December 2010
```

```
Mathematica version: 7.0 for Microsoft Windows (64-bit) (February 18, 2009)
```

```
Integration rules defined: 6537
```

```
Integration rules applied: 412478
```

Maximum steps used: 547 steps Optimal steps: 547 steps Integrand:  $x^3 (c + a^2 c x^2)^{5/2} \text{ArcTan}[a x]^3$

Largest result size: 3205 leaves Optimal size: 3205 leaves Integrand:  $\frac{x^3}{(a + b \text{Csc}[c + d \sqrt{x}])^2}$

Longest compute time: 32.916 seconds Result size: 798 leaves Integrand:  $x^3 (c + a^2 c x^2)^{5/2} \text{ArcTan}[a x]^3$

File	Optimal	Nonident	Unintegrable	Timeout	Invalid	Total	Int/sec	Time
0 Independent test suites\Apostol Problems	175	0	0	0	0	175	38.70	4.9
0 Independent test suites\Moses Problems	113	0	0	0	0	113	32.10	3.6
0 Independent test suites\Timofeev Problems	687	16	2	0	0	705	5.66	157.0
0 Independent test suites\Charlwood Problems	43	4	2	0	1	50	4.23	113.6
0 Independent test suites\Stewart Problems	376	0	0	0	0	376	34.60	11.2
0 Independent test suites\Hearn Problems	278	0	6	0	0	284	9.47	30.5
0 Independent test suites\Jeffrey Problems	4	3	2	0	0	9	0.83	10.9
0 Independent test suites\Hebisch Problems	3	0	4	0	0	7	2.67	2.7
0 Independent test suites\Wester Problems	8	0	0	0	0	8	36.70	0.2
0 Independent test suites\Welz Problems	54	8	8	0	0	70	1.99	47.1
0 Independent test suites\Bronstein Problems	12	0	2	0	0	14	5.16	2.7
0 Independent test suites\Bondarenko Problems	30	3	2	0	0	35	1.65	27.7
1 Algebraic functions\1.1 Binomial products\1.1.1 Linear\1.1.1.2 (a+b x)^m (c+d x)^n	1915	1	0	0	0	1916	5.68	363.8
1 Algebraic functions\1.1 Binomial products\1.1.1 Linear\1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p	3129	3	0	0	0	3132	4.33	803.1
1 Algebraic functions\1.1 Binomial products\1.1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q	82	0	1	0	0	83	1.84	45.6
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.2 (c x)^m (a+b x^2)^p	1069	2	0	0	0	1071	8.03	141.5
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.3 (a+b x^2)^p (c+d x^2)^q	334	1	0	0	0	335	3.24	105.5
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q	1154	0	0	0	0	1154	2.39	498.3
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r	115	0	0	0	0	115	1.67	69.7
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r	51	0	0	0	0	51	1.11	46.4
1 Algebraic functions\1.1 Binomial products\1.1.2 Quadratic\1.1.2.8 (c x)^m Pq(x) (a+b x^2)^p	174	0	0	0	0	174	3.83	45.9
1 Algebraic functions\1.1 Binomial products\1.1.3 General\1.1.3.2 (c x)^m (a+b x^n)^p	2909	3	0	0	0	2912	11.10	306.4
1 Algebraic functions\1.1 Binomial products\1.1.3 General\1.1.3.3 (a+b x^n)^p (c+d x^n)^q	285	0	0	0	0	285	2.90	99.7
1 Algebraic functions\1.1 Binomial products\1.1.3 General\1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q	728	0	0	0	0	728	2.91	257.6
1 Algebraic functions\1.1 Binomial products\1.1.3 General\1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r	46	0	0	0	0	46	1.10	42.4
1 Algebraic functions\1.1 Binomial products\1.1.3 General\1.1.3.8 (c x)^m Pq(x) (a+b x^n)^p	539	0	1	0	0	540	1.94	284.7
1 Algebraic functions\1.1 Binomial products\1.1.4 Improper\1.1.4.2 (c x)^m (a x^j+b x^k)^p	454	0	0	0	0	454	4.43	104.3
1 Algebraic functions\1.1 Binomial products\1.1.4 Improper\1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q	298	0	0	0	0	298	3.35	90.2
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.1 (a+b x+c x^2)^p	143	0	0	0	0	143	11.90	12.2
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.2 (d+e x)^m (a+b x+c x^2)^p	2584	0	0	0	0	2584	2.43	1140.5
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p	2645	0	0	0	0	2645	2.30	1235.9
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p	859	1	0	0	0	860	1.47	601.1
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q	123	0	0	0	0	123	1.16	107.1
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q	143	0	0	0	0	143	0.44	324.9
1 Algebraic functions\1.2 Trinomial products\1.2.1 Quadratic\1.2.1.9 (d+e x)^m Pq(x) (a+b x+c x^2)^p	384	0	0	0	0	384	1.56	249.0
1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic\1.2.2.2 (d x)^m (a+b x^2+c x^4)^p	1126	0	0	0	0	1126	3.84	304.4
1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic\1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p	330	0	1	0	0	331	2.94	113.8
1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic\1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p	373	1	0	0	0	374	1.14	330.5
1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic\1.2.2.5 Pq(x) (a+b x^2+c x^4)^p	111	0	0	0	0	111	1.07	103.9
1 Algebraic functions\1.2 Trinomial products\1.2.2 Quartic\1.2.2.6 (d x)^m Pq(x) (a+b x^2+c x^4)^p	145	0	0	0	0	145	0.88	165.2
1 Algebraic functions\1.2 Trinomial products\1.2.3 General\1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p	662	2	0	0	0	664	3.06	222.0
1 Algebraic functions\1.2 Trinomial products\1.2.3 General\1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p	96	0	0	0	0	96	0.93	104.1
1 Algebraic functions\1.2 Trinomial products\1.2.3 General\1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p	131	0	0	0	0	131	2.04	64.5
1 Algebraic functions\1.2 Trinomial products\1.2.3 General\1.2.3.5 (d x)^m Pq(x) (a+b x^n+c x^(2 n))^p	17	0	0	0	0	17	0.72	23.7
1 Algebraic functions\1.2 Trinomial products\1.2.4 Improper\1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p	140	0	0	0	0	140	3.11	45.4
1 Algebraic functions\1.3 Miscellaneous\1.3.1 Rational functions	292	1	3	0	0	296	2.61	114.3
1 Algebraic functions\1.3 Miscellaneous\1.3.2 Algebraic functions	449	4	0	0	0	457	3.56	130.4
1 Algebraic functions\1.3 Miscellaneous\1.3.3 Expansion problems	110	0	0	0	0	110	6.97	16.0
1 Algebraic functions\1.3 Miscellaneous\1.3.4 Substitution problems	361	3	0	0	0	364	3.92	94.3

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1 Algebraic functions\1.3 Miscellaneous\1.3.4 Substitution problems	361	3	0	0	0	364	3.92	34.3
1 Algebraic functions\1.3 Miscellaneous\1.3.5 Piecewise constant extraction	66	0	0	0	0	66	5.71	11.7
1 Algebraic functions\1.3 Miscellaneous\1.3.6 Derivative integration problems	95	0	2	0	0	97	13.90	7.1
2 Exponentials\2.1 u (F^(c (a+b x)))^n	98	0	0	0	0	98	3.64	27.1
2 Exponentials\2.2 (c+d x)^m (F^(g (e+f x)))^n (a+b (F^(g (e+f x)))^n)^p	88	0	0	0	0	88	3.04	29.1
2 Exponentials\2.3 Exponential functions	732	0	4	0	0	736	5.01	150.0
3 Logarithms\3.1 u (a+b log(c x^n))^p	657	0	0	0	0	657	2.97	226.1
3 Logarithms\3.2 u (a+b log(c (d+e x)^n))^p	540	0	2	0	0	542	1.30	421.5
3 Logarithms\3.3 u (a+b log(c (d+e x^m)^n))^p	436	0	0	0	0	436	2.25	196.6
3 Logarithms\3.4 u log(e (f (a+b x)^p (c+d x)^q)^r)^s	190	0	2	0	0	192	2.12	91.1
3 Logarithms\3.5 Logarithm functions	286	0	0	0	0	286	5.56	52.1
4 Trig functions\4.1a Sine\4.1.0 (a sin)^m (b trg)^n	538	0	0	0	0	538	9.93	56.0
4 Trig functions\4.1a Sine\4.1.1.1 (a+b sin)^n	72	0	0	0	0	72	9.12	8.0
4 Trig functions\4.1a Sine\4.1.1.2 (g cos)^p (a+b sin)^m	652	1	0	0	0	653	2.52	273.7
4 Trig functions\4.1a Sine\4.1.1.3 (g tan)^p (a+b sin)^m	208	0	0	0	0	208	2.91	72.0
4 Trig functions\4.1a Sine\4.1.2.1 (a+b sin)^m (c+d sin)^n	837	0	0	0	0	837	1.83	469.1
4 Trig functions\4.1a Sine\4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n	1560	0	3	0	0	1563	1.81	903.9
4 Trig functions\4.1a Sine\4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n	51	0	0	0	0	51	1.98	26.1
4 Trig functions\4.1a Sine\4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin)	356	0	0	0	0	356	1.57	229.5
4 Trig functions\4.1a Sine\4.1.4.1 (a+b sin)^m (A+B sin+C sin^2)	19	0	0	0	0	19	5.39	3.6
4 Trig functions\4.1a Sine\4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2)	34	0	0	0	0	34	0.86	40.0
4 Trig functions\4.1a Sine\4.1.7 (d trg)^m (a+b (c sin)^n)^p	589	0	3	0	0	592	3.65	166.5
4 Trig functions\4.1a Sine\4.1.8 (a+b sin)^m (c+d trg)^n	9	0	0	0	0	9	5.06	1.8
4 Trig functions\4.1a Sine\4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p	19	0	0	0	0	19	0.82	23.1
4 Trig functions\4.1a Sine\4.1.10 (c+d x)^m (a+b sin)^n	348	0	0	0	0	348	1.62	217.0
4 Trig functions\4.1a Sine\4.1.11 (e x)^m (a+b x^n)^p sin	113	0	0	0	0	113	0.93	122.8
4 Trig functions\4.1a Sine\4.1.12 (e x)^m (a+b sin(c+d x^n))^p	357	0	0	0	0	357	3.16	115.0
4 Trig functions\4.1a Sine\4.1.13 (d+e x)^m sin(a+b x+c x^2)^n	36	0	0	0	0	36	5.56	6.5
4 Trig functions\4.1b Cosine\4.1.0 (a cos)^m (b trg)^n	294	0	0	0	0	294	16.20	18.9
4 Trig functions\4.1b Cosine\4.1.1.1 (a+b cos)^n	62	0	0	0	0	62	9.62	6.6
4 Trig functions\4.1b Cosine\4.1.1.2 (g sin)^p (a+b cos)^m	88	0	0	0	0	88	1.20	74.1
4 Trig functions\4.1b Cosine\4.1.1.3 (g tan)^p (a+b cos)^m	22	0	0	0	0	22	5.36	4.1
4 Trig functions\4.1b Cosine\4.1.2.1 (a+b cos)^m (c+d cos)^n	932	0	0	0	0	932	2.50	384.3
4 Trig functions\4.1b Cosine\4.1.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n	4	0	0	0	0	4	1.05	3.9
4 Trig functions\4.1b Cosine\4.1.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n	1	0	0	0	0	1	4.27	0.2
4 Trig functions\4.1b Cosine\4.1.3.1 (a+b cos)^m (c+d cos)^n (A+B cos)	644	0	0	0	0	644	1.21	544.1
4 Trig functions\4.1b Cosine\4.1.4.1 (a+b cos)^m (A+B cos+C cos^2)	393	0	0	0	0	393	4.84	83.5
4 Trig functions\4.1b Cosine\4.1.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2)	1541	0	0	0	0	1541	0.95	1670.4
4 Trig functions\4.1b Cosine\4.1.7 (d trg)^m (a+b (c cos)^n)^p	98	0	0	0	0	98	6.39	15.5
4 Trig functions\4.1b Cosine\4.1.8 (a+b cos)^m (c+d trg)^n	21	0	0	0	0	21	2.29	9.2
4 Trig functions\4.1b Cosine\4.1.9 trig^m (a+b cos^n+c cos^(2 n))^p	20	0	0	0	0	20	0.53	37.7
4 Trig functions\4.1b Cosine\4.1.10 (c+d x)^m (a+b cos)^n	189	0	0	0	0	189	3.98	47.8
4 Trig functions\4.1b Cosine\4.1.12 (e x)^m (a+b cos(c+d x^n))^p	99	0	0	0	0	99	5.67	17.6
4 Trig functions\4.1b Cosine\4.1.13 (d+e x)^m cos(a+b x+c x^2)^n	34	0	0	0	0	34	5.86	5.8
4 Trig functions\4.2a Tangent\4.2.0 (a trg)^m (b tan)^n	387	0	0	0	0	387	6.53	60.6
4 Trig functions\4.2a Tangent\4.2.1.2 (d sec)^m (a+b tan)^n	700	0	0	0	0	700	3.78	190.6
4 Trig functions\4.2a Tangent\4.2.1.3 (d sin)^m (a+b tan)^n	93	0	0	0	0	93	1.62	58.1
4 Trig functions\4.2a Tangent\4.2.2.1 (a+b tan)^m (c+d tan)^n	1342	0	0	0	0	1342	1.60	862.4
4 Trig functions\4.2a Tangent\4.2.3.1 (a+b tan)^m (c+d tan)^n (A+B tan)	859	0	0	0	0	859	1.30	674.4
4 Trig functions\4.2a Tangent\4.2.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2)	127	0	0	0	0	127	0.29	440.5
4 Trig functions\4.2a Tangent\4.2.7 (d trg)^m (a+b (c tan)^n)^p	499	0	0	0	0	499	4.29	118.6
4 Trig functions\4.2a Tangent\4.2.9 trig^m (a+b tan^n+c tan^(2 n))^p	50	0	0	0	0	50	0.16	310.7
4 Trig functions\4.2a Tangent\4.2.10 (c+d x)^m (a+b tan)^n	62	0	1	0	0	63	2.35	27.0
4 Trig functions\4.2a Tangent\4.2.11 (e x)^m (a+b tan(c+d x^n))^p	72	0	0	0	0	72	1.73	41.9
4 Trig functions\4.2b Cotangent\4.2.0 (a trg)^m (b cot)^n	52	0	0	0	0	52	7.26	7.2
4 Trig functions\4.2b Cotangent\4.2.1.2 (d csc)^m (a+b cot)^n	23	0	0	0	0	23	10.40	2.2
4 Trig functions\4.2b Cotangent\4.2.1.3 (d cos)^m (a+b cot)^n	19	0	0	0	0	19	6.38	3.0
4 Trig functions\4.2b Cotangent\4.2.2.1 (a+b cot)^m (c+d cot)^n	106	0	0	0	0	106	1.47	72.4
4 Trig functions\4.2b Cotangent\4.2.7 (d trg)^m (a+b (c cot)^n)^p	64	0	0	0	0	64	6.01	10.7
4 Trig functions\4.2b Cotangent\4.2.9 trig^m (a+b cot^n+c cot^(2 n))^p	32	0	0	0	0	32	0.18	182.5



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4 Trig functions\4.2b Cotangent\4.2.10 (c+d x)^m (a+b cot)^n	61	0	0	0	0	61	2.23	27.5
4 Trig functions\4.3a Secant\4.3.0 (a sec)^m (b trg)^n	299	0	0	0	0	299	11.50	26.9
4 Trig functions\4.3a Secant\4.3.1.2 (d sec)^n (a+b sec)^m	879	0	0	0	0	879	2.35	384.4
4 Trig functions\4.3a Secant\4.3.1.3 (d sin)^n (a+b sec)^m	306	0	0	0	0	306	1.81	171.6
4 Trig functions\4.3a Secant\4.3.1.4 (d tan)^n (a+b sec)^m	363	1	1	0	0	365	2.96	128.8
4 Trig functions\4.3a Secant\4.3.2.1 (a+b sec)^m (c+d sec)^n	240	0	1	0	0	241	1.75	139.9
4 Trig functions\4.3a Secant\4.3.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n	286	0	0	0	0	286	2.37	122.4
4 Trig functions\4.3a Secant\4.3.3.1 (a+b sec)^m (d sec)^n (A+B sec)	633	0	0	0	0	633	1.26	511.1
4 Trig functions\4.3a Secant\4.3.4.1 (a+b sec)^m (A+B sec+C sec^2)	70	0	0	0	0	70	10.50	6.8
4 Trig functions\4.3a Secant\4.3.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2)	1373	0	0	0	0	1373	1.14	1242.7
4 Trig functions\4.3a Secant\4.3.7 (d trig)^m (a+b (c sec)^n)^p	469	0	2	0	0	471	3.20	150.2
4 Trig functions\4.3a Secant\4.3.10 (c+d x)^m (a+b sec)^n	46	0	0	0	0	46	2.04	22.7
4 Trig functions\4.3a Secant\4.3.11 (e x)^m (a+b sec(c+d x^n))^p	83	0	0	0	0	83	1.31	63.8
4 Trig functions\4.3b Cosecant\4.3.0 (a csc)^m (b trg)^n	70	0	0	0	0	70	23.90	3.0
4 Trig functions\4.3b Cosecant\4.3.1.2 (d csc)^n (a+b csc)^m	59	0	0	0	0	59	4.70	12.7
4 Trig functions\4.3b Cosecant\4.3.1.3 (d cos)^n (a+b csc)^m	16	0	0	0	0	16	5.80	2.8
4 Trig functions\4.3b Cosecant\4.3.1.4 (d cot)^n (a+b csc)^m	23	0	0	0	0	23	5.81	4.0
4 Trig functions\4.3b Cosecant\4.3.3.1 (a+b csc)^m (d csc)^n (A+B csc)	24	0	0	0	0	24	10.50	2.3
4 Trig functions\4.3b Cosecant\4.3.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2)	1	0	0	0	0	1	4.02	0.3
4 Trig functions\4.3b Cosecant\4.3.7 (d trig)^m (a+b (c csc)^n)^p	27	0	0	0	0	27	7.76	3.5
4 Trig functions\4.3b Cosecant\4.3.11 (e x)^m (a+b csc(c+d x^n))^p	84	0	0	0	0	84	1.32	64.2
4 Trig functions\4.4 Miscellaneous\4.4.1 (c trig)^m (d trig)^n	254	0	0	0	0	254	8.13	31.7
4 Trig functions\4.4 Miscellaneous\4.4.2 trig^m (a trig-b trig)^n	288	6	0	0	0	294	3.17	96.8
4 Trig functions\4.4 Miscellaneous\4.4.3 (c+d x)^m trig^n trig^p	397	0	0	0	0	397	1.94	206.8
4 Trig functions\4.4 Miscellaneous\4.4.4 x^m (a+b trig^n)^p	9	0	0	0	0	9	1.29	7.1
4 Trig functions\4.4 Miscellaneous\4.4.5 x^m trig(a+b log(c x^n))^p	268	0	0	0	0	268	11.10	24.8
4 Trig functions\4.4 Miscellaneous\4.4.6 f^n (a+b x+c x^2) trig(d+e x+f x^2)^n	140	0	2	0	0	142	2.18	65.6
4 Trig functions\4.4 Miscellaneous\4.4.7 Trig functions	886	4	5	0	0	895	4.98	185.1
5 Inverse trig functions\5.1a Inverse sine\5.1.2 (d x)^m (a+b arcsin(c x))^n	227	0	0	0	0	227	3.96	58.0
5 Inverse trig functions\5.1a Inverse sine\5.1.4a (f x)^m (d-c^2 d x^2)^p (a+b arcsin(c x))^n	502	0	0	0	0	502	2.14	239.0
5 Inverse trig functions\5.1a Inverse sine\5.1.4b (f x)^m (d+e x^2)^p (a+b arcsin(c x))^n	108	0	0	0	0	108	1.14	95.9
5 Inverse trig functions\5.1a Inverse sine\5.1.5 Inverse sine functions	491	0	1	0	0	492	1.37	364.3
5 Inverse trig functions\5.1b Inverse cosine\5.1.2 (d x)^m (a+b arccos(c x))^n	227	0	0	0	0	227	3.98	57.7
5 Inverse Trig functions\5.1b Inverse cosine\5.1.5 Inverse cosine functions	136	0	0	0	0	136	2.83	48.7
5 Inverse trig functions\5.2a Inverse tangent\5.2.1 u (a+b arctan(c x^n))^p	1390	1	0	0	0	1391	1.82	796.9
5 Inverse trig functions\5.2a Inverse tangent\5.2.2 u (a+b arctan(c+d x))^p	70	0	0	0	0	70	1.94	36.5
5 Inverse trig functions\5.2a Inverse tangent\5.2.3 Inverse tangent functions	119	0	0	0	0	119	4.87	24.7
5 Inverse trig functions\5.2a Inverse tangent\5.2.4 x^m (c+a^2 c x^2)^p E^n arctan(a x)	142	0	0	0	0	142	5.97	24.4
5 Inverse trig functions\5.2a Inverse tangent\5.2.5 Exponentials of inverse tangent	243	0	0	0	0	243	3.14	79.0
5 Inverse trig functions\5.2b Inverse cotangent\5.2.1 Inverse cotangent functions	228	0	0	0	0	228	2.81	82.7
5 Inverse trig functions\5.2b Inverse cotangent\5.2.2 Exponentials of inverse cotangent	12	0	0	0	0	12	4.93	2.5
5 Inverse trig functions\5.3a Inverse secant\5.3.1 u (a+b arcsec(c x))^n	174	0	0	0	0	174	1.45	121.6
5 Inverse trig functions\5.3a Inverse secant\5.3.2 Inverse secant functions	50	0	0	0	0	50	4.31	11.9
5 Inverse trig functions\5.3b Inverse cosecant\5.3.1 u (a+b arccsc(c x))^n	159	0	0	0	0	159	1.48	108.7
5 Inverse trig functions\5.3b Inverse cosecant\5.3.2 Inverse cosecant functions	49	0	0	0	0	49	4.18	11.9
6 Hyperbolic functions\6.1a Hyperbolic sine\6.1.1 (c+d x)^m (a+b sinh)^n	492	0	0	0	0	492	0.98	510.1
6 Hyperbolic functions\6.1a Hyperbolic sine\6.1.3 (e x)^m (a+b sinh(c+d x^n))^p	102	0	0	0	0	102	5.69	18.1
6 Hyperbolic functions\6.1a Hyperbolic sine\6.1.4 (d+e x)^m sinh(a+b x+c x^2)^n	33	0	0	0	0	33	6.08	5.5
6 Hyperbolic functions\6.1a Hyperbolic sine\6.1.7 hyper^m (a+b sinh^n)^p	525	0	0	0	0	525	3.28	163.2
6 Hyperbolic functions\6.1a Hyperbolic sine\6.1.5 Hyperbolic sine functions	362	0	0	0	0	362	5.56	66.1
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.1 (c+d x)^m (a+b cosh)^n	183	0	0	0	0	183	3.27	56.6
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.2 (e x)^m (a+b x^n)^p cosh	111	0	0	0	0	111	0.94	118.9
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.3 (e x)^m (a+b cosh(c+d x^n))^p	68	0	0	0	0	68	5.81	11.8
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.4 (d+e x)^m cosh(a+b x+c x^2)^n	33	0	0	0	0	33	6.20	5.4
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.7 hyper^m (a+b cosh^n)^p	85	0	0	0	0	85	4.99	17.1
6 Hyperbolic functions\6.1b Hyperbolic cosine\6.1.5 Hyperbolic cosine functions	329	0	0	0	0	329	5.32	62.8
6 Hyperbolic functions\6.2a Hyperbolic tangent\6.2.1 (c+d x)^m (a+b tanh)^n	77	0	3	0	0	80	2.61	30.9
6 Hyperbolic functions\6.2a Hyperbolic tangent\6.2.7 (d hyper)^m (a+b (c tanh)^n)^p	263	0	0	0	0	263	4.37	60.8
6 Hyperbolic functions\6.2a Hyperbolic tangent\6.2.2 Hyperbolic tangent functions	198	0	0	0	0	198	6.47	31.0
6 Hyperbolic functions\6.2b Hyperbolic cotangent\6.2.1 (c+d x)^m (a+b coth)^n	61	0	0	0	0	61	2.14	28.6

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6 Hyperbolic functions\6.2b Hyperbolic cotangent\6.2.7 (d hyper)^m (a-b (c coth)^n)^p	53	0	0	0	0	53	5.64	9.4
6 Hyperbolic functions\6.2b Hyperbolic cotangent\6.2.2 Hyperbolic cotangent functions	175	0	0	0	0	175	6.51	27.1
6 Hyperbolic functions\6.3a Hyperbolic secant\6.3.1 (c+d x)^m (a+b sech)^n	16	0	0	0	0	16	6.98	2.3
6 Hyperbolic functions\6.3a Hyperbolic secant\6.3.2 (e x)^m (a+b sech(c+d x^n))^p	84	0	0	0	0	84	1.41	60.0
6 Hyperbolic functions\6.3a Hyperbolic secant\6.3.7 (d hyper)^m (a-b (c sech)^n)^p	220	0	0	0	0	220	4.17	53.3
6 Hyperbolic functions\6.3a Hyperbolic secant\6.3.3 Hyperbolic secant functions	194	0	0	0	0	194	5.19	37.8
6 Hyperbolic functions\6.3b Hyperbolic cosecant\6.3.1 (c+d x)^m (a+b csch)^n	29	0	0	0	0	29	1.98	14.7
6 Hyperbolic functions\6.3b Hyperbolic cosecant\6.3.2 (e x)^m (a+b csch(c+d x^n))^p	83	0	0	0	0	83	1.47	56.7
6 Hyperbolic functions\6.3b Hyperbolic cosecant\6.3.7 (d hyper)^m (a-b (c csch)^n)^p	27	0	0	0	0	27	6.73	4.0
6 Hyperbolic functions\6.3b Hyperbolic cosecant\6.3.3 Hyperbolic cosecant functions	168	0	0	0	0	168	7.19	23.6
6 Hyperbolic functions\6.4 Miscellaneous\6.4.1 Hyperbolic functions	1059	0	0	0	0	1059	4.70	231.5
7 Inverse hyperbolic functions\7.1a Inverse hyperbolic sine\7.1.2 (d x)^m (a+b arcsinh(c x))^n	156	0	0	0	0	156	3.14	49.9
7 Inverse hyperbolic functions\7.1a Inverse hyperbolic sine\7.1.4a (f x)^m (d+c^2 d x^2)^p (a+b arcsinh(c x))^n	469	0	0	0	0	469	2.16	220.5
7 Inverse hyperbolic functions\7.1a Inverse hyperbolic sine\7.1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n	58	0	0	0	0	58	1.88	31.0
7 Inverse hyperbolic functions\7.1a Inverse hyperbolic sine\7.1.5 Inverse hyperbolic sine functions	370	0	1	0	0	371	2.04	183.9
7 Inverse hyperbolic functions\7.1b Inverse hyperbolic cosine\7.1.2 (d x)^m (a+b arccosh(c x))^n	166	0	0	0	0	166	2.05	81.2
7 Inverse hyperbolic functions\7.1b Inverse hyperbolic cosine\7.1.4a (f x)^m (d-c^2 d x^2)^p (a+b arccosh(c x))^n	453	0	0	0	0	453	1.27	359.3
7 Inverse hyperbolic functions\7.1b Inverse hyperbolic cosine\7.1.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n	109	0	0	0	0	109	1.01	108.6
7 Inverse hyperbolic functions\7.1b Inverse hyperbolic cosine\7.1.5 Inverse hyperbolic cosine functions	292	0	1	0	0	293	1.49	198.2
7 Inverse hyperbolic functions\7.2a Inverse hyperbolic tangent\7.2.1 u (a+b arctanh(c x^n))^p	706	0	0	0	0	706	2.23	321.8
7 Inverse hyperbolic functions\7.2a Inverse hyperbolic tangent\7.2.2 u (a+b arctanh(c+d x))^p	62	0	0	0	0	62	1.55	40.3
7 Inverse hyperbolic functions\7.2a Inverse hyperbolic tangent\7.2.3 Inverse hyperbolic tangent functions	327	0	0	0	0	327	6.87	48.4
7 Inverse hyperbolic functions\7.2a Inverse hyperbolic tangent\7.2.4 x^m (c-a^2 c x^2)^p E^(n arctanh(a x))	496	0	0	0	0	496	3.94	128.2
7 Inverse hyperbolic functions\7.2a Inverse hyperbolic tangent\7.2.5 Exponentials of inverse hyperbolic tangent functions	881	0	0	0	0	881	3.49	259.4
7 Inverse hyperbolic functions\7.2b Inverse hyperbolic cotangent\7.2.1 Inverse hyperbolic cotangent functions	293	0	0	0	0	293	2.97	99.7
7 Inverse hyperbolic functions\7.2b Inverse hyperbolic cotangent\7.2.2 Exponentials of inverse hyperbolic cotangent functions	935	0	0	0	0	935	2.99	320.9
7 Inverse hyperbolic functions\7.3a Inverse hyperbolic secant\7.3.1 u (a+b arcsech(c x))^n	190	0	0	0	0	190	1.60	119.8
7 Inverse hyperbolic functions\7.3a Inverse hyperbolic secant\7.3.2 Inverse hyperbolic secant functions	100	0	0	0	0	100	2.70	37.3
7 Inverse hyperbolic functions\7.3b Inverse hyperbolic cosecant\7.3.1 u (a+b arccsch(c x))^n	159	0	0	0	0	159	1.48	108.4
7 Inverse hyperbolic functions\7.3b Inverse hyperbolic cosecant\7.3.2 Inverse hyperbolic cosecant functions	71	0	0	0	0	71	3.76	19.0
8 Special functions\8.1 Error functions	290	0	0	0	0	290	5.38	54.7
8 Special functions\8.2 Fresnel integral functions	204	0	0	0	0	204	5.16	39.9
8 Special functions\8.3 Exponential integral functions	210	0	0	0	0	210	5.63	37.6
8 Special functions\8.4 Trig integral functions	122	0	0	0	0	122	3.09	39.8
8 Special functions\8.5 Hyperbolic integral functions	122	0	0	0	0	122	3.07	40.0
8 Special functions\8.6 Gamma functions	226	0	0	0	0	226	7.81	29.3
8 Special functions\8.7 Zeta function	14	0	0	0	0	14	23.00	0.6
8 Special functions\8.8 Polylogarithm function	53	0	0	0	0	53	3.50	15.3
8 Special functions\8.9 Product logarithm function	396	0	0	0	0	396	5.51	73.0
Totals	69145	69	72	0	1	69287	2.40	29801.0
Percentages	99.80%	0.10%	0.10%	0.00%	0.00%	100.00%		