

Rules for integrands of the form $(d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n$

1. $\int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$ when $e = c^2 d$

1. $\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx$ when $e = c^2 d$

1. $\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx$ when $e = c^2 d \wedge d > 0$

1: $\int \frac{1}{\sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])} dx$ when $e = c^2 d \wedge d > 0$

Derivation: Integration by substitution

Rule: If $e = c^2 d \wedge d > 0$, then

$$\int \frac{1}{\sqrt{d + e x^2} (a + b \operatorname{ArcSinh}[c x])} dx \rightarrow \frac{\operatorname{Log}[a + b \operatorname{ArcSinh}[c x]]}{b c \sqrt{d}}$$

Program code:

```
Int[1/(Sqrt[d_+e_.*x_^2]*(a_+b_.*ArcSinh[c_.*x_])),x_Symbol] :=
  Log[a+b*ArcSinh[c*x]]/(b*c*Sqrt[d]) /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[d,0]
```

```
Int[1/(Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]*(a_+b_.*ArcCosh[c_.*x_])),x_Symbol] :=
  Log[a+b*ArcCosh[c*x]]/(b*c*Sqrt[-d1*d2]) /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[d1,0] && LtQ[d2,0]
```

$$2: \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } e = c^2 d \wedge d > 0 \wedge n \neq -1$$

Derivation: Integration by substitution

Rule: If $e = c^2 d \wedge d > 0 \wedge n \neq -1$, then

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{(a + b \operatorname{ArcSinh}[c x])^{n+1}}{b c \sqrt{d} (n+1)}$$

Program code:

```
Int[(a_.+b_.*ArcSinh[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  (a+b*ArcSinh[c*x])^(n+1)/(b*c*Sqrt[d]*(n+1) /;
  FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && GtQ[d,0] && NeQ[n,-1]
```

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_/ (Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  (a+b*ArcCosh[c*x])^(n+1)/(b*c*Sqrt[-d1*d2]*(n+1) /;
  FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[d1,0] && LtQ[d2,0] && NeQ[n,-1]
```

$$2: \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx \text{ when } e = c^2 d \wedge d \neq 0$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } e = c^2 d, \text{ then } \partial_x \frac{\sqrt{1+c^2 x^2}}{\sqrt{d+e x^2}} = 0$$

Rule: If $e = c^2 d \wedge d \neq 0$, then

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{d + e x^2}} dx \rightarrow \frac{\sqrt{1 + c^2 x^2}}{\sqrt{d + e x^2}} \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{\sqrt{1 + c^2 x^2}} dx$$

Program code:

```
Int[(a_.+b_.*ArcSinh[c_.*x_])^n_/Sqrt[d_+e_.*x_^2],x_Symbol] :=
  Sqrt[1+c^2*x^2]/Sqrt[d+e*x^2]*Int[(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && Not[GtQ[d,0]]
```

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_/ (Sqrt[d1_+e1_.*x_] * Sqrt[d2_+e2_.*x_]),x_Symbol] :=
  Sqrt[1+c*x] * Sqrt[-1+c*x] / (Sqrt[d1+e1*x] * Sqrt[d2+e2*x]) * Int[(a+b*ArcCosh[c*x])^n/ (Sqrt[1+c*x] * Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && Not[GtQ[d1,0] && LtQ[d2,0]]
```

$$2. \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0$$

$$1: \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x]) dx \text{ when } e = c^2 d \wedge p \in \mathbb{Z}^+$$

Derivation: Integration by parts

Rule: If $e = c^2 d \wedge p \in \mathbb{Z}^+$, let $u = \int (d + e x^2)^p dx$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x]) dx \rightarrow u (a+b \operatorname{ArcSinh}[c x]) - b c \int \frac{u}{\sqrt{1+c^2 x^2}} dx$$

Program code:

```
Int[(d_+e_*x_^2)^p_.*(a_+b_*ArcSinh[c_*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcSinh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[p,0]
```

```
Int[(d_+e_*x_^2)^p_.*(a_+b_*ArcCosh[c_*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcCosh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x] /;
  FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p > 0$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p > 0 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Inverted integration by parts

Rule: If $e = c^2 d \wedge n > 0 \wedge p > 0 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{x (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n}{2p+1} + \frac{2dp}{2p+1} \int (d+e x^2)^{p-1} (a+b \operatorname{ArcSinh}[c x])^n dx - \frac{bcnd^p}{2p+1} \int x (1+c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
(* Int[(d+e.*x^2)^p.*(a.+b.*ArcSinh[c.*x])^n.,x_Symbol] :=
x*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(2*p+1) +
2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
b*c*n*d^p/(2*p+1)*Int[x*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(d+e.*x^2)^p.*(a.+b.*ArcCosh[c.*x])^n.,x_Symbol] :=
x*(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n/(2*p+1) +
2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d)^p/((2*p+1))*Int[x*(-1+c*x)^(p-1/2)*(1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && GtQ[p,0] && IntegerQ[p]
```

```
(* Int[(d1+e1.*x)^p*(d2+e2.*x)^p*(a.+b.*ArcCosh[c.*x])^n.,x_Symbol] :=
x*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(2*p+1) +
2*d1*d2*p/(2*p+1)*Int[(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^p/((2*p+1))*Int[x*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && GtQ[p,0] && IntegerQ[p-1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p > 0$$

$$1: \int \sqrt{d+e x^2} (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0$$

Derivation: Inverted integration by parts

Note: The piecewise constant factor in the second integral reduces the degree of d in the resulting antiderivative.

Rule: If $e = c^2 d \wedge n > 0$, then

$$\int \sqrt{d+e x^2} (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{x \sqrt{d+e x^2} (a+b \operatorname{ArcSinh}[c x])^n}{2} - \frac{b c n \sqrt{d+e x^2}}{2 \sqrt{1+c^2 x^2}} \int x (a+b \operatorname{ArcSinh}[c x])^{n-1} dx + \frac{\sqrt{d+e x^2}}{2 \sqrt{1+c^2 x^2}} \int \frac{(a+b \operatorname{ArcSinh}[c x])^n}{\sqrt{1+c^2 x^2}} dx$$

Program code:

```
Int[Sqrt[d_+e_.*x_^2]*(a_+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  x*Sqrt[d+e*x^2]*(a+b*ArcSinh[c*x])^n/2 -
  b*c*n*Sqrt[d+e*x^2]/(2*Sqrt[1+c^2*x^2])*Int[x*(a+b*ArcSinh[c*x])^(n-1),x] +
  Sqrt[d+e*x^2]/(2*Sqrt[1+c^2*x^2])*Int[(a+b*ArcSinh[c*x])^n/Sqrt[1+c^2*x^2],x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0]
```

```
Int[Sqrt[d1_+e1_.*x_]*Sqrt[d2_+e2_.*x_]*(a_+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  x*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]*(a+b*ArcCosh[c*x])^n/2 -
  b*c*n*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(2*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[x*(a+b*ArcCosh[c*x])^(n-1),x] -
  Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(2*Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(a+b*ArcCosh[c*x])^n/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0]
```

$$2: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p > 0$$

Derivation: Inverted integration by parts and piecewise constant extraction

Basis: If $e = c^2 d$, then $\partial_x \frac{(d+e x^2)^p}{(1+c^2 x^2)^p} = 0$

Rule: If $e = c^2 d \wedge n > 0 \wedge p > 0$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{x (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n}{2 p+1} + \frac{2 d p}{2 p+1} \int (d+e x^2)^{p-1} (a+b \operatorname{ArcSinh}[c x])^n dx - \frac{b c n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{(2 p+1) (1+c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1+c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
x*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n/(2*p+1) +
2*d*p/(2*p+1)*Int[(d+e*x^2)^(p-1)*(a+b*ArcSinh[c*x])^n,x] -
b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/((2*p+1)*(1+c^2*x^2)^FracPart[p])*
Int[x*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[p,0]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
x*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(2*p+1) +
2*d1*d2*p/(2*p+1)*Int[(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/((2*p+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
Int[x*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && GtQ[p,0] && IntegerQ[p-1/2]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
x*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n/(2*p+1) +
2*d1*d2*p/(2*p+1)*Int[(d1+e1*x)^(p-1)*(d2+e2*x)^(p-1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/((2*p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
Int[x*(-1+c*x)^(p-1/2)*(1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && GtQ[p,0]
```

3. $\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx$ when $e = c^2 d \wedge n > 0 \wedge p < -1$

$$1. \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } e = c^2 d \wedge n > 0$$

$$1: \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } e = c^2 d \wedge n > 0 \wedge d > 0$$

Derivation: Integration by parts

$$\text{Basis: } \frac{1}{(d+e x^2)^{3/2}} = \partial_x \frac{x}{d \sqrt{d+e x^2}}$$

Rule: If $e = c^2 d \wedge n > 0 \wedge d > 0$, then

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{(d + e x^2)^{3/2}} dx \rightarrow \frac{x (a + b \operatorname{ArcSinh}[c x])^n}{d \sqrt{d + e x^2}} - \frac{b c n}{\sqrt{d}} \int \frac{x (a + b \operatorname{ArcSinh}[c x])^{n-1}}{d + e x^2} dx$$

Program code:

```
(* Int[(a_.+b_.*ArcSinh[c_*x_])^n_/(d_+e_*x_^2)^(3/2),x_Symbol] :=
  x*(a+b*ArcSinh[c*x])^n/(d*Sqrt[d+e*x^2]) -
  b*c*n/Sqrt[d]*Int[x*(a+b*ArcSinh[c*x])^(n-1)/(d+e*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0] && GtQ[d,0] *)
```

```
(* Int[(a_.+b_.*ArcCosh[c_*x_])^n_/((d1_+e1_*x_)^(3/2)*(d2_+e2_*x_)^(3/2)),x_Symbol] :=
  x*(a+b*ArcCosh[c*x])^n/(d1*d2*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]) +
  b*c*n/Sqrt[-d1*d2]*Int[x*(a+b*ArcCosh[c*x])^(n-1)/(d1*d2+e1*e2*x^2),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

$$2: \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{(d + e x^2)^{3/2}} dx \text{ when } e = c^2 d \wedge n > 0$$

Derivation: Integration by parts and piecewise constant extraction

$$\text{Basis: } \frac{1}{(d+e x^2)^{3/2}} = \partial_x \frac{x}{d \sqrt{d+e x^2}}$$

Basis: If $e = c^2 d$, then $\partial_x \frac{\sqrt{1+c^2 x^2}}{\sqrt{d+e x^2}} = 0$

Rule: If $e = c^2 d \wedge n > 0$, then

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{(d + e x^2)^{3/2}} dx \rightarrow \frac{x (a + b \operatorname{ArcSinh}[c x])^n}{d \sqrt{d + e x^2}} - \frac{b c n \sqrt{1 + c^2 x^2}}{d \sqrt{d + e x^2}} \int \frac{x (a + b \operatorname{ArcSinh}[c x])^{n-1}}{1 + c^2 x^2} dx$$

Program code:

```
Int[(a_.+b_.*ArcSinh[c_.*x_])^n_./((d_+e_.*x_^2)^(3/2)),x_Symbol] :=
  x*(a+b*ArcSinh[c*x])^n/(d*Sqrt[d+e*x^2]) -
  b*c*n*Sqrt[1+c^2*x^2]/(d*Sqrt[d+e*x^2])*Int[x*(a+b*ArcSinh[c*x])^(n-1)/(1+c^2*x^2),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0]
```

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_./((d1_+e1_.*x_)^(3/2)*(d2_+e2_.*x_)^(3/2)),x_Symbol] :=
  x*(a+b*ArcCosh[c*x])^n/(d1*d2*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]) +
  b*c*n*Sqrt[1+c*x]*Sqrt[-1+c*x]/(d1*d2*Sqrt[d1+e1*x]*Sqrt[d2+e2*x])*Int[x*(a+b*ArcCosh[c*x])^(n-1)/(1-c^2*x^2),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0]
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p < -1 \wedge p \neq -\frac{3}{2}$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p < -1 \wedge p \neq -\frac{3}{2} \wedge (p \in \mathbb{Z} \vee d > 0)$$

Rule: If $e = c^2 d \wedge n > 0 \wedge p < -1 \wedge p \neq -\frac{3}{2} \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$

$$-\frac{x (d+e x^2)^{p+1} (a+b \operatorname{ArcSinh}[c x])^n}{2 d (p+1)} +$$

$$\frac{2 p+3}{2 d (p+1)} \int (d+e x^2)^{p+1} (a+b \operatorname{ArcSinh}[c x])^n dx + \frac{b c n d^p}{2 (p+1)} \int x (1+c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
(* Int[(d+_e_.*x_^2)^p_*(a_+_b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
-x*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*d*(p+1)) +
(2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] +
b*c*n*d^p/(2*(p+1))*Int[x*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(d+_e_.*x_^2)^p_*(a_+_b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
-x*(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d*(p+1)) +
(2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d)^p/(2*(p+1))*Int[x*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && GtQ[n,0] && LtQ[p,-1] && IntegerQ[p]
```

```
(* Int[(d1+_e1_.*x_)^p_*(d2+_e2_.*x_)^p_*(a_+_b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
-x*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d1*d2*(p+1)) +
(2*p+3)/(2*d1*d2*(p+1))*Int[(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^p/(2*(p+1))*Int[x*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2] &&
IntegerQ[p+1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

$$2: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n > 0 \wedge p < -1 \wedge p \neq -\frac{3}{2}$$

Rule: If $e = c^2 d \wedge n > 0 \wedge p < -1 \wedge p \neq -\frac{3}{2}$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$

$$-\frac{x (d+e x^2)^{p+1} (a+b \operatorname{ArcSinh}[c x])^n}{2 d (p+1)} + \frac{2 p+3}{2 d (p+1)} \int (d+e x^2)^{p+1} (a+b \operatorname{ArcSinh}[c x])^n dx +$$

$$\frac{b c n d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{2 (p+1) (1+c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1+c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n-1} dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
-x*(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n/(2*d*(p+1)) +
(2*p+3)/(2*d*(p+1))*Int[(d+e*x^2)^(p+1)*(a+b*ArcSinh[c*x])^n,x] +
b*c*n*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(2*(p+1)*(1+c^2*x^2)^FracPart[p])*
Int[x*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2]
```

```
Int[(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
-x*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d1*d2*(p+1)) +
(2*p+3)/(2*d1*d2*(p+1))*Int[(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^(p+1/2)*Sqrt[1+c*x]*Sqrt[-1+c*x]/(2*(p+1)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x])*
Int[x*(-1+c^2*x^2)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2] && IntegerQ[p+1/2]
```

```
Int[(d1_+e1_.*x_)^p_*(d2_+e2_.*x_)^p_*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
-x*(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n/(2*d1*d2*(p+1)) +
(2*p+3)/(2*d1*d2*(p+1))*Int[(d1+e1*x)^(p+1)*(d2+e2*x)^(p+1)*(a+b*ArcCosh[c*x])^n,x] -
b*c*n*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(2*(p+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
Int[x*(1+c*x)^(p+1/2)*(-1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n-1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && GtQ[n,0] && LtQ[p,-1] && NeQ[p,-3/2]
```

$$4: \int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{d + e x^2} dx \text{ when } e = c^2 d \wedge n \in \mathbb{Z}^+$$

Derivation: Integration by substitution

Basis: If $e = c^2 d$, then $\frac{1}{d+e x^2} = \frac{1}{c d} \operatorname{Subst}[\operatorname{Sech}[x], x, \operatorname{ArcSinh}[c x]] \partial_x \operatorname{ArcSinh}[c x]$

Note: If $n \in \mathbb{Z}^+$, then $(a + b x)^n \operatorname{Sech}[x]$ is integrable in closed-form.

Rule: If $e = c^2 d \wedge n \in \mathbb{Z}^+$, then

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^n}{d + e x^2} dx \rightarrow \frac{1}{c d} \operatorname{Subst}\left[\int (a + b x)^n \operatorname{Sech}[x] dx, x, \operatorname{ArcSinh}[c x]\right]$$

Program code:

```
Int[(a_+b_.*ArcSinh[c_.*x_])^n_./(d_+e_.*x_^2),x_Symbol] :=
  1/(c*d)*Subst[Int[(a+b*x)^n*Sech[x],x],x,ArcSinh[c*x] ] /;
FreeQ[{a,b,c,d,e},x] && EqQ[e,c^2*d] && IGtQ[n,0]
```

```
Int[(a_+b_.*ArcCosh[c_.*x_])^n_./(d_+e_.*x_^2),x_Symbol] :=
  -1/(c*d)*Subst[Int[(a+b*x)^n*Csch[x],x],x,ArcCosh[c*x] ] /;
FreeQ[{a,b,c,d,e},x] && EqQ[c^2*d+e,0] && IGtQ[n,0]
```

$$3. \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n < -1$$

$$1: \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge n < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by parts

Basis: $\frac{(a+b \operatorname{ArcSinh}[c x])^n}{\sqrt{1+c^2 x^2}} = \partial_x \frac{(a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)}$

Rule: If $e = c^2 d \wedge n < -1 \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{d^p (1+c^2 x^2)^{p+\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)} - \frac{c d^p (2p+1)}{b (n+1)} \int x (1+c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n+1} dx$$

Program code:

```
(* Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  d^p*(1+c^2*x^2)^(p+1/2)*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  c*d^p*(2*p+1)/(b*(n+1))*Int[x*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[e,c^2*d] && LtQ[n,-1] && (IntegerQ[p] || GtQ[d,0]) *)
```

```
Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (-d)^p*(-1+c*x)^(p+1/2)*(1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) -
  c*(-d)^p*(2*p+1)/(b*(n+1))*Int[x*(-1+c*x)^(p-1/2)*(1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[c^2*d+e,0] && LtQ[n,-1] && IntegerQ[p]
```

```
(* Int[(d1_+e1_.*x_^2)^p_.*(d2_+e2_.*x_^2)^p_.*(a_+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (-d1*d2)^p*(-1+c*x)^(p+1/2)*(1+c*x)^(p+1/2)*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) -
  c*(-d1*d2)^p*(2*p+1)/(b*(n+1))*Int[x*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && LtQ[n,-1] && IntegerQ[p-1/2] && (GtQ[d1,0] && LtQ[d2,0]) *)
```

2: $\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx$ when $e = c^2 d \wedge n < -1$

Derivation: Integration by parts and piecewise constant extraction

Basis: $\frac{(a+b \operatorname{ArcSinh}[c x])^n}{\sqrt{1+c^2 x^2}} = \partial_x \frac{(a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)}$

Basis: If $e = c^2 d$, then $\partial_x \frac{(d+e x^2)^p}{(1+c^2 x^2)^p} = 0$

Rule: If $e = c^2 d \wedge n < -1$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow$$

$$\frac{\sqrt{1+c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)} + \frac{c (2 p+1)}{b (n+1)} \int \frac{x (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^{n+1}}{\sqrt{1+c^2 x^2}} dx \rightarrow$$

$$\frac{\sqrt{1+c^2 x^2} (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^{n+1}}{b c (n+1)} - \frac{c (2 p+1) d^{\operatorname{IntPart}[p]} (d+e x^2)^{\operatorname{FracPart}[p]}}{b (n+1) (1+c^2 x^2)^{\operatorname{FracPart}[p]}} \int x (1+c^2 x^2)^{p-\frac{1}{2}} (a+b \operatorname{ArcSinh}[c x])^{n+1} dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  Sqrt[1+c^2*x^2]*(d+e*x^2)^p*(a+b*ArcSinh[c*x])^(n+1)/(b*c*(n+1)) -
  c*(2*p+1)*d^IntPart[p]*(d+e*x^2)^FracPart[p]/(b*(n+1)*(1+c^2*x^2)^FracPart[p])*
  Int[x*(1+c^2*x^2)^(p-1/2)*(a+b*ArcSinh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d,e,p},x] && EqQ[e,c^2*d] && LtQ[n,-1]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  Sqrt[1+c*x]*Sqrt[-1+c*x]*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) -
  c*(2*p+1)*(-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(b*(n+1)*Sqrt[1+c*x]*Sqrt[-1+c*x])*
  Int[x*(-1+c^2*x^2)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && LtQ[n,-1] && IntegerQ[p-1/2]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  Sqrt[1+c*x]*Sqrt[-1+c*x]*(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) -
  c*(2*p+1)*(-d1*d2)^IntPart[p]*(d1+e1*x)^FracPart[p]*(d2+e2*x)^FracPart[p]/(b*(n+1)*(1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
  Int[x*(1+c*x)^(p-1/2)*(-1+c*x)^(p-1/2)*(a+b*ArcCosh[c*x])^(n+1),x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,p},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && LtQ[n,-1]
```

$$4. \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge 2p \in \mathbb{Z}^+$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge 2p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Integration by substitution

Basis: If $e = c^2 d \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$(d+e x^2)^p = \frac{d^p}{c} \operatorname{Subst}[\operatorname{Cosh}[x]^{2p+1}, x, \operatorname{ArcSinh}[c x]] \partial_x \operatorname{ArcSinh}[c x]$$

Note: If $2p \in \mathbb{Z}^+$, then $(a+bx)^n \operatorname{cosh}[x]^{2p+1}$ is integrable in closed-form.

Rule: If $e = c^2 d \wedge 2p \in \mathbb{Z}^+ \wedge (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{d^p}{c} \operatorname{Subst}\left[\int (a+bx)^n \operatorname{Cosh}[x]^{2p+1} dx, x, \operatorname{ArcSinh}[c x]\right]$$

Program code:

```
Int[(d+e_.x^2)^p_.*(a_.+b_.*ArcSinh[c_.x])^n_,x_Symbol] :=
  d^p/c*Subst[Int[(a+b*x)^n*Cosh[x]^(2*p+1),x],x,ArcSinh[c*x] ] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && IGtQ[2*p,0] && (IntegerQ[p] || GtQ[d,0])
```

```
Int[(d+e_.x^2)^p_.*(a_.+b_.*ArcCosh[c_.x])^n_,x_Symbol] :=
  (-d)^p/c*Subst[Int[(a+b*x)^n*Sinh[x]^(2*p+1),x],x,ArcCosh[c*x] ] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[c^2*d+e,0] && IGtQ[p,0]
```

```
Int[(d1+e1_.x)^p_.*(d2+e2_.x)^p_.*(a_.+b_.*ArcCosh[c_.x])^n_,x_Symbol] :=
  (-d1*d2)^p/c*Subst[Int[(a+b*x)^n*Sinh[x]^(2*p+1),x],x,ArcCosh[c*x] ] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && IGtQ[p+1/2,0] && (GtQ[d1,0] && LtQ[d2,0])
```

$$2: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e = c^2 d \wedge 2p \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$$

Derivation: Piecewise constant extraction

$$\text{Basis: If } e = c^2 d, \text{ then } \partial_x \frac{\sqrt{d+e x^2}}{\sqrt{1+c^2 x^2}} = 0$$

Rule: If $e = c^2 d \wedge 2p \in \mathbb{Z}^+ \wedge \neg (p \in \mathbb{Z} \vee d > 0)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \rightarrow \frac{d^{p-\frac{1}{2}} \sqrt{d+e x^2}}{\sqrt{1+c^2 x^2}} \int (1+c^2 x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  d^(p-1/2)*Sqrt[d+e*x^2]/Sqrt[1+c^2*x^2]*Int[(1+c^2*x^2)^p*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n},x] && EqQ[e,c^2*d] && IGtQ[2*p,0] && Not[IntegerQ[p] || GtQ[d,0]]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  (-d1*d2)^(p-1/2)*Sqrt[d1+e1*x]*Sqrt[d2+e2*x]/(Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[(1+c*x)^p*(-1+c*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n},x] && EqQ[e1,c*d1] && EqQ[e2,-c*d2] && IGtQ[2*p,0] && Not[GtQ[d1,0] && LtQ[d2,0]]
```

$$2. \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n dx \text{ when } e \neq c^2 d$$

$$1: \int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x]) dx \text{ when } e \neq c^2 d \wedge (p \in \mathbb{Z}^+ \vee p + \frac{1}{2} \in \mathbb{Z}^-)$$

Derivation: Integration by parts

Note: If $p \in \mathbb{Z}^+ \vee p + \frac{1}{2} \in \mathbb{Z}^-$, then $\int (d+e x^2)^p dx$ is a rational function.

Rule: If $e \neq c^2 d \wedge (p \in \mathbb{Z}^+ \vee p + \frac{1}{2} \in \mathbb{Z}^-)$, let $u = \int (d+e x^2)^p dx$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x]) \, dx \rightarrow u (a+b \operatorname{ArcSinh}[c x]) - b c \int \frac{u}{\sqrt{1+c^2 x^2}} \, dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcSinh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/Sqrt[1+c^2*x^2],x],x] /;
  FreeQ[{a,b,c,d,e},x] && NeQ[e,c^2*d] && (IGtQ[p,0] || ILtQ[p+1/2,0])
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcCosh[c*x],u,x] - b*c*Int[SimplifyIntegrand[u/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x],x] /;
  FreeQ[{a,b,c,d,e},x] && NeQ[c^2*d+e,0] && (IGtQ[p,0] || ILtQ[p+1/2,0])
```

```
(* Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_]),x_Symbol] :=
  With[{u=IntHide[(d+e*x^2)^p,x]},
    Dist[a+b*ArcCosh[c*x],u,x] - b*c*Sqrt[1-c^2*x^2]/(Sqrt[1+c*x]*Sqrt[-1+c*x])*Int[SimplifyIntegrand[u/Sqrt[1-c^2*x^2],x],x] /;
  FreeQ[{a,b,c,d,e},x] && (IGtQ[p,0] || ILtQ[p+1/2,0]) *)
```

2: $\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n \, dx$ when $e \neq c^2 d \wedge p \in \mathbb{Z} \wedge (p > 0 \vee n \in \mathbb{Z}^+)$

Derivation: Algebraic expansion

Rule: If $e \neq c^2 d \wedge p \in \mathbb{Z} \wedge (p > 0 \vee n \in \mathbb{Z}^+)$, then

$$\int (d+e x^2)^p (a+b \operatorname{ArcSinh}[c x])^n \, dx \rightarrow \int (a+b \operatorname{ArcSinh}[c x])^n \operatorname{ExpandIntegrand}[(d+e x^2)^p, x] \, dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcSinh[c*x])^n,(d+e*x^2)^p,x],x] /;
  FreeQ[{a,b,c,d,e,n},x] && NeQ[e,c^2*d] && IntegerQ[p] && (p>0 || IGtQ[n,0])
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Int[ExpandIntegrand[(a+b*ArcCosh[c*x])^n,(d+e*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,n},x] && NeQ[c^2+d+e,0] && IntegerQ[p] && (p>0 || IGtQ[n,0])
```

$$\mathbf{x:} \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$$

Rule:

$$\int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx \rightarrow \int (d + e x^2)^p (a + b \operatorname{ArcSinh}[c x])^n dx$$

Program code:

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcSinh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(d+e*x^2)^p*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n,p},x]
```

```
Int[(d_+e_.*x_^2)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(d+e*x^2)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n,p},x] && IntegerQ[p]
```

```
Int[(d1_+e1_.*x_)^p_.*(d2_+e2_.*x_)^p_.*(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  Unintegrable[(d1+e1*x)^p*(d2+e2*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d1,e1,d2,e2,n,p},x]
```

Rules for integrands of the form $(d + e x)^p (f + g x)^q (a + b \operatorname{ArcSinh}[c x])^n$

$$\mathbf{1:} \int (d + e x)^p (f + g x)^q (a + b \operatorname{ArcSinh}[c x])^n dx \text{ when } e f + d g = 0 \wedge c^2 d^2 + e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{f}{e} < 0$$

Derivation: Algebraic expansion

Basis: If $ef + dg = 0 \wedge c^2 d^2 + e^2 = 0 \wedge d > 0 \wedge \frac{g}{e} < 0$, then

$$(d + ex)^p (f + gx)^q = \left(-\frac{d^2 g}{e}\right)^q (d + ex)^{p-q} (1 + c^2 x^2)^q$$

-

Rule: If $ef + dg = 0 \wedge c^2 d^2 + e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge d > 0 \wedge \frac{g}{e} < 0$, then

$$\int (d + ex)^p (f + gx)^q (a + b \operatorname{ArcSinh}[cx])^n dx \rightarrow \left(-\frac{d^2 g}{e}\right)^q \int (d + ex)^{p-q} (1 + c^2 x^2)^q (a + b \operatorname{ArcSinh}[cx])^n dx$$

Program code:

```
Int[(d+e.*x_)^p*(f+g.*x_)^q*(a.+b.*ArcSinh[c.*x_])^n.,x_Symbol] :=
  (-d^2*g/e)^q*Int[(d+e*x)^(p-q)*(1+c^2*x^2)^q*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2+e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0] && GtQ[d,0] && LtQ[g/e,0]
```

2: $\int (d + ex)^p (f + gx)^q (a + b \operatorname{ArcSinh}[cx])^n dx$ when $ef + dg = 0 \wedge c^2 d^2 + e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$

Derivation: Piecewise constant extraction

Basis: If $ef + dg = 0 \wedge c^2 d^2 + e^2 = 0$, then $\partial_x \frac{(d+ex)^q (f+gx)^q}{(1+c^2 x^2)^q} = 0$

-

Rule: If $ef + dg = 0 \wedge c^2 d^2 + e^2 = 0 \wedge (p | q) \in \mathbb{Z} + \frac{1}{2} \wedge p - q \geq 0 \wedge \neg (d > 0 \wedge \frac{g}{e} < 0)$, then

$$\int (d + ex)^p (f + gx)^q (a + b \operatorname{ArcSinh}[cx])^n dx \rightarrow \frac{(d + ex)^q (f + gx)^q}{(1 + c^2 x^2)^q} \int (d + ex)^{p-q} (1 + c^2 x^2)^q (a + b \operatorname{ArcSinh}[cx])^n dx$$

-

Program code:

```
Int[(d+e.*x_)^p*(f+g.*x_)^q*(a.+b.*ArcSinh[c.*x_])^n.,x_Symbol] :=
  (d+e*x)^(p-q)*(f+g*x)^q/(1+c^2*x^2)^q*Int[(d+e*x)^(p-q)*(1+c^2*x^2)^q*(a+b*ArcSinh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,f,g,n},x] && EqQ[e*f+d*g,0] && EqQ[c^2*d^2+e^2,0] && HalfIntegerQ[p,q] && GeQ[p-q,0]
```

1: $\int (d+e x)^p (f+g x)^p (a+b \operatorname{ArcSinh}[c x])^n dx$ when $e f + d g = 0 \wedge c^2 f^2 + g^2 = 0 \wedge p \notin \mathbb{Z}$

```
Int[(d+_e_.*x_^2)^p_*(a_+_b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  (-d)^IntPart[p]*(d+e*x^2)^FracPart[p]/((1+c*x)^FracPart[p]*(-1+c*x)^FracPart[p])*
  Int[(1+c*x)^p*(-1+c*x)^p*(a+b*ArcCosh[c*x])^n,x] /;
FreeQ[{a,b,c,d,e,n,p},x] && EqQ[c^2*d+e,0] && Not[IntegerQ[p]]
```