

Timofeev Integration Test Suite

The following is a list of the 705 example integration problems from the nine chapters in A. F. Timofeev's 1948 book *Integration of Functions*. Each integral along with its optimal antiderivative (that is, the best antiderivative found so far) is shown. These problems are also available in machine readable form from the *Rubi* home page expressed in *Axiom*, *Maple*, *Mathematica* and *Maxima* syntax.

Chapter 1 (p. 9-42) Basic Integration Problems

Problems 1 - 5 (p. 25)

■ Problem #1

$$\int \frac{1}{a^2 - b^2 x^2} dx = \frac{1}{a b} \operatorname{ArcTanh}\left[\frac{b x}{a}\right]$$

■ Problem #2

$$\int \frac{1}{a^2 + b^2 x^2} dx = \frac{1}{a b} \operatorname{ArcTan}\left[\frac{b x}{a}\right]$$

■ Problem #3

$$\int \sec[2 a x] dx = \frac{\operatorname{ArcTanh}[\sin[2 a x]]}{2 a}$$

■ Problem #4

$$\int \frac{1}{4 \sin\left[\frac{x}{3}\right]} dx = -\frac{3}{4} \operatorname{ArcTanh}\left[\cos\left[\frac{x}{3}\right]\right]$$

■ Problem #5

$$\int \frac{1}{\cos\left[\frac{3\pi}{4} - 2 x\right]} dx = -\frac{1}{2} \operatorname{ArcTanh}\left[\sin\left[\frac{\pi}{4} + 2 x\right]\right]$$

Problems 6 - 11 (p. 25-26)

■ Problem #6

$$\int \sec[x] \tan[x] dx = \sec[x]$$

■ Problem #7

$$\int \csc[x] \cot[x] dx = -\csc[x]$$

■ Problem #8

$$\int \frac{\tan[x]}{\sin[2x]} dx = \frac{\tan[x]}{2}$$

■ Problem #9

$$\int \frac{1}{1 + \cos[x]} dx = \frac{\sin[x]}{1 + \cos[x]}$$

■ Problem #10

$$\int \frac{1}{1 - \cos[x]} dx = -\frac{\sin[x]}{1 - \cos[x]}$$

■ Problem #11

$$\int \frac{\sin[x]}{a - b \cos[x]} dx = \frac{\log[a - b \cos[x]]}{b}$$

Problems 12 - 16 (p. 26)

■ Problem #12

$$\int \frac{\cos[x]}{a^2 + b^2 \sin[x]^2} dx = \frac{1}{ab} \operatorname{ArcTan}\left[\frac{b \sin[x]}{a}\right]$$

■ Problem #13

$$\int \frac{\cos[x]}{a^2 - b^2 \sin[x]^2} dx = \frac{1}{ab} \operatorname{ArcTanh}\left[\frac{b \sin[x]}{a}\right]$$

■ Problem #14a

$$\int \frac{\sin[2x]}{a^2 + b^2 \sin[x]^2} dx = \frac{\log[a^2 + b^2 \sin[x]^2]}{b^2}$$

■ **Problem #14b**

$$\int \frac{\sin[2x]}{a^2 - b^2 \sin[x]^2} dx = -\frac{\text{Log}[a^2 - b^2 \sin[x]^2]}{b^2}$$

■ **Problem #15a**

$$\int \frac{\sin[2x]}{a^2 + b^2 \cos[x]^2} dx = -\frac{\text{Log}[a^2 + b^2 \cos[x]^2]}{b^2}$$

■ **Problem #15b**

$$\int \frac{\sin[2x]}{a^2 - b^2 \cos[x]^2} dx = \frac{\text{Log}[a^2 - b^2 \cos[x]^2]}{b^2}$$

■ **Problem #16**

$$\int \frac{1}{4 - \cos[x]^2} dx = \frac{x}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} \text{ArcTan}\left[\frac{\cos[x] \sin[x]}{3 + 2\sqrt{3} + \sin[x]^2}\right]$$

Problems 17 - 21 (p. 26)

■ **Problem #17**

$$\int \frac{e^x}{e^{2x} - 1} dx = -\text{ArcTanh}[e^x]$$

■ **Problem #18**

$$\int \frac{1}{x \text{Log}[x]} dx = \text{Log}[\text{Log}[x]]$$

■ **Problem #19**

$$\int \frac{1}{x(1 + \text{Log}[x]^2)} dx = \text{ArcTan}[\text{Log}[x]]$$

■ **Problem #20**

$$\int \frac{1}{x(1 - \text{Log}[x])} dx = -\text{Log}[1 - \text{Log}[x]]$$

■ **Problem #21**

$$\int \frac{1}{x(1 + \text{Log}[\frac{x}{a}])} dx = \text{Log}\left[1 + \text{Log}\left[\frac{x}{a}\right]\right]$$

Problems 22 - 26 (p. 27)

■ Problem #22

$$\int \frac{(1 - \sqrt{x} + x)^2}{x^2} dx = -\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3 \operatorname{Log}[x]$$

■ Problem #23

$$\int \frac{(2 - x^{2/3})(x + \sqrt{x})}{x^{3/2}} dx = 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2 \operatorname{Log}[x]$$

■ Problem #24

$$\int \frac{2x - 1}{2x + 3} dx = x - 2 \operatorname{Log}[2x + 3]$$

■ Problem #25

$$\int \frac{2x - 5}{3x^2 - 2} dx = \frac{1}{12} (4 - 5\sqrt{6}) \operatorname{Log}[\sqrt{2} - \sqrt{3}x] + \frac{1}{12} (4 + 5\sqrt{6}) \operatorname{Log}[\sqrt{2} + \sqrt{3}x]$$

■ Problem #26

$$\int \frac{2x - 5}{3x^2 + 2} dx = -\frac{5}{\sqrt{6}} \operatorname{ArcTan}\left[\sqrt{\frac{3}{2}}x\right] + \frac{1}{3} \operatorname{Log}[3x^2 + 2]$$

Problems 27 - 33 (p. 27-28)

■ Problem #27

$$\int \sin\left[\frac{x}{4}\right] \sin[x] dx = \frac{2}{3} \sin\left[\frac{3x}{4}\right] - \frac{2}{5} \sin\left[\frac{5x}{4}\right]$$

■ Problem #28

$$\int \cos[3x] \cos[4x] dx = \frac{\sin[x]}{2} + \frac{1}{14} \sin[7x]$$

■ Problem #29

$$\int \tan[x] \tan[x - a] dx = -x + \operatorname{Cot}[a] \operatorname{Log}[\cos[x - a]] - \operatorname{Cot}[a] \operatorname{Log}[\cos[x]]$$

■ Problem #30a

$$\int \sin[x]^2 dx = \frac{x}{2} - \frac{1}{2} \cos[x] \sin[x]$$

■ **Problem #30b**

$$\int \cos [x]^2 dx = \frac{x}{2} + \frac{1}{2} \cos [x] \sin [x]$$

■ **Problem #31**

$$\int \sin [x] \cos [x]^3 dx = -\frac{1}{4} \cos [x]^4$$

■ **Problem #32**

$$\int \frac{\cos [x]^3}{\sin [x]^4} dx = \frac{1}{\sin [x]} - \frac{1}{3 \sin [x]^3}$$

■ **Problem #33**

$$\int \frac{1}{\sin [x]^2 \cos [x]^2} dx = \tan [x] - \cot [x]$$

Problems 34 - 37 (p. 28)

■ **Problem #34**

$$\int \cot \left[\frac{3x}{4} \right]^2 dx = -x - \frac{4}{3} \cot \left[\frac{3x}{4} \right]$$

■ **Problem #35**

$$\int (1 + \tan [2x])^2 dx = -\log [\cos [2x]] + \frac{1}{2} \tan [2x]$$

■ **Problem #36**

$$\int (\tan [x] - \cot [x])^2 dx = -4x - \cot [x] + \tan [x]$$

■ **Problem #37**

$$\int (\tan [x] - \sec [x])^2 dx = -x - 2 \sec [x] (1 - \sin [x])$$

Problems 38 - 40 (p. 28)

■ **Problem #38**

$$\int \frac{\sin [x]}{1 + \sin [x]} dx = x + \frac{\cos [x]}{1 + \sin [x]}$$

■ **Problem #39**

$$\int \frac{\cos[x]}{1 - \cos[x]} dx = -x - \frac{\sin[x]}{1 - \cos[x]}$$

■ **Problem #40**

$$\int (e^{x/2} - 1)^3 e^{-\frac{x}{2}} dx = \frac{2}{e^{x/2}} - 6 e^{x/2} + e^x + 3x$$

Problems 41 - 43 (p. 35)

■ **Problem #41**

$$\int \frac{1}{5 - 6x + x^2} dx = -\frac{1}{4} \operatorname{Log}[1 - x] + \frac{1}{4} \operatorname{Log}[5 - x]$$

■ **Problem #42**

$$\int \frac{x^2}{13 - 6x^3 + x^6} dx = -\frac{1}{6} \operatorname{ArcTan}\left[\frac{3}{2} - \frac{x^3}{2}\right]$$

■ **Problem #43**

$$\int \frac{2 + x}{-1 - 4x + x^2} dx = \frac{1}{10} (5 - 4\sqrt{5}) \operatorname{Log}[2 - \sqrt{5} - x] + \frac{1}{10} (5 + 4\sqrt{5}) \operatorname{Log}[2 + \sqrt{5} - x]$$

Problems 44 - 48 (p. 35-36)

■ **Problem #44**

$$\int \frac{1}{1 + (1 + x)^{1/3}} dx = -3 (1 + x)^{1/3} + \frac{3}{2} (1 + x)^{2/3} + 3 \operatorname{Log}[1 + (1 + x)^{1/3}]$$

■ **Problem #45**

$$\int \frac{1}{\sqrt{x} (ax + b)} dx = \frac{2}{\sqrt{a} \sqrt{b}} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}}\right]$$

■ **Problem #46**

$$\int x^3 \sqrt{1 + x^2} dx = -\frac{1}{3} (1 + x^2)^{3/2} + \frac{1}{5} (1 + x^2)^{5/2}$$

■ **Problem #47**

$$\int \frac{x}{\sqrt{a^4 - x^4}} dx = \frac{1}{2} \operatorname{ArcTan} \left[\frac{x^2}{\sqrt{a^4 - x^4}} \right]$$

■ **Problem #48a**

$$\int \frac{1}{x \sqrt{x^2 - a^2}} dx = \frac{1}{a} \operatorname{ArcTan} \left[\frac{\sqrt{x^2 - a^2}}{a} \right]$$

■ **Problem #48b**

$$\int \frac{1}{x \sqrt{a^2 - x^2}} dx = -\frac{1}{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a^2 - x^2}}{a} \right]$$

■ **Problem #48c**

$$\int \frac{1}{x \sqrt{x^2 + a^2}} dx = -\frac{1}{a} \operatorname{ArcTanh} \left[\frac{\sqrt{x^2 + a^2}}{a} \right]$$

Problems 49 - 54 (p. 36)

■ **Problem #49**

$$\int \frac{1}{\sqrt{2 + x - x^2}} dx = -\operatorname{ArcSin} \left[\frac{1}{3} (1 - 2x) \right]$$

■ **Problem #50**

$$\int \frac{1}{\sqrt{5 - 4x + 3x^2}} dx = -\frac{1}{\sqrt{3}} \operatorname{ArcSinh} \left[\frac{2 - 3x}{\sqrt{11}} \right]$$

■ **Problem #51**

$$\int \frac{1}{\sqrt{x - x^2}} dx = -\operatorname{ArcSin} [1 - 2x]$$

■ **Problem #52**

$$\int \frac{1 + 2x}{\sqrt{2 + x - x^2}} dx = -2 \sqrt{2 + x - x^2} - 2 \operatorname{ArcSin} \left[\frac{1}{3} (1 - 2x) \right]$$

■ **Problem #53**

$$\int \frac{1}{x \sqrt{2 + x - x^2}} dx = -\frac{1}{\sqrt{2}} \operatorname{ArcTanh} \left[\frac{4 + x}{2\sqrt{2} \sqrt{2 + x - x^2}} \right]$$

■ **Problem #54**

$$\int \frac{1}{(x-2)\sqrt{2+x-x^2}} dx = \frac{2\sqrt{2+x-x^2}}{3(x-2)}$$

Problems 55 - 60 (p. 36-37)

■ **Problem #55**

$$\int \frac{2+3\sin[x]}{\sin[x](1-\cos[x])} dx = -\text{ArcTanh}[\cos[x]] - \frac{1}{1-\cos[x]} - \frac{3\sin[x]}{1-\cos[x]}$$

■ **Problem #56**

$$\int \frac{1}{2+3\cos[x]^2} dx = \frac{x}{\sqrt{10}} - \frac{1}{\sqrt{10}} \text{ArcTan}\left[\frac{3\cos[x]\sin[x]}{2+\sqrt{10}+3\cos[x]^2}\right]$$

■ **Problem #57**

$$\int \frac{1-\tan[x]}{\sin[2x]} dx = \frac{1}{2} \text{Log}[\tan[x]] - \frac{\tan[x]}{2}$$

■ **Problem #58**

$$\int \frac{1+\tan[x]^2}{1-\tan[x]^2} dx = \frac{1}{2} \text{ArcTanh}[2\cos[x]\sin[x]]$$

■ **Problem #59**

$$\int (a^2 - 4\cos[x]^2)^{3/4} \sin[2x] dx = \frac{1}{7} (a^2 - 4\cos[x]^2)^{7/4}$$

■ **Problem #60**

$$\int \frac{\sin[2x]}{(a^2 - 4\sin[x]^2)^{1/3}} dx = -\frac{3}{8} (a^2 - 4\sin[x]^2)^{2/3}$$

Problems 61 - 65 (p. 37)

■ **Problem #61**

$$\int \frac{1}{\sqrt{a^{2x}-1}} dx = \frac{\text{ArcTan}[\sqrt{a^{2x}-1}]}{\text{Log}[a]}$$

■ **Problem #62**

$$\int \frac{e^{x/2}}{\sqrt{e^x - 1}} dx = 2 \operatorname{ArcTanh} \left[\frac{e^{x/2}}{\sqrt{e^x - 1}} \right]$$

■ **Problem #63**

$$\int \frac{\operatorname{ArcTan}[x]^n}{1+x^2} dx = \frac{\operatorname{ArcTan}[x]^{n+1}}{n+1}$$

■ **Problem #64**

$$\int \frac{\operatorname{ArcSin} \left[\frac{x}{a} \right]^{3/2}}{\sqrt{a^2 - x^2}} dx = \frac{2a}{5\sqrt{a^2 - x^2}} \sqrt{1 - \frac{x^2}{a^2}} \operatorname{ArcSin} \left[\frac{x}{a} \right]^{5/2}$$

■ **Problem #65**

$$\int \frac{1}{\operatorname{ArcCos}[x]^3 \sqrt{1-x^2}} dx = \frac{1}{2 \operatorname{ArcCos}[x]^2}$$

Problems 66 - 68 (p. 41)

■ **Problem #66**

$$\int x \operatorname{Log}[x]^2 dx = \frac{x^2}{4} - \frac{1}{2} x^2 \operatorname{Log}[x] + \frac{1}{2} x^2 \operatorname{Log}[x]^2$$

■ **Problem #67**

$$\int \frac{\operatorname{Log}[x]}{x^5} dx = -\frac{1}{16x^4} - \frac{\operatorname{Log}[x]}{4x^4}$$

■ **Problem #68**

$$\int x^2 \operatorname{Log} \left[\frac{x-1}{x} \right] dx = -\frac{x}{3} - \frac{x^2}{6} + \frac{1}{3} x^3 \operatorname{Log} \left[\frac{x-1}{x} \right] - \frac{1}{3} \operatorname{Log}[x-1]$$

Problems 69 - 71 (p. 41)

■ **Problem #69**

$$\int \operatorname{Cos}[x]^5 dx = \operatorname{Sin}[x] - \frac{2 \operatorname{Sin}[x]^3}{3} + \frac{\operatorname{Sin}[x]^5}{5}$$

■ Problem #70

$$\int \sin[x]^2 \cos[x]^4 dx = \frac{x}{16} + \frac{1}{16} \cos[x] \sin[x] + \frac{1}{24} \cos[x]^3 \sin[x] - \frac{1}{6} \cos[x]^5 \sin[x]$$

■ Problem #71

$$\int \frac{1}{\sin[x]^5} dx = -\frac{3}{8} \operatorname{ArcTanh}[\cos[x]] - \frac{3}{8} \cot[x] \csc[x] - \frac{1}{4} \cot[x] \csc[x]^3$$

Problems 72 - 76 (p. 42)

■ Problem #72

$$\int \frac{\sin[x]}{e^x} dx = -\frac{\cos[x]}{2e^x} - \frac{\sin[x]}{2e^x}$$

■ Problem #73

$$\int e^{2x} \sin[3x] dx = -\frac{3}{13} e^{2x} \cos[3x] + \frac{2}{13} e^{2x} \sin[3x]$$

■ Problem #74

$$\int a^x \cos[x] dx = \frac{a^x \cos[x] \operatorname{Log}[a]}{1 + \operatorname{Log}[a]^2} + \frac{a^x \sin[x]}{1 + \operatorname{Log}[a]^2}$$

■ Problem #75

$$\int \cos[\operatorname{Log}[x]] dx = \frac{1}{2} x \cos[\operatorname{Log}[x]] + \frac{1}{2} x \sin[\operatorname{Log}[x]]$$

■ Problem #76

$$\int \sec[x]^2 \operatorname{Log}[\cos[x]] dx = -x + \tan[x] + \operatorname{Log}[\cos[x]] \tan[x]$$

Problems 77 - 81 (p. 42)

■ Problem #77

$$\int x \tan[x]^2 dx = -\frac{x^2}{2} + \operatorname{Log}[\cos[x]] + x \tan[x]$$

■ Problem #78

$$\int \frac{\operatorname{ArcSin}[x]}{x^2} dx = -\frac{\operatorname{ArcSin}[x]}{x} - \operatorname{ArcTanh}[\sqrt{1-x^2}]$$

■ **Problem #79**

$$\int \text{ArcSin}[x]^2 dx = -2x + 2\sqrt{1-x^2} \text{ArcSin}[x] + x \text{ArcSin}[x]^2$$

■ **Problem #80**

$$\int \frac{\text{ArcTan}[x] x^2}{1+x^2} dx = x \text{ArcTan}[x] - \frac{\text{ArcTan}[x]^2}{2} - \frac{1}{2} \text{Log}[1+x^2]$$

■ **Problem #81**

$$\int \text{ArcCos}\left[\sqrt{\frac{x}{1+x}}\right] dx = (1+x) \left(\sqrt{\frac{1}{1+x}} \sqrt{\frac{x}{1+x}} + \text{ArcCos}\left[\sqrt{\frac{x}{1+x}}\right] \right)$$

Chapter 2 (p. 43-97)

Rational Function Integration Problems

Problems 1 - 3 (p. 60)

■ Problem #1

$$\int (3x^2 + 2x)^3 dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

■ Problem #2

$$\int (3x^2 + 2x - 1)^2 (x - 1) dx = -x + \frac{5x^2}{2} - \frac{2x^3}{3} - \frac{7x^4}{2} + \frac{3x^5}{5} + \frac{3x^6}{2}$$

■ Problem #3

$$\int (a + bx^k)^n x^{k-1} dx = \frac{(a + bx^k)^{1+n}}{bk(1+n)}$$

Problems 4 - 9 (p. 62-63)

■ Problem #4

$$\int \frac{x^3}{1+2x} dx = \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \text{Log}[1+2x]$$

■ Problem #5

$$\int \frac{x^6}{2+3x^2} dx = \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27} \sqrt{\frac{2}{3}} \text{ArcTan}\left[\sqrt{\frac{3}{2}}x\right]$$

■ Problem #6

$$\int \frac{1}{3x^2 - 7x + 2} dx = -\frac{1}{5} \text{Log}[1-3x] + \frac{1}{5} \text{Log}[2-x]$$

■ Problem #7

$$\int \frac{3x-1}{x^2-x+1} dx = -\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{3}{2} \text{Log}[1-x+x^2]$$

■ **Problem #8**

$$\int \frac{x^2}{5 + 2x + x^2} dx = x - \frac{3}{2} \operatorname{ArcTan}\left[\frac{1}{2} + \frac{x}{2}\right] - \operatorname{Log}[5 + 2x + x^2]$$

■ **Problem #9**

$$\int \frac{6x^4 - 5x^3 + 4x^2}{2x^2 - x + 1} dx = -\frac{x^2}{2} + x^3 - \frac{\operatorname{ArcTan}\left[\frac{1-4x}{\sqrt{7}}\right]}{2\sqrt{7}} + \frac{1}{4} \operatorname{Log}[1 - x + 2x^2]$$

Problems 10 - 14 (p. 63)

■ **Problem #10**

$$\int \frac{x^2 + x - 1}{x^3 + x^2 - 6x} dx = \frac{1}{2} \operatorname{Log}[2 - x] + \frac{\operatorname{Log}[x]}{6} + \frac{1}{3} \operatorname{Log}[3 + x]$$

■ **Problem #11**

$$\int \frac{5x^2 - 7ax + 11a^2}{x^3 - 6ax^2 + 11a^2x - 6a^3} dx = \frac{9}{2} \operatorname{Log}[a - x] - 17 \operatorname{Log}[2a - x] + \frac{35}{2} \operatorname{Log}[3a - x]$$

■ **Problem #12**

$$\int \frac{x^2 - x + 2}{x^4 - 5x^2 + 4} dx = -\frac{1}{3} \operatorname{Log}[1 - x] + \frac{1}{3} \operatorname{Log}[2 - x] + \frac{2}{3} \operatorname{Log}[1 + x] - \frac{2}{3} \operatorname{Log}[2 + x]$$

■ **Problem #13**

$$\int \frac{2x^2 - 5}{x^4 - 5x^2 + 6} dx = -\frac{\operatorname{ArcTanh}\left[\frac{x}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}}$$

■ **Problem #14**

$$\int \frac{1}{(x-1)(x-2)(x-3)(x-4)} dx = -\frac{1}{6} \operatorname{Log}[1 - x] + \frac{1}{2} \operatorname{Log}[2 - x] - \frac{1}{2} \operatorname{Log}[3 - x] + \frac{1}{6} \operatorname{Log}[4 - x]$$

Problems 15 - 17 (p. 64)

■ **Problem #15**

$$\int \frac{x^2 + 1}{(x-1)^3} dx = -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \operatorname{Log}[1-x]$$

■ **Problem #16**

$$\int \frac{x^5}{(3+x)^2} dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \operatorname{Log}[3+x]$$

■ **Problem #17**

$$\int \frac{5x^3 - 2}{x^4 - 8x^3 + 18x^2 - 27} dx = -\frac{133}{8(3-x)^2} + \frac{407}{16(3-x)} + \frac{313}{64} \operatorname{Log}[3-x] + \frac{7}{64} \operatorname{Log}[1+x]$$

Problems 18 - 20 (p. 65)

■ **Problem #18**

$$\int \frac{x^3 - 6x^2 + 3x - 9}{(x+3)^2(x+4)^2} dx = \frac{99}{3+x} + \frac{181}{4+x} + 264 \operatorname{Log}[3+x] - 263 \operatorname{Log}[4+x]$$

■ **Problem #19**

$$\int \frac{x^3 + x^2 + 2}{x(x^2 - 1)^2} dx = \frac{x(1+3x)}{2(1-x^2)} - \frac{3}{4} \operatorname{Log}[1-x] + 2 \operatorname{Log}[x] - \frac{5}{4} \operatorname{Log}[1+x]$$

■ **Problem #20**

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6} dx = \frac{1}{2(1-x)} - \frac{1}{2x^2} - \frac{1}{x} - \frac{7}{4} \operatorname{Log}[1-x] + 2 \operatorname{Log}[x] - \frac{1}{4} \operatorname{Log}[1+x]$$

Problems 21 - 25 (p. 66)

■ **Problem #21**

$$\int \frac{x^4 + 1}{x^3 - x^2 + x - 1} dx = x + \frac{x^2}{2} - \operatorname{ArcTan}[x] + \operatorname{Log}[1-x] - \frac{1}{2} \operatorname{Log}[1+x^2]$$

■ **Problem #22**

$$\int \frac{1}{x(1+x)(1+x^2)} dx = -\frac{\operatorname{ArcTan}[x]}{2} + \operatorname{Log}[x] - \frac{1}{2} \operatorname{Log}[1+x] - \frac{1}{4} \operatorname{Log}[1+x^2]$$

■ **Problem #23**

$$\int \frac{x^2}{x^4 + x^2 - 2} dx = \frac{1}{3} \sqrt{2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{2}}\right] - \frac{\operatorname{ArcTanh}[x]}{3}$$

■ **Problem #24**

$$\int \frac{x^3 + 4x^2 + 6x}{x^4 + 2x^3 + 3x^2 + 4x + 2} dx = \frac{1}{1+x} + \frac{4}{3} \sqrt{2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{2}}\right] - \frac{1}{3} \operatorname{Log}[1+x] + \frac{2}{3} \operatorname{Log}[2+x^2]$$

■ **Problem #25**

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx = \frac{2}{5(1+2x)} + \frac{\operatorname{ArcTan}[x]}{50} - \frac{1}{2} \operatorname{Log}[1+x] + \frac{16}{25} \operatorname{Log}[1+2x] - \frac{7}{100} \operatorname{Log}[1+x^2]$$

Problems 26 - 27 (p. 67)

■ **Problem #26**

$$\int \frac{3x^2 + x - 2}{(x-1)^3(x^2+1)} dx = -\frac{1}{2(1-x)^2} + \frac{5}{2(1-x)} - \operatorname{ArcTan}[x] - \frac{3}{2} \operatorname{Log}[1-x] + \frac{3}{4} \operatorname{Log}[1+x^2]$$

■ **Problem #27**

$$\int \frac{1}{x^4 + x^2 + 1} dx = -\frac{\operatorname{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}} - \frac{1}{4} \operatorname{Log}[1-x+x^2] + \frac{1}{4} \operatorname{Log}[1+x+x^2]$$

Problems 28 - 32 (p. 68)

■ **Problem #28**

$$\int \frac{2x^3 + 3}{x^5 - 9x} dx = \frac{\operatorname{ArcTan}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\operatorname{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\operatorname{Log}[x]}{3} + \frac{1}{12} \operatorname{Log}[9-x^4]$$

■ **Problem #29**

$$\int \frac{5x^3 + 8x - 20}{(x-4)^3(x^2-4x+8)} dx = -\frac{83}{4(4-x)^2} + \frac{41}{4(4-x)} - \frac{3}{16} \operatorname{ArcTan}\left[1 - \frac{x}{2}\right] - \frac{45}{16} \operatorname{Log}[4-x] + \frac{45}{32} \operatorname{Log}[8-4x+x^2]$$

■ **Problem #30**

$$\int \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)} dx = -\frac{1}{12} \operatorname{ArcTan}\left[\frac{x}{2}\right] + \frac{\operatorname{ArcTan}[x]}{6} - \frac{\operatorname{ArcTan}\left[\frac{x}{\sqrt{2}}\right]}{2\sqrt{2}} + \frac{\operatorname{ArcTan}\left[\frac{x}{\sqrt{3}}\right]}{2\sqrt{3}}$$

■ **Problem #31**

$$\int \frac{x}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)} dx = \frac{1}{12} \operatorname{Log}[1+x^2] - \frac{1}{4} \operatorname{Log}[2+x^2] + \frac{1}{4} \operatorname{Log}[3+x^2] - \frac{1}{12} \operatorname{Log}[4+x^2]$$

■ **Problem #32**

$$\int \frac{1}{a^3 + x^3} dx = -\frac{\text{ArcTan}\left[\frac{a-2x}{\sqrt{3}a}\right]}{\sqrt{3}a^2} + \frac{\text{Log}[a+x]}{3a^2} - \frac{\text{Log}[a^2 - ax + x^2]}{6a^2}$$

Problems 33 - 44 (p. 69)

■ **Problem #33**

$$\int \frac{x}{a^3 + x^3} dx = -\frac{\text{ArcTan}\left[\frac{a-2x}{\sqrt{3}a}\right]}{\sqrt{3}a} - \frac{\text{Log}[a+x]}{3a} + \frac{\text{Log}[a^2 - ax + x^2]}{6a}$$

■ **Problem #34**

$$\int \frac{x^2}{a^3 + x^3} dx = \frac{1}{3} \text{Log}[a^3 + x^3]$$

■ **Problem #35**

$$\int \frac{1}{x(a^3 + x^3)} dx = \frac{\text{Log}[x]}{a^3} - \frac{\text{Log}[a^3 + x^3]}{3a^3}$$

■ **Problem #36**

$$\int \frac{1}{x^2(a^3 + x^3)} dx = -\frac{1}{a^3 x} + \frac{\text{ArcTan}\left[\frac{a-2x}{\sqrt{3}a}\right]}{\sqrt{3}a^4} + \frac{\text{Log}[a+x]}{3a^4} - \frac{\text{Log}[a^2 - ax + x^2]}{6a^4}$$

■ **Problem #37.3**

$$\int \frac{1}{x^3(a^3 + x^3)} dx = -\frac{1}{2a^3 x^2} + \frac{\text{ArcTan}\left[\frac{a-2x}{\sqrt{3}a}\right]}{\sqrt{3}a^5} - \frac{\text{Log}[a+x]}{3a^5} + \frac{\text{Log}[a^2 - ax + x^2]}{6a^5}$$

■ **Problem #37.4**

$$\int \frac{1}{x^4(a^3 + x^3)} dx = -\frac{1}{3a^3 x^3} - \frac{\text{Log}[x]}{a^6} + \frac{\text{Log}[a^3 + x^3]}{3a^6}$$

■ **Problem #37.5**

$$\int \frac{1}{x^5(a^3 + x^3)} dx = -\frac{1}{4a^3 x^4} + \frac{1}{a^6 x} - \frac{\text{ArcTan}\left[\frac{a-2x}{\sqrt{3}a}\right]}{\sqrt{3}a^7} - \frac{\text{Log}[a+x]}{3a^7} + \frac{\text{Log}[a^2 - ax + x^2]}{6a^7}$$

■ **Problem #37.m**

$$\int \frac{1}{x^m (a^3 + x^3)} dx = \frac{x^{1-m}}{a^3 (1-m)} \text{Hypergeometric2F1} \left[1, \frac{1-m}{3}, \frac{4-m}{3}, -\frac{x^3}{a^3} \right]$$

■ **Problem #38**

$$\int \frac{1}{a^4 - x^4} dx = \frac{\text{ArcTan} \left[\frac{x}{a} \right]}{2 a^3} + \frac{\text{ArcTanh} \left[\frac{x}{a} \right]}{2 a^3}$$

■ **Problem #39**

$$\int \frac{x}{a^4 - x^4} dx = \frac{\text{ArcTanh} \left[\frac{x^2}{a^2} \right]}{2 a^2}$$

■ **Problem #40**

$$\int \frac{1}{x (a^4 - x^4)} dx = \frac{\text{Log}[x]}{a^4} - \frac{\text{Log}[a^4 - x^4]}{4 a^4}$$

■ **Problem #41.2**

$$\int \frac{1}{x^2 (a^4 - x^4)} dx = -\frac{1}{a^4 x} - \frac{\text{ArcTan} \left[\frac{x}{a} \right]}{2 a^5} + \frac{\text{ArcTanh} \left[\frac{x}{a} \right]}{2 a^5}$$

■ **Problem #41.3**

$$\int \frac{1}{x^3 (a^4 - x^4)} dx = -\frac{1}{2 a^4 x^2} + \frac{\text{ArcTanh} \left[\frac{x^2}{a^2} \right]}{2 a^6}$$

■ **Problem #41.4**

$$\int \frac{1}{x^4 (a^4 - x^4)} dx = -\frac{1}{3 a^4 x^3} + \frac{\text{ArcTan} \left[\frac{x}{a} \right]}{2 a^7} + \frac{\text{ArcTanh} \left[\frac{x}{a} \right]}{2 a^7}$$

■ **Problem #41.m**

$$\int \frac{1}{x^m (a^4 - x^4)} dx = \frac{x^{1-m}}{a^4 (1-m)} \text{Hypergeometric2F1} \left[1, \frac{1-m}{4}, \frac{5-m}{4}, \frac{x^4}{a^4} \right]$$

■ **Problem #42**

$$\int \frac{x}{a^4 + x^4} dx = \frac{\text{ArcTan} \left[\frac{x^2}{a^2} \right]}{2 a^2}$$

■ Problem #43

$$\int \frac{x^2}{a^4 + x^4} dx = -\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}x}{a}\right]}{2\sqrt{2}a} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}x}{a}\right]}{2\sqrt{2}a} + \frac{\text{Log}\left[a^2 - \sqrt{2}ax + x^2\right]}{4\sqrt{2}a} - \frac{\text{Log}\left[a^2 + \sqrt{2}ax + x^2\right]}{4\sqrt{2}a}$$

■ Problem #44

$$\int \frac{1}{a^5 + x^5} dx = -\frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \text{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a^4} - \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a^4} +$$

$$\frac{\text{Log}[a + x]}{5a^4} - \frac{(1 - \sqrt{5}) \text{Log}\left[2a^2 - (1 - \sqrt{5})ax + 2x^2\right]}{20a^4} - \frac{(1 + \sqrt{5}) \text{Log}\left[2a^2 - (1 + \sqrt{5})ax + 2x^2\right]}{20a^4}$$

Problems 45 - 50 (p. 71-72)

■ Problem #45

$$\int \frac{x}{a^5 + x^5} dx = \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \text{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a^3} - \frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a^3} -$$

$$\frac{\text{Log}[a + x]}{5a^3} + \frac{(1 + \sqrt{5}) \text{Log}\left[a^2 - \frac{1}{2}(1 - \sqrt{5})ax + x^2\right]}{20a^3} + \frac{(1 - \sqrt{5}) \text{Log}\left[a^2 - \frac{1}{2}(1 + \sqrt{5})ax + x^2\right]}{20a^3}$$

■ Problem #46

$$\int \frac{x^2}{a^5 + x^5} dx = \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \text{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a^2} - \frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a^2} +$$

$$\frac{\text{Log}[a + x]}{5a^2} - \frac{(1 + \sqrt{5}) \text{Log}\left[a^2 - \frac{1}{2}(1 - \sqrt{5})ax + x^2\right]}{20a^2} - \frac{(1 - \sqrt{5}) \text{Log}\left[a^2 - \frac{1}{2}(1 + \sqrt{5})ax + x^2\right]}{20a^2}$$

■ Problem #47

$$\int \frac{x^3}{a^5 + x^5} dx = - \frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a} - \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a} -$$

$$\frac{\operatorname{Log}[a + x]}{5a} + \frac{(1 - \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 - \sqrt{5})ax + x^2\right]}{20a} + \frac{(1 + \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 + \sqrt{5})ax + x^2\right]}{20a}$$

■ Problem #48

$$\int \frac{x^4}{a^5 + x^5} dx = \frac{1}{5} \operatorname{Log}[a^5 + x^5]$$

■ Problem #49

$$\int \frac{1}{x(a^5 + x^5)} dx = \frac{\operatorname{Log}[x]}{a^5} - \frac{\operatorname{Log}[a^5 + x^5]}{5a^5}$$

■ Problem #50.2

$$\int \frac{1}{x^2(a^5 + x^5)} dx = - \frac{1}{a^5 x} + \frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a^6} + \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a^6} +$$

$$\frac{\operatorname{Log}[a + x]}{5a^6} - \frac{(1 - \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 - \sqrt{5})ax + x^2\right]}{20a^6} - \frac{(1 + \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 + \sqrt{5})ax + x^2\right]}{20a^6}$$

■ Problem #50.3

$$\int \frac{1}{x^3(a^5 + x^5)} dx = - \frac{1}{2a^5 x^2} - \frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{ArcTan}\left[\frac{(1 - \sqrt{5})a - 4x}{\sqrt{2(5 + \sqrt{5})}a}\right]}{5a^7} + \frac{\sqrt{\frac{1}{2}(5 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5 + \sqrt{5})}((1 + \sqrt{5})a - 4x)}{2a}\right]}{5a^7} -$$

$$\frac{\operatorname{Log}[a + x]}{5a^7} + \frac{(1 + \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 - \sqrt{5})ax + x^2\right]}{20a^7} + \frac{(1 - \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2}(1 + \sqrt{5})ax + x^2\right]}{20a^7}$$

■ **Problem #50.4**

$$\int \frac{1}{x^4 (a^5 + x^5)} dx = -\frac{1}{3 a^5 x^3} - \frac{\sqrt{\frac{1}{2} (5 - \sqrt{5})} \operatorname{ArcTan}\left[\frac{(1 - \sqrt{5}) a - 4 x}{\sqrt{2 (5 + \sqrt{5})} a}\right]}{5 a^8} + \frac{\sqrt{\frac{1}{2} (5 + \sqrt{5})} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{10} (5 + \sqrt{5})} ((1 + \sqrt{5}) a - 4 x)}{2 a}\right]}{5 a^8} +$$

$$\frac{\operatorname{Log}[a + x]}{5 a^8} - \frac{(1 + \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2} (1 - \sqrt{5}) a x + x^2\right]}{20 a^8} - \frac{(1 - \sqrt{5}) \operatorname{Log}\left[a^2 - \frac{1}{2} (1 + \sqrt{5}) a x + x^2\right]}{20 a^8}$$

■ **Problem #50.m**

$$\int \frac{1}{x^m (a^5 + x^5)} dx = \frac{x^{1-m}}{a^5 (1-m)} \operatorname{Hypergeometric2F1}\left[1, \frac{1-m}{5}, \frac{6-m}{5}, -\frac{x^5}{a^5}\right]$$

Problems 51 - 57 (p. 77-79)

■ **Problem #51**

$$\int \frac{x^4 + 1}{x^6 + 1} dx = -\frac{1}{3} \operatorname{ArcTan}[\sqrt{3} - 2x] + \frac{2 \operatorname{ArcTan}[x]}{3} + \frac{1}{3} \operatorname{ArcTan}[\sqrt{3} + 2x]$$

■ **Problem #52**

$$\int \frac{1}{(x^2 + 3x + 5)^3} dx = \frac{3 + 2x}{22 (5 + 3x + x^2)^2} + \frac{3 (3 + 2x)}{121 (5 + 3x + x^2)} + \frac{12 \operatorname{ArcTan}\left[\frac{3+2x}{\sqrt{11}}\right]}{121 \sqrt{11}}$$

■ **Problem #53**

$$\int \frac{x^4 + x^2 + 1}{(x^2 + 1)^4} dx = \frac{x}{6 (1 + x^2)^3} - \frac{x}{24 (1 + x^2)^2} + \frac{7x}{16 (1 + x^2)} + \frac{7 \operatorname{ArcTan}[x]}{16}$$

■ **Problem #54**

$$\int \frac{Ax + B}{(ax^2 + 2bx + c)^2} dx = -\frac{bB - Ac - (Ab - aB)x}{2 (b^2 - ac) (c + 2bx + ax^2)} - \frac{(Ab - aB) \operatorname{ArcTanh}\left[\frac{b+ax}{\sqrt{b^2-ac}}\right]}{2 (b^2 - ac)^{3/2}}$$

■ **Problem #55**

$$\int \frac{5x^3 - 27x^2 + 55x - 41}{(x^2 - 4x + 5)^2} dx = \frac{1-x}{5-4x+x^2} - 2 \operatorname{ArcTan}[2-x] + \frac{5}{2} \operatorname{Log}[5-4x+x^2]$$

■ Problem #56

$$\int \frac{1}{(x^3 - 1)^2} dx = \frac{x}{3(1 - x^3)} + \frac{2 \operatorname{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{3\sqrt{3}} - \frac{2}{9} \operatorname{Log}[1 - x] + \frac{1}{9} \operatorname{Log}[1 + x + x^2]$$

■ Problem #57

$$\int \frac{3x^4 + 4}{x^2(x^2 + 1)^3} dx = -\frac{4}{x} - \frac{7x}{4(1 + x^2)^2} - \frac{25x}{8(1 + x^2)} - \frac{57 \operatorname{ArcTan}[x]}{8}$$

Problems 58 - 65 (p. 80-81)

■ Problem #58

$$\int \frac{x}{x^6 + 1} dx = -\frac{\operatorname{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]}{2\sqrt{3}} + \frac{1}{6} \operatorname{Log}[1 + x^2] - \frac{1}{12} \operatorname{Log}[1 - x^2 + x^4]$$

■ Problem #59

$$\int \frac{x^{n-1} - 1}{x^n - nx} dx = \frac{\operatorname{Log}[x^n - nx]}{n}$$

■ Problem #60

$$\int \frac{x^3}{3x^4 - 2x^2 + 1} dx = -\frac{\operatorname{ArcTan}\left[\frac{1-3x^2}{\sqrt{2}}\right]}{6\sqrt{2}} + \frac{1}{12} \operatorname{Log}[1 - 2x^2 + 3x^4]$$

■ Problem #61

$$\int \frac{x^5}{3x^4 + x^2 - 4} dx = \frac{x^2}{6} + \frac{1}{14} \operatorname{Log}[1 - x^2] - \frac{8}{63} \operatorname{Log}[4 + 3x^2]$$

■ Problem #62

$$\int \frac{x^2}{9 - 10x^3 + x^6} dx = -\frac{1}{24} \operatorname{Log}[1 - x^3] + \frac{1}{24} \operatorname{Log}[9 - x^3]$$

■ Problem #63

$$\int \frac{x^3 - 4x^2 + 1}{(x - 2)^4} dx = -\frac{7}{3(2 - x)^3} + \frac{2}{(2 - x)^2} + \frac{2}{2 - x} + \operatorname{Log}[2 - x]$$

■ Problem #64

$$\int \frac{x^3}{(x - 1)^{12}} dx = \frac{1}{11(1 - x)^{11}} - \frac{3}{10(1 - x)^{10}} + \frac{1}{3(1 - x)^9} - \frac{1}{8(1 - x)^8}$$

■ **Problem #65**

$$\int \frac{x^4 - 3x}{(1+2x)^5} dx = -\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32} \text{Log}[1+2x]$$

Problems 66 - 70 (p. 83-78)

■ **Problem #66**

$$\int \frac{1}{(x+1)^3(x-1)^2} dx = \frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3 \text{ArcTanh}[x]}{8}$$

■ **Problem #67**

$$\int \frac{1}{x^2(5-6x)^2} dx = \frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125} \text{Log}[5-6x] + \frac{12 \text{Log}[x]}{125}$$

■ **Problem #68**

$$\int \frac{1}{(x^2-2x-3)^3} dx = \frac{1-x}{16(3+2x-x^2)^2} + \frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{512} \text{Log}[3-x] - \frac{3}{512} \text{Log}[1+x]$$

■ **Problem #69**

$$\int \frac{1}{(x^2-4x+13)^3} dx = -\frac{2-x}{36(13-4x+x^2)^2} - \frac{2-x}{216(13-4x+x^2)} - \frac{1}{648} \text{ArcTan}\left[\frac{2-x}{3}\right]$$

■ **Problem #70**

$$\int \frac{1}{(x+2)^3(x+3)^4} dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \text{Log}[2+x] - 10 \text{Log}[3+x]$$

Problems 71 - 82 (p. 86-87)

■ **Problem #71**

$$\int \frac{x^6}{(x^2-2)^2} dx = 4x + \frac{x^3}{3} - \frac{2x}{x^2-2} - 5\sqrt{2} \text{ArcTanh}\left[\frac{x}{\sqrt{2}}\right]$$

■ **Problem #72**

$$\int \frac{x^8}{(x^2+4)^4} dx = \frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{8} \text{ArcTan}\left[\frac{x}{2}\right]$$

■ **Problem #73**

$$\int \frac{7x - 4}{(3x^2 + 2x + 5)^2} dx = -\frac{39 + 19x}{28(5 + 2x + 3x^2)} - \frac{19 \operatorname{ArcTan}\left[\frac{1+3x}{\sqrt{14}}\right]}{28\sqrt{14}}$$

■ **Problem #74**

$$\int \frac{5 - 4x}{(3x^2 - 4x - 2)^2} dx = -\frac{18 - 7x}{20(2 + 4x - 3x^2)} - \frac{7 \operatorname{ArcTanh}\left[\frac{2-3x}{\sqrt{10}}\right]}{20\sqrt{10}}$$

■ **Problem #75**

$$\int \frac{x^5}{(x^4 + 1)^3} dx = -\frac{x^2}{8(1 + x^4)^2} + \frac{x^2}{16(1 + x^4)} + \frac{\operatorname{ArcTan}[x^2]}{16}$$

■ **Problem #76**

$$\int \frac{x(x^2 + 1)^3}{(x^4 + 2x^2 + 2)^2} dx = \frac{1}{4(x^4 + 2x^2 + 2)} + \frac{1}{4} \operatorname{Log}[x^4 + 2x^2 + 2]$$

■ **Problem #77**

$$\int \frac{x^3}{(a^4 + x^4)^3} dx = -\frac{1}{8(a^4 + x^4)^2}$$

■ **Problem #78**

$$\int \frac{1}{x(a^4 + x^4)^3} dx = \frac{1}{8a^4(a^4 + x^4)^2} + \frac{1}{4a^8(a^4 + x^4)} + \frac{\operatorname{Log}[x]}{a^{12}} - \frac{\operatorname{Log}[a^4 + x^4]}{4a^{12}}$$

■ **Problem #79**

$$\int \frac{1}{x^2(a^4 + x^4)^3} dx = -\frac{45}{32a^{12}x} + \frac{1}{8a^4x(a^4 + x^4)^2} + \frac{9}{32a^8x(a^4 + x^4)} + \frac{45 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}x}{a}\right]}{64\sqrt{2}a^{13}} - \frac{45 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}x}{a}\right]}{64\sqrt{2}a^{13}} - \frac{45 \operatorname{Log}[a^2 - \sqrt{2}ax + x^2]}{128\sqrt{2}a^{13}} + \frac{45 \operatorname{Log}[a^2 + \sqrt{2}ax + x^2]}{128\sqrt{2}a^{13}}$$

■ **Problem #80**

$$\int \frac{1}{x^3(a^4 + x^4)^3} dx = -\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2(a^4 + x^4)^2} + \frac{5}{16a^8x^2(a^4 + x^4)} - \frac{15 \operatorname{ArcTan}\left[\frac{x^2}{a^2}\right]}{16a^{14}}$$

■ Problem #81

$$\int \frac{x^{14}}{(3+2x^5)^3} dx = -\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40} \operatorname{Log}[3+2x^5]$$

■ Problem #82

$$\int \frac{x^6}{(3+2x^5)^3} dx = -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} + \frac{\sqrt{5-\sqrt{5}} \operatorname{ArcTan}\left[\frac{3^{1/5}(1-\sqrt{5})-4 \times 2^{1/5}x}{3^{1/5}\sqrt{2(5+\sqrt{5})}}\right]}{250 \times 2^{9/10} 3^{3/5}} - \frac{\sqrt{5+\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} (3^{1/5}(1+\sqrt{5})-4 \times 2^{1/5}x)}{2 \times 3^{1/5}}\right]}{250 \times 2^{9/10} 3^{3/5}} -$$

$$\frac{\operatorname{Log}[3^{1/5}+2^{1/5}x]}{250 \times 2^{2/5} 3^{3/5}} + \frac{(1+\sqrt{5}) \operatorname{Log}\left[3^{2/5} - \frac{3^{1/5}(1-\sqrt{5})x}{2^{4/5}} + 2^{2/5}x^2\right]}{1000 \times 2^{2/5} 3^{3/5}} + \frac{(1-\sqrt{5}) \operatorname{Log}\left[3^{2/5} - \frac{3^{1/5}(1+\sqrt{5})x}{2^{4/5}} + 2^{2/5}x^2\right]}{1000 \times 2^{2/5} 3^{3/5}}$$

Problems 83 - 87 (p. 90-91)

■ Problem #83

$$\int \frac{9}{5x^2(3-2x^2)^3} dx = -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\operatorname{ArcTanh}\left[\sqrt{\frac{2}{3}}x\right]}{4\sqrt{6}}$$

■ Problem #84

$$\int \frac{3x^4+4}{x^2(x^2+1)^3} dx = -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57 \operatorname{ArcTan}[x]}{8}$$

■ Problem #85

$$\int \frac{5-3x+6x^2+5x^3-x^4}{x^5-x^4-2x^3+2x^2+x-1} dx = -\frac{3}{2(1-x)^2} + \frac{2}{1-x} + \frac{1}{1+x} + \operatorname{Log}[1-x] - 2 \operatorname{Log}[1+x]$$

■ Problem #86

$$\int \frac{x^2+1}{x(x^3+1)^2} dx = \frac{x(x-x^2)}{3(x^3+1)} - \frac{\operatorname{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{3\sqrt{3}} + \operatorname{Log}[x] - \frac{4}{9} \operatorname{Log}[1+x] - \frac{5}{18} \operatorname{Log}[1-x+x^2]$$

■ Problem #87

$$\int \frac{x^2-3x-2}{(x+1)^2(x^2+x+1)^2} dx = -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25 \operatorname{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{3\sqrt{3}} - \operatorname{Log}[1+x] + \frac{1}{2} \operatorname{Log}[1+x+x^2]$$

Problems 88 - 90 (p. 97)■ **Problem #88**

$$\int \frac{1}{(2-3x)(1-4x)^3} dx = \frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125} \text{Log}[1-4x] + \frac{9}{125} \text{Log}[2-3x]$$

■ **Problem #89**

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

■ **Problem #90**

$$\int \frac{x^7}{(2-5x^2)^3} dx = -\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625} \text{Log}[2-5x^2]$$

Chapter 3 (p. 98-113)

Rational Function Integration Problems

Problems 1 - 3 (p. 101)

■ Problem #1

$$\int \frac{1}{(x-2)^3 (x+1)^2} dx = -\frac{1}{18(x-2)^2} + \frac{2}{27(x-2)} + \frac{1}{27(x+1)} + \frac{1}{27} \operatorname{Log}[x-2] - \frac{1}{27} \operatorname{Log}[x+1]$$

■ Problem #2

$$\int \frac{1}{(x+2)^3 (x+3)^4} dx = -\frac{1}{2(x+2)^2} + \frac{4}{x+2} + \frac{1}{3(x+3)^3} + \frac{3}{2(x+3)^2} + \frac{6}{x+3} + 10 \operatorname{Log}[x+2] - 10 \operatorname{Log}[x+3]$$

■ Problem #3

$$\int \frac{x^5}{(3+x)^2} dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \operatorname{Log}[3+x]$$

Problems 4 - 9 (p. 105)

■ Problem #4.1

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^1 dx = a b_1 x + \frac{1}{2} (2b b_1 + a c_1) x^2 + \frac{1}{3} (b_1 c + 2b c_1) x^3 + \frac{1}{4} c c_1 x^4$$

■ Problem #4.2

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^2 dx = a^2 b_1 x + \frac{1}{2} a (4b b_1 + a c_1) x^2 + \frac{2}{3} (2b^2 b_1 + a b_1 c + 2a b c_1) x^3 + \frac{1}{2} (2b b_1 c + 2b^2 c_1 + a c c_1) x^4 + \frac{1}{5} c (b_1 c + 4b c_1) x^5 + \frac{1}{6} c^2 c_1 x^6$$

■ Problem #4.3

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^3 dx = a^3 b_1 x + \frac{1}{2} a^2 (6b b_1 + a c_1) x^2 + a (4b^2 b_1 + a b_1 c + 2a b c_1) x^3 + \frac{1}{4} (8b^3 b_1 + 12a b b_1 c + 12a b^2 c_1 + 3a^2 c c_1) x^4 + \frac{1}{5} (12b^2 b_1 c + 3a b_1 c^2 + 8b^3 c_1 + 12a b c c_1) x^5 + \frac{1}{2} c (2b b_1 c + 4b^2 c_1 + a c c_1) x^6 + \frac{1}{7} c^2 (b_1 c + 6b c_1) x^7 + \frac{1}{8} c^3 c_1 x^8$$

■ **Problem #4.4**

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^4 dx = a^4 b_1 x + \frac{1}{2} a^3 (8bb_1 + ac_1) x^2 + \frac{4}{3} a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^3 + a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 c c_1) x^4 + \frac{2}{5} (8b^4 b_1 + 24ab^2 b_1 c + 3a^2 b_1 c^2 + 16ab^3 c_1 + 12a^2 bc c_1) x^5 + \frac{1}{3} (16b^3 b_1 c + 12abb_1 c^2 + 8b^4 c_1 + 24ab^2 c c_1 + 3a^2 c^2 c_1) x^6 + \frac{4}{7} c (6b^2 b_1 c + ab_1 c^2 + 8b^3 c_1 + 6abc c_1) x^7 + \frac{1}{2} c^2 (2bb_1 c + 6b^2 c_1 + ac c_1) x^8 + \frac{1}{9} c^3 (b_1 c + 8bc_1) x^9 + \frac{1}{10} c^4 c_1 x^{10}$$

■ **Problem #4.n**

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^n dx = \frac{c_1 (a + 2bx + cx^2)^{n+1}}{2c(n+1)} - \frac{(b_1 c - bc_1) (a + 2bx + cx^2)^{n+1}}{2c(n+1) \sqrt{b^2 - ac}} \operatorname{Hypergeometric2F1} \left[-n, 1+n, 2+n, \frac{b+cx + \sqrt{b^2 - ac}}{2\sqrt{b^2 - ac}} \right]$$

■ **Problem #5.1**

$$\int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^1} dx = -\frac{(b_1 c - bc_1) \operatorname{ArcTanh} \left[\frac{b+cx}{\sqrt{b^2 - ac}} \right]}{c \sqrt{b^2 - ac}} + \frac{c_1 \operatorname{Log} [a + 2bx + cx^2]}{2c}$$

■ **Problem #5.2**

$$\int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^2} dx = -\frac{bb_1 - ac_1 + (b_1 c - bc_1) x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(b_1 c - bc_1) \operatorname{ArcTanh} \left[\frac{b+cx}{\sqrt{b^2 - ac}} \right]}{2(b^2 - ac)^{3/2}}$$

■ **Problem #5.3**

$$\int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^3} dx = -\frac{bb_1 - ac_1 + (b_1 c - bc_1) x}{4(b^2 - ac)(a + 2bx + cx^2)^2} + \frac{3(b_1 c - bc_1)(b+cx)}{8(b^2 - ac)^2(a + 2bx + cx^2)} - \frac{3c(b_1 c - bc_1) \operatorname{ArcTanh} \left[\frac{b+cx}{\sqrt{b^2 - ac}} \right]}{8(b^2 - ac)^{5/2}}$$

■ **Problem #5.4**

$$\int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^4} dx = -\frac{bb_1 - ac_1 + (b_1 c - bc_1) x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b_1 c - bc_1)(b+cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{5c(b_1 c - bc_1)(b+cx)}{16(b^2 - ac)^3(a + 2bx + cx^2)} + \frac{5c^2(b_1 c - bc_1) \operatorname{ArcTanh} \left[\frac{b+cx}{\sqrt{b^2 - ac}} \right]}{16(b^2 - ac)^{7/2}}$$

■ **Problem #5.n**

$$\int \frac{b1 + c1 x}{(a + 2 b x + c x^2)^n} dx = -\frac{c1}{2 c (n-1) (a + 2 b x + c x^2)^{n-1}} + \frac{(b1 c - b c1) \left(-\frac{b - \sqrt{b^2 - a c} + c x}{2 \sqrt{b^2 - a c}} \right)^{n-1}}{2 c (n-1) \sqrt{b^2 - a c} (a + 2 b x + c x^2)^{n-1}} \text{Hypergeometric2F1} \left[1 - n, n, 2 - n, \frac{b + c x + \sqrt{b^2 - a c}}{2 \sqrt{b^2 - a c}} \right]$$

■ **Problem #6**

$$\int \frac{x}{3 + 6 x + 2 x^2} dx = \frac{1}{4} (1 - \sqrt{3}) \text{Log} [3 - \sqrt{3} + 2 x] + \frac{1}{4} (1 + \sqrt{3}) \text{Log} [3 + \sqrt{3} + 2 x]$$

■ **Problem #7**

$$\int \frac{2 x - 3}{(3 + 6 x + 2 x^2)^3} dx = \frac{5 + 4 x}{4 (3 + 6 x + 2 x^2)^2} - \frac{3 + 2 x}{2 (3 + 6 x + 2 x^2)} + \frac{1}{\sqrt{3}} \text{ArcTanh} \left[\frac{3 + 2 x}{\sqrt{3}} \right]$$

■ **Problem #8**

$$\int \frac{x - 1}{(x^2 + 5 x + 4)^2} dx = \frac{7 x + 13}{9 (x^2 + 5 x + 4)} + \frac{7}{27} \text{Log} [x + 1] - \frac{7}{27} \text{Log} [x + 4]$$

■ **Problem #9**

$$\int \frac{1}{(x^2 + 3 x + 2)^5} dx = -\frac{2 x + 3}{4 (x^2 + 3 x + 2)^4} + \frac{7 (2 x + 3)}{6 (x^2 + 3 x + 2)^3} - \frac{35 (2 x + 3)}{6 (x^2 + 3 x + 2)^2} + \frac{35 (2 x + 3)}{x^2 + 3 x + 2} - 140 \text{ArcTanh} [2 x + 3]$$

Problems 10 - 12 (p. 109)

■ **Problem #10**

$$\int \frac{1}{x^3 (7 - 6 x + 2 x^2)^2} dx = -\frac{1}{490 x^2} - \frac{69}{1715 x} - \frac{2 - 3 x}{35 x^2 (7 - 6 x + 2 x^2)} - \frac{234}{12005 \sqrt{5}} \text{ArcTan} \left[\frac{3 - 2 x}{\sqrt{5}} \right] + \frac{80 \text{Log} [x]}{2401} - \frac{40 \text{Log} [7 - 6 x + 2 x^2]}{2401}$$

■ **Problem #11**

$$\int \frac{x^9}{(x^2 + 3 x + 2)^5} dx = 735 x + \frac{x^8 (4 + 3 x)}{4 (2 + 3 x + x^2)^4} - \frac{x^6 (110 + 81 x)}{12 (2 + 3 x + x^2)^3} + \frac{x^4 (184 + 135 x)}{2 (2 + 3 x + x^2)^2} - \frac{x^2 (2206 + 1593 x)}{2 (2 + 3 x + x^2)} - 1471 \text{Log} [1 + x] + 1472 \text{Log} [2 + x]$$

■ **Problem #12**

$$\int \frac{(1 + 2 x)^2}{(3 + 5 x + 2 x^2)^5} dx = \frac{(1 + 2 x) (7 + 6 x)}{4 (3 + 5 x + 2 x^2)^4} + \frac{73 + 62 x}{3 (3 + 5 x + 2 x^2)^3} - \frac{155 (5 + 4 x)}{3 (3 + 5 x + 2 x^2)^2} + \frac{620 (5 + 4 x)}{3 + 5 x + 2 x^2} - 4960 \text{ArcTanh} [5 + 4 x]$$

Problems 13 - 14 (p. 113)

■ Problem #13

$$\int \frac{(a - b x^2)^3}{x^7} dx = -\frac{a^3}{6 x^6} + \frac{3 a^2 b}{4 x^4} - \frac{3 a b^2}{2 x^2} - b^3 \operatorname{Log}[x]$$

■ Problem #14

$$\int \frac{x^{13}}{(a^4 + x^4)^5} dx = -\frac{x^{10}}{16 (a^4 + x^4)^4} - \frac{5 x^6}{96 (a^4 + x^4)^3} - \frac{5 x^2}{128 (a^4 + x^4)^2} + \frac{5 x^2}{256 a^4 (a^4 + x^4)} + \frac{5}{256 a^6} \operatorname{ArcTan}\left[\frac{x^2}{a^2}\right]$$

Chapter 4 (p. 114-200)

Algebraic Function Integration Problems

Problems 1 - 9 (p. 115-116)

■ **Problem #1**

$$\int x^{3/2} (1+x^2) (2\sqrt{x} - x)^2 dx = \frac{8x^{7/2}}{7} - x^4 + \frac{2x^{9/2}}{9} + \frac{8x^{11/2}}{11} - \frac{2x^6}{3} + \frac{2x^{13/2}}{13}$$

■ **Problem #2**

$$\int (x^{3/2} - 3x^{3/5})^2 \left(4x^{3/2} - \frac{x^{2/3}}{3}\right) dx = -\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}$$

■ **Problem #3**

$$\int \frac{1}{1+\sqrt{1+x}} dx = 2\sqrt{1+x} - 2\text{Log}[1+\sqrt{1+x}]$$

■ **Problem #4**

$$\int \frac{x}{1+\sqrt{1+x}} dx = \frac{2}{3}(1+x)^{3/2} - x$$

■ **Problem #5**

$$\int \frac{\sqrt{1+x} + 1}{\sqrt{1+x} - 1} dx = x + 4\sqrt{1+x} + 4\text{Log}[1 - \sqrt{1+x}]$$

■ **Problem #6**

$$\int \frac{1}{(1+x)^{2/3} - \sqrt{1+x}} dx = 6(1+x)^{1/6} + 3(1+x)^{1/3} + 6\text{Log}[1 - (1+x)^{1/6}]$$

■ **Problem #7**

$$\int \frac{(1+x^{1/4})^{1/3}}{\sqrt{x}} dx = \frac{12}{7}(1+x^{1/4})^{7/3} - 3(1+x^{1/4})^{4/3}$$

■ **Problem #8**

$$\int \frac{1}{x^3(1+x)^{3/2}} dx = \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} - \frac{15}{4}\text{ArcTanh}[\sqrt{1+x}]$$

■ Problem #9

$$\int \frac{1}{x^5 (1-x)^{7/2}} dx = \frac{2}{5 (1-x)^{5/2} x^4} + \frac{26}{15 (1-x)^{3/2} x^4} + \frac{286}{15 \sqrt{1-x} x^4} - \frac{429 \sqrt{1-x}}{20 x^4} - \frac{1001 \sqrt{1-x}}{40 x^3} - \frac{1001 \sqrt{1-x}}{32 x^2} - \frac{3003 \sqrt{1-x}}{64 x} - \frac{3003}{64} \operatorname{ArcTanh}[\sqrt{1-x}]$$

Problems 10 - 12 (p. 117-118)

■ Problem #10

$$\int \frac{1}{x^5 (x-1)^{2/3}} dx = \frac{(x-1)^{1/3}}{4 x^4} + \frac{11 (x-1)^{1/3}}{36 x^3} + \frac{11 (x-1)^{1/3}}{27 x^2} + \frac{55 (x-1)^{1/3}}{81 x} - \frac{110}{81 \sqrt{3}} \operatorname{ArcTan}\left[\frac{1-2(x-1)^{1/3}}{\sqrt{3}}\right] + \frac{55}{81} \operatorname{Log}[1+(x-1)^{1/3}] - \frac{55 \operatorname{Log}[x]}{243}$$

■ Problem #11

$$\int \sqrt{\frac{1-x}{1+x}} dx = (1+x) \sqrt{\frac{1-x}{1+x}} - 2 \operatorname{ArcTan}\left[\sqrt{\frac{1-x}{1+x}}\right]$$

■ Problem #12

$$\int \sqrt{\frac{x-a}{b-x}} x dx = -\frac{(x-a)^2}{2 \sqrt{\frac{x-a}{b-x}}} - \frac{1}{4} (a+3b) (b-x) \sqrt{\frac{x-a}{b-x}} - \frac{1}{4} (a-b) (a+3b) \operatorname{ArcTan}\left[\sqrt{\frac{x-a}{b-x}}\right]$$

Problems 13 - 15 (p. 119-120)

■ Problem #13

$$\int \frac{\sqrt{x-5} \sqrt{x+3}}{(x-1)(x^2-25)} dx = -\frac{1}{3} \operatorname{ArcTan}\left[\frac{\sqrt{x+3}}{\sqrt{x-5}}\right] + \frac{1}{3 \sqrt{5}} \operatorname{ArcTanh}\left[\frac{\sqrt{5} \sqrt{x+3}}{\sqrt{x-5}}\right]$$

■ Problem #14

$$\int \frac{x^2 (1-x^2)^{1/4} \sqrt{1+x}}{\sqrt{1-x} (\sqrt{1-x} - \sqrt{1+x})} dx = \frac{5}{16} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{16} (1-x)^{1/4} (1+x)^{3/4} + \frac{1}{24} (1-x)^{5/4} (1+x)^{3/4} + \frac{7 (1-x^2)^{5/4}}{24 \sqrt{1-x}} + \frac{x (1-x^2)^{5/4}}{6 \sqrt{1-x}} + \frac{1}{6} \sqrt{1+x} (1-x^2)^{5/4} - \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} + \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}}$$

■ **Problem #15**

$$\int \frac{x(1+x)^{2/3} \sqrt{1-x}}{\sqrt{1+x} (1-x)^{2/3} - (1+x)^{1/3} (1-x)^{5/6}} dx =$$

$$-\frac{1}{12} (1-3x) (1-x)^{2/3} (1+x)^{1/3} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4} (1-x) (3+x) + \frac{1}{12} (1-x)^{1/3} (1+x)^{2/3} (1+3x) + \frac{1}{12} (1-x)^{1/6} (1+x)^{5/6} (2+3x) -$$

$$\frac{1}{12} (1-x)^{5/6} (1+x)^{1/6} (10+3x) + \frac{1}{6} \operatorname{ArcTan} \left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}} \right] - \frac{4 \operatorname{ArcTan} \left[\frac{(1-x)^{1/3} - 2(1+x)^{1/3}}{\sqrt{3} (1-x)^{1/3}} \right]}{3\sqrt{3}} - \frac{5}{6} \operatorname{ArcTan} \left[\frac{(1-x)^{1/3} - (1+x)^{1/3}}{(1-x)^{1/6} (1+x)^{1/6}} \right] + \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{3} (1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} + (1+x)^{1/3}} \right]}{6\sqrt{3}}$$

Problems 16 - 21 (p. 127)

■ **Problem #16**

$$\int \frac{1}{((x+1)^2 (x-1)^4)^{1/3}} dx = -\frac{3(x-1)(x+1)}{2((x+1)^2 (x-1)^4)^{1/3}}$$

■ **Problem #17**

$$\int \frac{1}{((x-1)^3 (x+2)^5)^{1/4}} dx = \frac{4(x-1)(2+x)}{3((x-1)^3 (x+2)^5)^{1/4}}$$

■ **Problem #18**

$$\int \frac{1}{((x+1)^2 (x-1)^7)^{1/3}} dx = -\frac{3(x-1)(x+1)}{8((x+1)^2 (x-1)^7)^{1/3}} + \frac{9(x-1)^2(x+1)}{16((x+1)^2 (x-1)^7)^{1/3}}$$

■ **Problem #19**

$$\int \frac{1}{((x-1)^2 (x+1))^{1/3}} dx = \sqrt{3} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} \left(1 + \frac{2(x-1)}{((x-1)^2 (x+1))^{1/3}} \right) \right] - \frac{1}{2} \operatorname{Log}[x+1] - \frac{3}{2} \operatorname{Log} \left[1 - \frac{x-1}{((x-1)^2 (x+1))^{1/3}} \right]$$

■ **Problem #20**

$$\int \frac{x + \frac{1}{x}}{\sqrt{(x+1)^3 (x-2)}} dx = -\frac{4(x-2)(x+1)}{3\sqrt{(x+1)^3 (x-2)}} + \frac{2\sqrt{x-2}(x+1)^{3/2} \operatorname{ArcSinh} \left[\frac{\sqrt{x-2}}{\sqrt{3}} \right]}{\sqrt{(x+1)^3 (x-2)}} - \frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}} \right]}{\sqrt{(x+1)^3 (x-2)}}$$

■ **Problem #21**

$$\int \frac{((x-1)^2(x+1))^{1/3}}{x^2} dx = -\frac{((x-1)^2(x+1))^{1/3}}{x} - \frac{1}{\sqrt{3}} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} \left(1 - \frac{2(x-1)}{((x-1)^2(x+1))^{1/3}} \right) \right] - \sqrt{3} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} \left(1 + \frac{2(x-1)}{((x-1)^2(x+1))^{1/3}} \right) \right] + \frac{\operatorname{Log}[x]}{6} - \frac{2}{3} \operatorname{Log}[x+1] - \frac{3}{2} \operatorname{Log} \left[1 - \frac{x-1}{((x-1)^2(1+x))^{1/3}} \right] - \frac{1}{2} \operatorname{Log} \left[1 + \frac{x-1}{((x-1)^2(1+x))^{1/3}} \right]$$

Problems 22 - 27 (p. 128)

■ **Problem #22**

$$\int \frac{1}{(x^2 - 2x - 3)^{5/2}} dx = \frac{1-x}{12(x^2 - 2x - 3)^{3/2}} - \frac{1-x}{24\sqrt{x^2 - 2x - 3}}$$

■ **Problem #23**

$$\int \frac{1}{\sqrt{x^3 - 5x^2 + 3x + 9}} dx = \frac{(3-x)\sqrt{1+x} \operatorname{ArcTanh} \left[\frac{\sqrt{1+x}}{2} \right]}{\sqrt{x^3 - 5x^2 + 3x + 9}}$$

■ **Problem #24**

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{3/2}} dx = \frac{(3-x)(1+x)}{8(x^3 - 5x^2 + 3x + 9)^{3/2}} + \frac{5(3-x)^2(1+x)}{64(x^3 - 5x^2 + 3x + 9)^{3/2}} - \frac{15(3-x)^3(1+x)}{256(x^3 - 5x^2 + 3x + 9)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{1+x}}{2} \right]}{512(x^3 - 5x^2 + 3x + 9)^{3/2}}$$

■ **Problem #25**

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{1/3}} dx = \sqrt{3} \operatorname{ArcTan} \left[\frac{1}{\sqrt{3}} \left(1 + \frac{2(x-3)}{(x^3 - 5x^2 + 3x + 9)^{1/3}} \right) \right] - \frac{1}{2} \operatorname{Log}[x+1] - \frac{3}{2} \operatorname{Log} \left[1 - \frac{x-3}{(x^3 - 5x^2 + 3x + 9)^{1/3}} \right]$$

■ **Problem #26**

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{2/3}} dx = \frac{3(3-x)(1+x)}{4(x^3 - 5x^2 + 3x + 9)^{2/3}}$$

■ **Problem #27**

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{4/3}} dx = \frac{3(3-x)(1+x)}{20(x^3 - 5x^2 + 3x + 9)^{4/3}} + \frac{9(3-x)^2(1+x)}{80(x^3 - 5x^2 + 3x + 9)^{4/3}} - \frac{27(3-x)^3(1+x)}{320(x^3 - 5x^2 + 3x + 9)^{4/3}}$$

Problems 28 - 37 (p. 143-144)

■ Problem #28

$$\int \frac{1}{\sqrt{4 + 3x - 2x^2}} dx = -\frac{1}{\sqrt{2}} \operatorname{ArcSin}\left[\frac{3 - 4x}{\sqrt{41}}\right]$$

■ Problem #29

$$\int \frac{1}{\sqrt{-3 + 4x - x^2}} dx = -\operatorname{ArcSin}[2 - x]$$

■ Problem #30

$$\int \frac{1}{\sqrt{-2 - 5x - 3x^2}} dx = \frac{\operatorname{ArcSin}[5 + 6x]}{\sqrt{3}}$$

■ Problem #31

$$\int \frac{1}{(x^2 + 4)\sqrt{1 - x^2}} dx = \frac{\operatorname{ArcTan}\left[\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right]}{2\sqrt{5}}$$

■ Problem #32

$$\int \frac{1}{(x^2 + 4)\sqrt{4x^2 + 1}} dx = \frac{1}{2\sqrt{15}} \operatorname{ArcTanh}\left[\frac{\sqrt{15}x}{2\sqrt{1 + 4x^2}}\right]$$

■ Problem #33

$$\int \frac{x}{(3 - x^2)\sqrt{5 - x^2}} dx = \frac{1}{\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sqrt{5 - x^2}}{\sqrt{2}}\right]$$

■ Problem #34

$$\int \frac{x}{(5 - x^2)\sqrt{3 - x^2}} dx = -\frac{1}{\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{3 - x^2}}{\sqrt{2}}\right]$$

■ Problem #35

$$\int \frac{1}{(x^4 - 1)\sqrt{x^2 + 2}} dx = -\frac{1}{2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{2 + x^2}}\right] - \frac{1}{2\sqrt{3}} \operatorname{ArcTanh}\left[\frac{\sqrt{3}x}{\sqrt{2 + x^2}}\right]$$

■ Problem #36

$$\int \frac{x}{(x^2 - 1)\sqrt{x^2 + 2x + 4}} dx = -\frac{1}{2\sqrt{7}} \operatorname{ArcTanh}\left[\frac{5 + 2x}{\sqrt{7}\sqrt{x^2 + 2x + 4}}\right] - \frac{1}{2\sqrt{3}} \operatorname{ArcTanh}\left[\frac{\sqrt{x^2 + 2x + 4}}{\sqrt{3}}\right]$$

■ Problem #37

$$\int \frac{1}{(x^3 - 8) \sqrt{x^2 + 2x + 5}} dx = -\frac{1}{4\sqrt{3}} \operatorname{ArcTan}\left[\frac{1+x}{\sqrt{3}\sqrt{x^2+2x+5}}\right] - \frac{1}{12\sqrt{13}} \operatorname{ArcTanh}\left[\frac{7+3x}{\sqrt{13}\sqrt{x^2+2x+5}}\right] + \frac{1}{12} \operatorname{ArcTanh}\left[\sqrt{x^2+2x+5}\right]$$

Problems 38 - 42 (p. 145-146)

■ Problem #38

$$\int \frac{x}{(x^2 + x + 4) \sqrt{4x^2 + 4x + 5}} dx = \frac{1}{\sqrt{11}} \operatorname{ArcTan}\left[\frac{\sqrt{4x^2 + 4x + 5}}{\sqrt{11}}\right] - \frac{1}{\sqrt{165}} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2 + 4x + 5}}\right]$$

■ Problem #39

$$\int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}} dx = 2\sqrt{2} \operatorname{ArcTan}\left[\frac{x-1}{\sqrt{2}\sqrt{x^2+x+1}}\right] + \sqrt{2} \operatorname{ArcTanh}\left[\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right]$$

■ Problem #40

$$\int \frac{2x+1}{(3x^2+4x+4)\sqrt{x^2+6x-1}} dx = -\frac{5}{6\sqrt{14}} \operatorname{ArcTan}\left[\frac{\sqrt{7}(2-x)}{2\sqrt{2}\sqrt{x^2+6x-1}}\right] - \frac{1}{3\sqrt{7}} \operatorname{ArcTanh}\left[\frac{\sqrt{7}(1+x)}{\sqrt{x^2+6x-1}}\right]$$

■ Problem #41

$$\int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}} dx = -\frac{2A+B}{\sqrt{35}} \operatorname{ArcTan}\left[\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right] - \frac{A+B}{2\sqrt{35}} \operatorname{ArcTanh}\left[\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right]$$

■ Problem #42

$$\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}} dx = \frac{1}{2\sqrt{35}} \operatorname{ArcTanh}\left[\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right]$$

Problems 43 - 49 (p. 163)

■ Problem #43

$$\int x^4 \sqrt{5-x^2} dx = -\frac{25}{16} x \sqrt{5-x^2} - \frac{5}{24} x^3 \sqrt{5-x^2} + \frac{1}{6} x^5 \sqrt{5-x^2} + \frac{125}{16} \operatorname{ArcSin}\left[\frac{x}{\sqrt{5}}\right]$$

■ **Problem #44**

$$\int \frac{1}{x^6 \sqrt{x^2 + 2}} dx = -\frac{\sqrt{x^2 + 2}}{10 x^5} + \frac{\sqrt{x^2 + 2}}{15 x^3} - \frac{\sqrt{x^2 + 2}}{15 x}$$

■ **Problem #45**

$$\int \frac{1}{(2x^2 + 3)^{7/2}} dx = \frac{x}{15 (2x^2 + 3)^{5/2}} + \frac{4x}{135 (2x^2 + 3)^{3/2}} + \frac{8x}{405 \sqrt{2x^2 + 3}}$$

■ **Problem #46**

$$\int \frac{x}{1 + x^2 + a \sqrt{1 + x^2}} dx = \text{Log} \left[a + \sqrt{1 + x^2} \right]$$

■ **Problem #47**

$$\int \frac{x^2 - x + 1}{(1 + x^2) \sqrt{1 + x^2}} dx = \frac{1}{\sqrt{1 + x^2}} + \text{ArcSinh}[x]$$

■ **Problem #48**

$$\int \frac{\sqrt{1 + x^2}}{2 + x^2} dx = \text{ArcSinh}[x] - \frac{1}{\sqrt{2}} \text{ArcTanh} \left[\frac{x}{\sqrt{2} \sqrt{1 + x^2}} \right]$$

■ **Problem #49**

$$\int \frac{1}{(2 + x^2)^2 \sqrt{1 + x^2}} dx = -\frac{x \sqrt{1 + x^2}}{4 (2 + x^2)} + \frac{3}{4 \sqrt{2}} \text{ArcTanh} \left[\frac{x}{\sqrt{2} \sqrt{1 + x^2}} \right]$$

Problems 50 - 62 (p. 164)

■ **Problem #50**

$$\int \frac{x^2}{(x^2 - 6) \sqrt{x^2 - 2}} dx = \text{ArcTanh} \left[\frac{x}{\sqrt{x^2 - 2}} \right] - \sqrt{\frac{3}{2}} \text{ArcTanh} \left[\frac{\sqrt{\frac{2}{3}} x}{\sqrt{x^2 - 2}} \right]$$

■ **Problem #51**

$$\int \frac{x^2 + 5}{(1 + x^2)^2 \sqrt{1 - x^2}} dx = \frac{x \sqrt{1 - x^2}}{1 + x^2} + 2 \sqrt{2} \text{ArcTan} \left[\frac{\sqrt{2} x}{\sqrt{1 - x^2}} \right]$$

■ **Problem #52**

$$\int \frac{4x - \sqrt{1-x^2}}{5 + \sqrt{1-x^2}} dx = -x - 4\sqrt{1-x^2} + 5 \operatorname{ArcSin}[x] + \frac{25 \operatorname{ArcTan}\left[\frac{x}{2\sqrt{6}}\right]}{2\sqrt{6}} - \frac{25 \operatorname{ArcTan}\left[\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right]}{2\sqrt{6}} + 20 \operatorname{Log}\left[5 + \sqrt{1-x^2}\right]$$

■ **Problem #53**

$$\int \frac{(2 - \sqrt{x^2+1})x^2}{\sqrt{x^2+1}((x^2+1)^{3/2} - x^3 + 1)} dx = \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{x^2+1}}{9} - \frac{1}{6}x\sqrt{x^2+1} - \frac{41 \operatorname{ArcSinh}[x]}{54} + \frac{4}{27}\sqrt{2} \operatorname{ArcTan}\left[\frac{1+3x}{2\sqrt{2}}\right] + \frac{4}{27}\sqrt{2} \operatorname{ArcTan}\left[\frac{1+x}{\sqrt{2}\sqrt{x^2+1}}\right] + \frac{7}{27} \operatorname{ArcTanh}\left[\frac{1-x}{2\sqrt{x^2+1}}\right] - \frac{7}{54} \operatorname{Log}[3 + 2x + 3x^2]$$

■ **Problem #54**

$$\int x\sqrt{2rx-x^2} dx = -\frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2} + r^3 \operatorname{ArcTan}\left[\frac{x}{\sqrt{2rx-x^2}}\right]$$

■ **Problem #55**

$$\int x^2\sqrt{2rx-x^2} dx = -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r(2rx-x^2)^{3/2} - \frac{1}{4}x(2rx-x^2)^{3/2} + \frac{5}{4}r^4 \operatorname{ArcTan}\left[\frac{x}{\sqrt{2rx-x^2}}\right]$$

■ **Problem #56**

$$\int x^3\sqrt{2rx-x^2} dx = -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2} - \frac{7}{12}r^2(2rx-x^2)^{3/2} - \frac{7}{20}rx(2rx-x^2)^{3/2} - \frac{1}{5}x^2(2rx-x^2)^{3/2} + \frac{7}{4}r^5 \operatorname{ArcTan}\left[\frac{x}{\sqrt{2rx-x^2}}\right]$$

■ **Problem #57**

$$\int \frac{1}{(x^2-1)\sqrt{2x+x^2}} dx = -\frac{1}{2} \operatorname{ArcTan}\left[\sqrt{2x+x^2}\right] - \frac{1}{2\sqrt{3}} \operatorname{ArcTanh}\left[\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right]$$

■ **Problem #58**

$$\int \frac{3x-2}{(x+1)^3\sqrt{2x-x^2}} dx = -\frac{5\sqrt{2x-x^2}}{6(1+x)^2} - \frac{2\sqrt{2x-x^2}}{3(1+x)} + \frac{1}{2\sqrt{3}} \operatorname{ArcTan}\left[\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right]$$

■ **Problem #59**

$$\int \frac{1}{\sqrt{1+x+x^2}} dx = \operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #60**

$$\int \frac{x^3}{\sqrt{1+x+x^2}} dx = \frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7}{16} \operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #61**

$$\int \frac{1}{(1+x+x^2)^{3/2}} dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

■ **Problem #62**

$$\int \frac{x}{(1+x+x^2)^{3/2}} dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Problems 63 - 72 (p. 165)

■ **Problem #63**

$$\int \frac{x^3}{(1+x+x^2)^{3/2}} dx = -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2}\operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #64**

$$\int x^2 \sqrt{1+x+x^2} dx = \frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{3}{128}\operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #65**

$$\int (1+x+x^2)^{3/2} dx = \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{27}{128}\operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #66**

$$\int (1+x+x^2)^{5/2} dx = \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} + \frac{135}{1024}\operatorname{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

■ **Problem #67**

$$\int \frac{1}{x^2 \sqrt{1+x+x^2}} dx = -\frac{\sqrt{1+x+x^2}}{x} + \frac{1}{2}\operatorname{ArcTanh}\left[\frac{2+x}{2\sqrt{1+x+x^2}}\right]$$

■ **Problem #68**

$$\int \frac{1}{x^3 \sqrt{1+x+x^2}} dx = -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{8}\operatorname{ArcTanh}\left[\frac{2+x}{2\sqrt{1+x+x^2}}\right]$$

■ **Problem #69**

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}} dx = \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + \frac{3}{2}\operatorname{ArcTanh}\left[\frac{2+x}{2\sqrt{1+x+x^2}}\right]$$

■ Problem #70

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}} dx = \frac{2(1-x)}{3x^2 \sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8} \operatorname{ArcTanh} \left[\frac{2+x}{2\sqrt{1+x+x^2}} \right]$$

■ Problem #71

$$\int \frac{1}{(x+1)\sqrt{1+x+x^2}} dx = -\operatorname{ArcTanh} \left[\frac{1-x}{2\sqrt{1+x+x^2}} \right]$$

■ Problem #72

$$\int \frac{1}{(x^3-x)\sqrt{x^2+2x+4}} dx = \frac{1}{2} \operatorname{ArcTanh} \left[\frac{4+x}{2\sqrt{x^2+2x+4}} \right] - \frac{1}{2\sqrt{7}} \operatorname{ArcTanh} \left[\frac{5+2x}{\sqrt{7}\sqrt{x^2+2x+4}} \right] - \frac{1}{2\sqrt{3}} \operatorname{ArcTanh} \left[\frac{\sqrt{x^2+2x+4}}{\sqrt{3}} \right]$$

Problems 73 - 79 (p. 166)

■ Problem #73

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2} dx = \frac{\sqrt{x^2+2x+4}}{1-x} + \operatorname{ArcSinh} \left[\frac{1+x}{\sqrt{3}} \right] - \frac{2}{\sqrt{7}} \operatorname{ArcTanh} \left[\frac{5+2x}{\sqrt{7}\sqrt{x^2+2x+4}} \right]$$

■ Problem #74

$$\int \frac{2x+3}{(x^2+2x+3)^2 \sqrt{x^2+2x+4}} dx = \frac{(x-3)\sqrt{x^2+2x+4}}{4(x^2+2x+3)} - \frac{1}{4\sqrt{2}} \operatorname{ArcTan} \left[\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}} \right] + \operatorname{ArcTanh} \left[\sqrt{x^2+2x+4} \right]$$

■ Problem #75

$$\int \frac{2x^3+3x^2}{(2x^2+x-3)\sqrt{x^2+2x-3}} dx = \sqrt{x^2+2x-3} + \frac{\sqrt{x^2+2x-3}}{2(1-x)}$$

■ Problem #76

$$\int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}} dx = -\frac{7}{4}\sqrt{x^2+x+2} + \frac{1}{2}x\sqrt{x^2+x+2} - \frac{1}{8}\operatorname{ArcSinh} \left[\frac{1+2x}{\sqrt{7}} \right] + \frac{1}{\sqrt{3}} \operatorname{ArcTan} \left[\frac{1+2x}{\sqrt{3}\sqrt{x^2+x+2}} \right] - \operatorname{ArcTanh} \left[\sqrt{x^2+x+2} \right]$$

■ Problem #77

$$\int \frac{1}{(x^2+2x+4)^{7/2}} dx = \frac{1+x}{15(x^2+2x+4)^{5/2}} + \frac{4(1+x)}{135(x^2+2x+4)^{3/2}} + \frac{8(1+x)}{405\sqrt{x^2+2x+4}}$$

■ **Problem #78**

$$\int \frac{1}{(3x^2 + 8x + 1)^{5/2}} dx = -\frac{4 + 3x}{39(3x^2 + 8x + 1)^{3/2}} + \frac{2(4 + 3x)}{169\sqrt{3x^2 + 8x + 1}}$$

■ **Problem #79**

$$\int \frac{1}{(5 + 4x - 3x^2)^{5/2}} dx = -\frac{2 - 3x}{57(5 + 4x - 3x^2)^{3/2}} - \frac{2(2 - 3x)}{361\sqrt{5 + 4x - 3x^2}}$$

Problems 80 - 84 (p. 167)

■ **Problem #80**

$$\int \frac{1}{1 + \sqrt{x^2 + 2x + 2}} dx = \frac{1}{1 + x} - \frac{\sqrt{x^2 + 2x + 2}}{1 + x} + \text{ArcSinh}[1 + x]$$

■ **Problem #81**

$$\int \frac{1}{x + \sqrt{1 + x + x^2}} dx = -x + \sqrt{1 + x + x^2} - \frac{3}{2} \text{ArcSinh}\left[\frac{1 + 2x}{\sqrt{3}}\right] + 2 \text{Log}\left[x + \sqrt{1 + x + x^2}\right]$$

■ **Problem #82**

$$\int \frac{x^2}{2x + 1 + 2\sqrt{1 + x + x^2}} dx = -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1 + 2x)\sqrt{1 + x + x^2} - \frac{5}{36}(1 + x + x^2)^{3/2} + \frac{1}{6}x(1 + x + x^2)^{3/2} + \frac{1}{64} \text{ArcSinh}\left[\frac{1 + 2x}{\sqrt{3}}\right]$$

■ **Problem #83**

$$\int \frac{\sqrt{1 + x + x^2} - 3x}{\sqrt{1 + x + x^2} - 1} dx = x - 3\sqrt{1 + x + x^2} + \frac{5}{2} \text{ArcSinh}\left[\frac{1 + 2x}{\sqrt{3}}\right] + 4 \text{ArcTanh}\left[\frac{1 - x}{2\sqrt{1 + x + x^2}}\right] - \text{ArcTanh}\left[\frac{2 + x}{2\sqrt{1 + x + x^2}}\right] + \text{Log}[x] - 4 \text{Log}[1 + x]$$

■ **Problem #84**

$$\int \frac{x + 1}{\sqrt{x^2 + 2x + 4} - \sqrt{x^2 + x + 1}} dx = -2\sqrt{x^2 + x + 1} + \frac{1}{4}(1 + 2x)\sqrt{x^2 + x + 1} - 2\sqrt{x^2 + 2x + 4} + \frac{1}{2}(1 + x)\sqrt{x^2 + 2x + 4} + \frac{11}{2} \text{ArcSinh}\left[\frac{1 + x}{\sqrt{3}}\right] + \frac{43}{8} \text{ArcSinh}\left[\frac{1 + 2x}{\sqrt{3}}\right] - 2\sqrt{7} \text{ArcTanh}\left[\frac{1 + 5x}{2\sqrt{7}\sqrt{x^2 + x + 1}}\right] + 2\sqrt{7} \text{ArcTanh}\left[\frac{1 - 2x}{\sqrt{7}\sqrt{x^2 + 2x + 4}}\right]$$

Problems 85 - 91 (p. 177)

■ Problem #85

$$\int \frac{1}{x^3 \sqrt{x-1}} dx = \frac{\sqrt{x-1}}{2x^2} + \frac{3\sqrt{x-1}}{4x} + \frac{3}{4} \text{ArcTan}[\sqrt{x-1}]$$

■ Problem #86

$$\int \frac{1}{x^2 \left(1 - \frac{3}{x}\right)^{4/3}} dx = -\frac{1}{\left(1 - \frac{3}{x}\right)^{1/3}}$$

■ Problem #87

$$\int \frac{(3x-1)^{4/3}}{x^2} dx = 12(3x-1)^{1/3} - \frac{(3x-1)^{4/3}}{x} + 4\sqrt{3} \text{ArcTan}\left[\frac{1-2(3x-1)^{1/3}}{\sqrt{3}}\right] + 2 \text{Log}[x] - 6 \text{Log}[1+(3x-1)^{1/3}]$$

■ Problem #88

$$\int (4-3x)^{4/3} x^2 dx = -\frac{16}{63} (4-3x)^{7/3} + \frac{4}{45} (4-3x)^{10/3} - \frac{1}{117} (4-3x)^{13/3}$$

■ Problem #89

$$\int \frac{(1-2x^{1/3})^{3/4}}{x} dx = 4(1-2x^{1/3})^{3/4} + 6 \text{ArcTan}[(1-2x^{1/3})^{1/4}] - 6 \text{ArcTanh}[(1-2x^{1/3})^{1/4}]$$

■ Problem #90

$$\int \frac{x}{(3-2\sqrt{x})^{3/4}} dx = -\frac{27}{2} (3-2\sqrt{x})^{1/4} + \frac{27}{10} (3-2\sqrt{x})^{5/4} - \frac{1}{2} (3-2\sqrt{x})^{9/4} + \frac{1}{26} (3-2\sqrt{x})^{13/4}$$

■ Problem #91

$$\int \frac{(2\sqrt{x}-1)^{5/4}}{x^2} dx = -\frac{(2\sqrt{x}-1)^{5/4}}{x} - \frac{5(2\sqrt{x}-1)^{1/4}}{2\sqrt{x}} - \frac{5 \text{ArcTan}[1-\sqrt{2}(2\sqrt{x}-1)^{1/4}]}{2\sqrt{2}} +$$

$$\frac{5 \text{ArcTan}[1+\sqrt{2}(2\sqrt{x}-1)^{1/4}]}{2\sqrt{2}} - \frac{5 \text{Log}[1-\sqrt{2}(2\sqrt{x}-1)^{1/4} + \sqrt{2\sqrt{x}-1}]}{4\sqrt{2}} + \frac{5 \text{Log}[1+\sqrt{2}(2\sqrt{x}-1)^{1/4} + \sqrt{2\sqrt{x}-1}]}{4\sqrt{2}}$$

Problems 92 - 100 (p. 178)

■ Problem #92

$$\int (x^7 + 1)^{1/3} x^6 dx = \frac{3}{28} (x^7 + 1)^{4/3}$$

■ Problem #93

$$\int \frac{x^6}{(x^7 + 1)^{5/3}} dx = -\frac{3}{14 (x^7 + 1)^{2/3}}$$

■ Problem #94

$$\int \frac{1}{x (2x^7 - 27)^{2/3}} dx = -\frac{1}{21\sqrt{3}} \operatorname{ArcTan}\left[\frac{3 - 2(2x^7 - 27)^{1/3}}{3\sqrt{3}}\right] - \frac{\operatorname{Log}[x]}{18} + \frac{1}{42} \operatorname{Log}[3 + (2x^7 - 27)^{1/3}]$$

■ Problem #95

$$\int \frac{(x^7 + 1)^{2/3}}{x^8} dx = -\frac{(x^7 + 1)^{2/3}}{7x^7} + \frac{2}{7\sqrt{3}} \operatorname{ArcTan}\left[\frac{1 + 2(x^7 + 1)^{1/3}}{\sqrt{3}}\right] - \frac{\operatorname{Log}[x]}{3} + \frac{1}{7} \operatorname{Log}[1 - (x^7 + 1)^{1/3}]$$

■ Problem #96

$$\int \frac{(3 + 4x^4)^{1/4}}{x^2} dx = -\frac{(3 + 4x^4)^{1/4}}{x} - \frac{1}{\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right] + \frac{1}{\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right]$$

■ Problem #97

$$\int x^2 (3 + 4x^4)^{5/4} dx = \frac{15}{32} x^3 (3 + 4x^4)^{1/4} + \frac{1}{8} x^3 (3 + 4x^4)^{5/4} - \frac{45 \operatorname{ArcTan}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right]}{128\sqrt{2}} + \frac{45 \operatorname{ArcTanh}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right]}{128\sqrt{2}}$$

■ Problem #98

$$\int x^6 (3 + 4x^4)^{1/4} dx = \frac{3}{128} x^3 (3 + 4x^4)^{1/4} + \frac{1}{8} x^7 (3 + 4x^4)^{1/4} + \frac{27 \operatorname{ArcTan}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right]}{512\sqrt{2}} - \frac{27 \operatorname{ArcTanh}\left[\frac{\sqrt{2}x}{(3 + 4x^4)^{1/4}}\right]}{512\sqrt{2}}$$

■ Problem #99

$$\int (x(1 - x^2))^{1/3} dx = \frac{1}{2} x (x(1 - x^2))^{1/3} + \frac{1}{2\sqrt{3}} \operatorname{ArcTan}\left[\frac{2x - (x(1 - x^2))^{1/3}}{\sqrt{3}(x(1 - x^2))^{1/3}}\right] + \frac{\operatorname{Log}[x]}{12} - \frac{1}{4} \operatorname{Log}[x + (x(1 - x^2))^{1/3}]$$

■ **Problem #100**

$$\int \sqrt{x(1+x^{1/3})} dx =$$

$$\frac{7}{64} \sqrt{x(1+x^{1/3})} - \frac{21 \sqrt{x(1+x^{1/3})}}{128 x^{1/3}} - \frac{7}{80} x^{1/3} \sqrt{x(1+x^{1/3})} + \frac{3}{40} x^{2/3} \sqrt{x(1+x^{1/3})} + \frac{3}{5} x \sqrt{x(1+x^{1/3})} + \frac{21}{128} \operatorname{ArcTanh} \left[\frac{x^{2/3}}{\sqrt{x(1+x^{1/3})}} \right]$$

Problems 101 - 112 (p. 193-194)

■ **Problem #101**

$$\int \frac{x^3}{(x^4-1)\sqrt{2x^8+1}} dx = -\frac{1}{4\sqrt{3}} \operatorname{ArcTanh} \left[\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}} \right]$$

■ **Problem #102**

$$\int x^9 \sqrt{1+x^5+x^{10}} dx = -\frac{1}{40} (1+2x^5) \sqrt{1+x^5+x^{10}} + \frac{1}{15} (1+x^5+x^{10})^{3/2} - \frac{3}{80} \operatorname{ArcSinh} \left[\frac{1+2x^5}{\sqrt{3}} \right]$$

■ **Problem #103**

$$\int \frac{1}{x^5 \sqrt{4+2x^2+x^4}} dx = -\frac{\sqrt{4+2x^2+x^4}}{16x^4} + \frac{3\sqrt{4+2x^2+x^4}}{64x^2} + \frac{1}{128} \operatorname{ArcTanh} \left[\frac{4+x^2}{2\sqrt{4+2x^2+x^4}} \right]$$

■ **Problem #104**

$$\int \frac{x^2-1}{x\sqrt{1+3x^2+x^4}} dx = \operatorname{ArcTanh} \left[\frac{1+x^2}{\sqrt{1+3x^2+x^4}} \right]$$

■ **Problem #105**

$$\int (x^4-3x^2)^{3/5} (2x^3-3x) dx = \frac{5}{16} (x^4-3x^2)^{8/5}$$

■ **Problem #106**

$$\int \frac{3x^8-2x^5-x^2(3x^3-1)^{2/3}}{(3x^3-1)^{3/4}} dx = -\frac{4}{27} (3x^3-1)^{1/4} - \frac{4}{33} (3x^3-1)^{11/12} + \frac{4}{243} (3x^3-1)^{9/4}$$

■ **Problem #107**

$$\int \frac{1}{(x^3-1)(x^3+2)^{1/3}} dx = -\frac{\operatorname{ArcTan} \left[\frac{2 \times 3^{1/3} x + (2+x^3)^{1/3}}{\sqrt{3}(2+x^3)^{1/3}} \right]}{3^{5/6}} - \frac{\operatorname{Log}[-1+x^3]}{6 \times 3^{1/3}} + \frac{\operatorname{Log}[3^{1/3} x - (2+x^3)^{1/3}]}{2 \times 3^{1/3}}$$

■ **Problem #108**

$$\int \frac{1}{(x^4 + 1)(x^4 + 2)^{1/4}} dx = -\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}x}{(x^4+2)^{1/4}}\right]}{2\sqrt{2}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}x}{(x^4+2)^{1/4}}\right]}{2\sqrt{2}} - \frac{\text{Log}\left[1 + \frac{x^2}{\sqrt{x^4+2}} - \frac{\sqrt{2}x}{(x^4+2)^{1/4}}\right]}{4\sqrt{2}} + \frac{\text{Log}\left[1 + \frac{x^2}{\sqrt{x^4+2}} + \frac{\sqrt{2}x}{(x^4+2)^{1/4}}\right]}{4\sqrt{2}}$$

■ **Problem #109**

$$\int \frac{x^3 - 1}{(x^3 + 2)^{1/3}} dx = \frac{1}{3}x(x^3 + 2)^{2/3} - \frac{5}{3\sqrt{3}}\text{ArcTan}\left[\frac{1}{\sqrt{3}}\left(1 + \frac{2x}{(x^3 + 2)^{1/3}}\right)\right] + \frac{5}{6}\text{Log}[x - (x^3 + 2)^{1/3}]$$

■ **Problem #110**

$$\int \frac{(x^4 + 1)^{3/4}}{(x^4 + 2)^2} dx = \frac{x(x^4 + 1)^{3/4}}{8(x^4 + 2)} + \frac{3\text{ArcTan}\left[\frac{x}{2^{1/4}(x^4+1)^{1/4}}\right]}{16 \times 2^{3/4}} + \frac{3\text{ArcTanh}\left[\frac{x}{2^{1/4}(x^4+1)^{1/4}}\right]}{16 \times 2^{3/4}}$$

■ **Problem #111**

$$\int \frac{(x^5 - 2)^2}{(x^5 + 3)^3 (x^5 + 3)^{1/5}} dx = -\frac{5x(x^5 - 2)}{33(x^5 + 3)^{11/5}} + \frac{5x}{297(x^5 + 3)^{6/5}} + \frac{97x}{891(x^5 + 3)^{1/5}}$$

■ **Problem #112**

$$\int \frac{1}{(x^3 + 3x^2 + 3x)(x^3 + 3x^2 + 3x + 3)^{1/3}} dx = -\frac{1}{3^{5/6}}\text{ArcTan}\left[\frac{2 \times 3^{1/3}(x+1) + (x^3 + 3x^2 + 3x + 3)^{1/3}}{\sqrt{3}(x^3 + 3x^2 + 3x + 3)^{1/3}}\right] - \frac{\text{Log}[x^3 + 3x^2 + 3x]}{6 \times 3^{1/3}} + \frac{\text{Log}[3^{1/3}(x+1) - (x^3 + 3x^2 + 3x + 3)^{1/3}]}{2 \times 3^{1/3}}$$

Problems 113 - 122 (p. 195-196)

■ **Problem #113**

$$\int \frac{1 - x^2}{(1 + x^2)\sqrt{1 + x^4}} dx = \frac{1}{\sqrt{2}}\text{ArcTan}\left[\frac{\sqrt{2}x}{\sqrt{1 + x^4}}\right]$$

■ **Problem #114**

$$\int \frac{1 + x^2}{(1 - x^2)\sqrt{1 + x^4}} dx = \frac{1}{\sqrt{2}}\text{ArcTanh}\left[\frac{\sqrt{2}x}{\sqrt{1 + x^4}}\right]$$

■ **Problem #115**

$$\int \frac{x^2 + 1}{x \sqrt{1 + x^4}} dx = \text{ArcTanh} \left[\frac{x^2 - 1}{\sqrt{1 + x^4}} \right]$$

■ **Problem #116**

$$\int \frac{x^2 - 1}{x \sqrt{1 + x^4}} dx = \text{ArcTanh} \left[\frac{x^2 + 1}{\sqrt{1 + x^4}} \right]$$

■ **Problem #117**

$$\int \frac{1 + x^2}{(1 - x^2) \sqrt{1 + x^2 + x^4}} dx = \frac{1}{\sqrt{3}} \text{ArcTanh} \left[\frac{\sqrt{3} x}{\sqrt{1 + x^2 + x^4}} \right]$$

■ **Problem #118**

$$\int \frac{1 - x^2}{(1 + x^2) \sqrt{1 + x^2 + x^4}} dx = \text{ArcTan} \left[\frac{x}{\sqrt{1 + x^2 + x^4}} \right]$$

■ **Problem #119**

$$\int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx = \frac{\sqrt{x^4 + x^2 + 1}}{x}$$

■ **Problem #120**

$$\int \frac{1 - x^2}{(1 + 2ax + x^2) \sqrt{1 + 2ax + 2bx^2 + 2ax^3 + x^4}} dx = \frac{1}{\sqrt{2} \sqrt{1 - b}} \text{ArcTan} \left[\frac{a + 2(a^2 - b + 1)x + ax^2}{\sqrt{2} \sqrt{1 - b} \sqrt{1 + 2ax + 2bx^2 + 2ax^3 + x^4}} \right]$$

■ **Problem #121**

$$\int \frac{1}{(1 + x^4) \sqrt{\sqrt{1 + x^4} - x^2}} dx = \text{ArcTan} \left[\frac{x}{\sqrt{\sqrt{1 + x^4} - x^2}} \right]$$

■ **Problem #122**

$$\int \frac{1}{(1 + x^{2n}) \sqrt{(1 + x^{2n})^{1/n} - x^2}} dx = \text{ArcTan} \left[\frac{x}{\sqrt{(1 + x^{2n})^{1/n} - x^2}} \right]$$

Chapter 5 (p. 201-309)

Trigonometric Function Integration Problems

Problems 1 - 3 (p. 202-203)

■ Problem #1

$$\int \cos[x]^2 dx = \frac{x}{2} + \frac{1}{2} \cos[x] \sin[x]$$

■ Problem #2

$$\int \cos[x]^3 dx = \sin[x] - \frac{\sin[x]^3}{3}$$

■ Problem #3

$$\int \sin[x]^4 dx = \frac{3x}{8} - \frac{3}{8} \cos[x] \sin[x] - \frac{1}{4} \cos[x] \sin[x]^3$$

Problems 4 - 7 (p. 208)

■ Problem #4

$$\int \cos[x]^6 dx = \frac{5x}{16} + \frac{5}{16} \cos[x] \sin[x] + \frac{5}{24} \cos[x]^3 \sin[x] + \frac{1}{6} \cos[x]^5 \sin[x]$$

■ Problem #5

$$\int \sin[x]^8 dx = \frac{35x}{128} - \frac{35}{128} \cos[x] \sin[x] - \frac{35}{192} \cos[x] \sin[x]^3 - \frac{7}{48} \cos[x] \sin[x]^5 - \frac{1}{8} \cos[x] \sin[x]^7$$

■ Problem #6

$$\int \cos\left[\frac{\pi}{4} + \frac{x}{2}\right]^4 dx = \frac{3x}{8} + \frac{\cos[x]}{2} - \frac{1}{8} \cos[x] \sin[x]$$

■ Problem #7

$$\int \sin\left[3x - \frac{\pi}{12}\right]^3 dx = -\frac{1}{3} \cos\left[\frac{\pi}{12} - 3x\right] + \frac{1}{9} \cos\left[\frac{\pi}{12} - 3x\right]^3$$

Problems 8 - 11 (p. 211)

■ Problem #8

$$\int \frac{1}{\sin[x]^6} dx = -\cot[x] - \frac{2 \cot[x]^3}{3} - \frac{\cot[x]^5}{5}$$

■ Problem #9

$$\int \csc[x]^7 dx = -\frac{5}{16} \operatorname{ArcTanh}[\cos[x]] - \frac{5}{16} \cot[x] \csc[x] - \frac{5}{24} \cot[x] \csc[x]^3 - \frac{1}{6} \cot[x] \csc[x]^5$$

■ Problem #10

$$\int \frac{1}{\cos[x]^{12}} dx = \tan[x] + \frac{5 \tan[x]^3}{3} + 2 \tan[x]^5 + \frac{10 \tan[x]^7}{7} + \frac{5 \tan[x]^9}{9} + \frac{\tan[x]^{11}}{11}$$

■ Problem #11

$$\int \frac{1}{\cos\left[\frac{\pi}{4} + 3x\right]^3} dx = \frac{1}{6} \operatorname{ArcTanh}\left[\sin\left[\frac{\pi}{4} + 3x\right]\right] + \frac{1}{6} \operatorname{Sec}\left[\frac{\pi}{4} + 3x\right] \tan\left[\frac{\pi}{4} + 3x\right]$$

Problems 12 - 14 (p. 213)

■ Problem #12

$$\int \tan[x]^6 dx = -x + \tan[x] - \frac{\tan[x]^3}{3} + \frac{\tan[x]^5}{5}$$

■ Problem #13

$$\int \frac{1}{\tan[x]^5} dx = \frac{\cot[x]^2}{2} - \frac{\cot[x]^4}{4} + \operatorname{Log}[\sin[x]]$$

■ Problem #14

$$\int \cot\left[\frac{x}{3} - \frac{3\pi}{4}\right]^4 dx = x + 3 \cot\left[\frac{\pi}{4} + \frac{x}{3}\right] - \cot\left[\frac{\pi}{4} + \frac{x}{3}\right]^3$$

Problems 15 - 20 (p. 219-220)

■ Problem #15

$$\int \sin[x]^4 \cos[x]^6 dx = \frac{3x}{256} + \frac{3}{256} \cos[x] \sin[x] + \frac{1}{128} \cos[x]^3 \sin[x] + \frac{1}{160} \cos[x]^5 \sin[x] - \frac{3}{80} \cos[x]^7 \sin[x] - \frac{1}{10} \cos[x]^7 \sin[x]^3$$

■ **Problem #16**

$$\int \sin[x]^7 \cos[x]^6 dx = -\frac{1}{7} \cos[x]^7 + \frac{\cos[x]^9}{3} - \frac{3 \cos[x]^{11}}{11} + \frac{\cos[x]^{13}}{13}$$

■ **Problem #17**

$$\int \frac{\sin[x]^{11}}{\cos[x]} dx = \frac{5 \cos[x]^2}{2} - \frac{5 \cos[x]^4}{2} + \frac{5 \cos[x]^6}{3} - \frac{5 \cos[x]^8}{8} + \frac{\cos[x]^{10}}{10} - \text{Log}[\cos[x]]$$

■ **Problem #18**

$$\int \frac{1}{\sin[x]^6 \cos[x]^6} dx = -10 \cot[x] - \frac{5 \cot[x]^3}{3} - \frac{\cot[x]^5}{5} + 10 \tan[x] + \frac{5 \tan[x]^3}{3} + \frac{\tan[x]^5}{5}$$

■ **Problem #19.1**

$$\int \sin[x]^2 \cos[x]^2 dx = \frac{x}{8} + \frac{1}{8} \cos[x] \sin[x] - \frac{1}{4} \cos[x]^3 \sin[x]$$

■ **Problem #19.2**

$$\int \sin[x]^4 \cos[x]^4 dx = \frac{3x}{128} + \frac{3}{128} \cos[x] \sin[x] + \frac{1}{64} \cos[x]^3 \sin[x] - \frac{1}{16} \cos[x]^5 \sin[x] - \frac{1}{8} \cos[x]^5 \sin[x]^3$$

■ **Problem #19.3**

$$\int \sin[x]^6 \cos[x]^6 dx = \frac{5x}{1024} + \frac{5 \cos[x] \sin[x]}{1024} + \frac{5 \cos[x]^3 \sin[x]}{1536} + \frac{1}{384} \cos[x]^5 \sin[x] - \frac{1}{64} \cos[x]^7 \sin[x] - \frac{1}{24} \cos[x]^7 \sin[x]^3 - \frac{1}{12} \cos[x]^7 \sin[x]^5$$

■ **Problem #19.4**

$$\int \sin[x]^8 \cos[x]^8 dx = \frac{35x}{32768} + \frac{35 \cos[x] \sin[x]}{32768} + \frac{35 \cos[x]^3 \sin[x]}{49152} + \frac{7 \cos[x]^5 \sin[x]}{12288} + \frac{\cos[x]^7 \sin[x]}{2048} - \frac{1}{256} \cos[x]^9 \sin[x] - \frac{5}{384} \cos[x]^9 \sin[x]^3 - \frac{1}{32} \cos[x]^9 \sin[x]^5 - \frac{1}{16} \cos[x]^9 \sin[x]^7$$

■ **Problem #19.m**

$$\int \sin[x]^{2m} \cos[x]^{2m} dx = -\frac{\cos[x]^{1+2m} \sin[x]^{1+2m} (\sin[x]^2)^{\frac{1}{2}(-1-2m)}}{1+2m} \text{Hypergeometric2F1}\left[\frac{1}{2}(1-2m), \frac{1}{2}(1+2m), \frac{1}{2}(3+2m), \cos[x]^2\right]$$

■ **Problem #20**

$$\int \frac{1}{\sin\left[\frac{\pi}{4} + 2x\right]^3 \cos\left[\frac{\pi}{4} + 2x\right]} dx = -\frac{1}{4} \cot\left[\frac{\pi}{4} + 2x\right]^2 + \frac{1}{2} \text{Log}\left[\tan\left[\frac{\pi}{4} + 2x\right]\right]$$

Problems 21 - 29 (p. 223)

■ Problem #21

$$\int \tan[x]^2 \sec[x]^2 dx = \frac{\tan[x]^3}{3}$$

■ Problem #22

$$\int \cot[x]^3 \csc[x] dx = \csc[x] - \frac{\csc[x]^3}{3}$$

■ Problem #23

$$\int \tan[x] \sec[x]^3 dx = \frac{\sec[x]^3}{3}$$

■ Problem #24

$$\int \cot[x]^2 \csc[x]^3 dx = \frac{1}{8} \operatorname{ArcTanh}[\cos[x]] - \frac{1}{8} \cot[x] \csc[x] - \frac{1}{4} \cot[x]^3 \csc[x]$$

■ Problem #25

$$\int \frac{\cos[x]^3}{\sin[x]^7} dx = -\frac{1}{4} \cot[x]^4 - \frac{\cot[x]^6}{6}$$

■ Problem #26

$$\int \tan[x]^5 \sec[x]^{3/2} dx = \frac{2}{3} \sec[x]^{3/2} - \frac{4}{7} \sec[x]^{7/2} + \frac{2}{11} \sec[x]^{11/2}$$

■ Problem #27

$$\int \tan[x]^{3/2} \sec[x]^4 dx = \frac{2}{5} \tan[x]^{5/2} + \frac{2}{9} \tan[x]^{9/2}$$

■ Problem #28

$$\int \cot[x]^4 \csc[x]^3 dx = -\frac{1}{16} \operatorname{ArcTanh}[\cos[x]] + \frac{1}{16} \cot[x] \csc[x] - \frac{1}{24} \cot[x]^3 \csc[x] - \frac{1}{6} \cot[x]^5 \csc[x]$$

■ Problem #29

$$\int \tan\left[\frac{\pi}{4} + \frac{x}{2}\right]^2 \sec\left[\frac{\pi}{4} + \frac{x}{2}\right]^3 dx = -\frac{1}{4} \operatorname{ArcTanh}\left[\sin\left[\frac{\pi}{4} + \frac{x}{2}\right]\right] + \frac{1}{4} \sec\left[\frac{\pi}{4} + \frac{x}{2}\right] \tan\left[\frac{\pi}{4} + \frac{x}{2}\right] + \frac{1}{2} \sec\left[\frac{\pi}{4} + \frac{x}{2}\right] \tan\left[\frac{\pi}{4} + \frac{x}{2}\right]^3$$

Problems 30 - 32 (p. 228)

■ Problem #30

$$\int (a \operatorname{Sec}[x]^2 - \sin[2x])^2 (\operatorname{Cot}[x]^3 + 1) dx = \frac{x}{2} + 4ax + 2\cos[x]^2 + \cos[x]^4 + 4a\operatorname{Cot}[x] - \frac{1}{2}a^2\operatorname{Cot}[x]^2 +$$

$$(4-a)a\operatorname{Log}[\cos[x]] + (4+a^2)\operatorname{Log}[\sin[x]] + \frac{1}{2}\cos[x]\sin[x] - \cos[x]^3\sin[x] + a^2\tan[x] + \frac{1}{3}a^2\tan[x]^3$$

■ Problem #31

$$\int \left(1 - \frac{\sin[x]}{2}\right)^4 (4 - 3\cos[x]) dx =$$

$$\frac{227x}{32} + 10\cos[x] - \frac{15\cos[x]^2}{4} - \frac{2\cos[x]^3}{3} + \frac{3\cos[x]^4}{8} - 3\sin[x] - \frac{99}{32}\cos[x]\sin[x] - \frac{3\sin[x]^3}{2} - \frac{1}{16}\cos[x]\sin[x]^3 - \frac{3\sin[x]^5}{80}$$

■ Problem #32

$$\int (3 - 2\operatorname{Cot}[x])^3 \left(\frac{1}{2} - 3\operatorname{Cot}[x]\right) dx = -\frac{285x}{2} + 5(3 - 2\operatorname{Cot}[x])^2 + (3 - 2\operatorname{Cot}[x])^3 - 42\operatorname{Cot}[x] + 4\operatorname{Log}[\sin[x]]$$

Problems 33 - 36 (p. 229)

■ Problem #33

$$\int \frac{\cos[5x]}{\cos[x]^5} dx = 16x - 15\tan[x] + \frac{5\tan[x]^3}{3}$$

■ Problem #34

$$\int \frac{\cos[4x]}{\cos[x]} dx = \operatorname{ArcTanh}[\sin[x]] - \frac{8\sin[x]^3}{3}$$

■ Problem #35

$$\int \cos[4x]\cos[x] dx = \frac{1}{6}\sin[3x] + \frac{1}{10}\sin[5x]$$

■ Problem #36

$$\int \frac{\cos[4x]}{\cos[x]^5} dx = \frac{35}{8}\operatorname{ArcTanh}[\sin[x]] - \frac{29}{8}\sec[x]\tan[x] + \frac{1}{4}\sec[x]^3\tan[x]$$

Problems 37 - 39 (p. 233)

■ Problem #37

$$\int \cos[4x] \cos[x]^4 dx = \frac{x}{16} + \frac{1}{8} \sin[2x] + \frac{3}{32} \sin[4x] + \frac{1}{24} \sin[6x] + \frac{1}{128} \sin[8x]$$

■ Problem #38

$$\int \frac{\cos[5x]}{\sin[x]^5} dx = 6 \operatorname{Csc}[x]^2 - \frac{\operatorname{Csc}[x]^4}{4} + 16 \operatorname{Log}[\sin[x]]$$

■ Problem #39

$$\int \frac{\sin[4x]}{\sin[x]^4} dx = -2 \operatorname{Csc}[x]^2 - 8 \operatorname{Log}[\sin[x]]$$

Problems 40 - 49 (p. 254-255)

■ Problem #40

$$\int \frac{\cos[x]}{\sin[x] (2 + \sin[2x])} dx = -\frac{x}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} \operatorname{ArcTan}\left[\frac{1 - 2\cos[x]^2}{2 + \sqrt{3} + 2\cos[x]\sin[x]}\right] + \frac{1}{2} \operatorname{Log}[\sin[x]] - \frac{1}{4} \operatorname{Log}[1 + \cos[x]\sin[x]]$$

■ Problem #41

$$\int \frac{\cos[x]^2}{\sin[x] \cos[3x]} dx = -\frac{1}{2} \operatorname{Log}[\operatorname{Csc}[x]^2 - 4]$$

■ Problem #42

$$\int \frac{\sin[2x]}{\cos[x]^4 + \sin[x]^4} dx = -\operatorname{ArcTan}[\cos[2x]]$$

■ Problem #43

$$\int \frac{1}{4 + \sqrt{3} \cos[x] + \sin[x]} dx = \frac{x}{2\sqrt{3}} + \frac{1}{\sqrt{3}} \operatorname{ArcTan}\left[\frac{\cos[x] - \sqrt{3} \sin[x]}{2(2 + \sqrt{3}) + \sqrt{3} \cos[x] + \sin[x]}\right]$$

■ Problem #44

$$\int \frac{1}{3 + 4 \cos[x] + 4 \sin[x]} dx = -\frac{1}{\sqrt{23}} \operatorname{ArcTanh}\left[\frac{\sqrt{23} (\cos[x] - \sin[x])}{8 + 3 \cos[x] + 3 \sin[x]}\right]$$

■ Problem #45

$$\int \frac{1}{4 - 3 \cos[x]^2 + 5 \sin[x]^2} dx = \frac{x}{3} + \frac{1}{3} \operatorname{ArcTan}\left[\frac{2 \cos[x] \sin[x]}{1 + 2 \sin[x]^2}\right]$$

■ Problem #46

$$\int \frac{1}{4 + \tan[x] + 4 \cot[x]} dx = \frac{4x}{25} - \frac{3}{25} \operatorname{Log}[2 \cos[x] + \sin[x]] + \frac{2}{5(2 + \tan[x])}$$

■ Problem #47

$$\int \frac{1}{(\sin[x] + 2 \sec[x])^2} dx = \frac{8x}{15\sqrt{15}} - \frac{8}{15\sqrt{15}} \operatorname{ArcTan}\left[\frac{1 - 2 \cos[x]^2}{4 + \sqrt{15} + 2 \cos[x] \sin[x]}\right] + \frac{1 + 4 \tan[x]}{15(2 + \tan[x] + 2 \tan[x]^2)}$$

■ Problem #48

$$\int \frac{1}{(\cos[x] + 2 \sec[x])^2} dx = \frac{x}{6\sqrt{6}} - \frac{1}{6\sqrt{6}} \operatorname{ArcTan}\left[\frac{\cos[x] \sin[x]}{2 + \sqrt{6} + \cos[x]^2}\right] + \frac{\tan[x]}{6(3 + 2 \tan[x]^2)}$$

■ Problem #49

$$\int \frac{5 - \tan[x] - 6 \tan[x]^2}{(1 + 3 \tan[x])^3} dx = -\frac{67x}{250} - \frac{28}{125} \operatorname{Log}[\cos[x] + 3 \sin[x]] - \frac{7}{10(1 + 3 \tan[x])^2} - \frac{29}{50(1 + 3 \tan[x])}$$

Problems 50 - 56 (p. 260)

■ Problem #50

$$\int \frac{\cos[x]^2}{\cos[3x]} dx = \frac{1}{2} \operatorname{ArcTanh}[2 \sin[x]]$$

■ Problem #51

$$\int \frac{\sin[x]}{\cos[2x]} dx = \frac{\operatorname{ArcTanh}[\sqrt{2} \cos[x]]}{\sqrt{2}}$$

■ Problem #52

$$\int \frac{\sin[x]^2}{\cos[2x]} dx = -\frac{x}{2} + \frac{1}{4} \operatorname{ArcTanh}[2 \cos[x] \sin[x]]$$

■ Problem #53

$$\int \frac{\sin[x]^3}{\cos[3x]} dx = \frac{1}{3} \operatorname{Log}[\cos[x]] - \frac{1}{24} \operatorname{Log}[3 - 4 \cos[x]^2]$$

■ Problem #54

$$\int \frac{\cos[x]}{\sin[3x]} dx = \frac{1}{3} \operatorname{Log}[\sin[x]] - \frac{1}{6} \operatorname{Log}[3 - 4 \sin[x]^2]$$

■ **Problem #55**

$$\int \frac{\sin[x]}{\sin[4x]} dx = -\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] + \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{2\sqrt{2}}$$

■ **Problem #56**

$$\int \frac{\sin[x]^3}{\sin[4x]} dx = -\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] + \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{4\sqrt{2}}$$

Problems 57 - 61 (p. 266)

■ **Problem #57a**

$$\int \sqrt{1 + \sin[2x]} dx = -\frac{\cos[2x]}{\sqrt{1 + \sin[2x]}}$$

■ **Problem #57b**

$$\int \sqrt{1 - \sin[2x]} dx = \frac{\cos[2x]}{\sqrt{1 - \sin[2x]}}$$

■ **Problem #58a**

$$\int \frac{1}{\sqrt{1 + \cos[2x]}} dx = \frac{1}{\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sin[2x]}{\sqrt{2} \sqrt{1 + \cos[2x]}}\right]$$

■ **Problem #58b**

$$\int \frac{1}{\sqrt{1 - \cos[2x]}} dx = -\frac{1}{\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sin[2x]}{\sqrt{2} \sqrt{1 - \cos[2x]}}\right]$$

■ **Problem #59**

$$\int \frac{1}{(1 - \cos[3x])^{3/2}} dx = -\frac{1}{6\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sin[3x]}{\sqrt{2} \sqrt{1 - \cos[3x]}}\right] - \frac{\sin[3x]}{6(1 - \cos[3x])^{3/2}}$$

■ **Problem #60**

$$\int \left(1 - \sin\left[\frac{2x}{3}\right]\right)^{5/2} dx = \frac{32 \cos\left[\frac{2x}{3}\right]}{5\sqrt{1 - \sin\left[\frac{2x}{3}\right]}} + \frac{8}{5} \cos\left[\frac{2x}{3}\right] \sqrt{1 - \sin\left[\frac{2x}{3}\right]} + \frac{3}{5} \cos\left[\frac{2x}{3}\right] \left(1 - \sin\left[\frac{2x}{3}\right]\right)^{3/2}$$

■ **Problem #61**

$$\int \frac{(2(1+2\sin[x])^{1/4} - \cos[x]^2) \cos[x]}{(1+2\sin[x])^{3/2}} dx = \frac{3}{4\sqrt{1+2\sin[x]}} - \frac{4}{(1+2\sin[x])^{1/4}} - \frac{1}{2}\sqrt{1+2\sin[x]} + \frac{1}{12}(1+2\sin[x])^{3/2}$$

Problems 62 - 66 (p. 268)

■ **Problem #62**

$$\int \sqrt{\tan[x]} dx = -\frac{\operatorname{ArcTan}\left[\frac{1-\tan[x]}{\sqrt{2}\sqrt{\tan[x]}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh}\left[\frac{1+\tan[x]}{\sqrt{2}\sqrt{\tan[x]}}\right]}{\sqrt{2}}$$

■ **Problem #63**

$$\int \frac{1}{\tan[5x]^{1/3}} dx = -\frac{1}{10}\sqrt{3}\operatorname{ArcTan}\left[\frac{1-2\tan[5x]^{2/3}}{\sqrt{3}}\right] + \frac{3}{20}\operatorname{Log}[1+\tan[5x]^{2/3}] - \frac{1}{20}\operatorname{Log}[1+\tan[5x]^2]$$

■ **Problem #64**

$$\int \frac{1}{(4+3\tan[2x])^{3/2}} dx = -\frac{9\operatorname{ArcTan}\left[\frac{1-3\tan[2x]}{\sqrt{2}\sqrt{4+3\tan[2x]}}\right]}{250\sqrt{2}} + \frac{13\operatorname{ArcTanh}\left[\frac{3+\tan[2x]}{\sqrt{2}\sqrt{4+3\tan[2x]}}\right]}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3\tan[2x]}}$$

■ **Problem #65**

$$\int \frac{3\tan[x] - \sqrt{4-3\tan[x]}}{\cos[x]^2(4-3\tan[x])^{3/2}} dx = \frac{1}{3}\operatorname{Log}[4-3\tan[x]] + \frac{8}{3\sqrt{4-3\tan[x]}} + \frac{2}{3}\sqrt{4-3\tan[x]}$$

■ **Problem #66**

$$\int \frac{\tan[x]}{(\sqrt{\tan[x]} - 1)^2} dx = -\frac{x}{2} + \frac{\operatorname{ArcTan}\left[\frac{1-\tan[x]}{\sqrt{2}\sqrt{\tan[x]}}\right]}{\sqrt{2}} + \frac{\operatorname{ArcTanh}\left[\frac{1+\tan[x]}{\sqrt{2}\sqrt{\tan[x]}}\right]}{\sqrt{2}} + \frac{1}{2}\operatorname{Log}[\cos[x]] + \operatorname{Log}[1-\sqrt{\tan[x]}] + \frac{1}{1-\sqrt{\tan[x]}}$$

Problems 67 - 75 (p. 272-273)

■ **Problem #67a**

$$\int \frac{\sin[x]}{\sqrt{\sin[2x]}} dx = -\frac{1}{2}\operatorname{ArcSin}[\cos[x] - \sin[x]] - \frac{1}{2}\operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}]$$

■ **Problem #67b**

$$\int \frac{\cos[x]}{\sqrt{\sin[2x]}} dx = -\frac{1}{2} \operatorname{ArcSin}[\cos[x] - \sin[x]] + \frac{1}{2} \operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}]$$

■ **Problem #68**

$$\int \sqrt{\sin[2x]} \sin[x] dx = -\frac{1}{4} \operatorname{ArcSin}[\cos[x] - \sin[x]] + \frac{1}{4} \operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}] - \frac{1}{2} \cos[x] \sqrt{\sin[2x]}$$

■ **Problem #69**

$$\int (\cos[x] - \sin[x]) \sqrt{\sin[2x]} dx = -\frac{1}{2} \operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}] + \frac{1}{2} \cos[x] \sqrt{\sin[2x]} + \frac{1}{2} \sin[x] \sqrt{\sin[2x]}$$

■ **Problem #70**

$$\int \frac{\sin[x]^7}{\sin[2x]^{7/2}} dx = -\frac{1}{16} \operatorname{ArcSin}[\cos[x] - \sin[x]] + \frac{1}{16} \operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}] + \frac{\sin[x]^5}{5 \sin[2x]^{5/2}} - \frac{\sin[x]}{4 \sqrt{\sin[2x]}}$$

■ **Problem #71**

$$\int \frac{\cos[x]^7}{\sin[2x]^{7/2}} dx = -\frac{1}{16} \operatorname{ArcSin}[\cos[x] - \sin[x]] - \frac{1}{16} \operatorname{Log}[\cos[x] + \sin[x] + \sqrt{\sin[2x]}] - \frac{\cos[x]^5}{5 \sin[2x]^{5/2}} + \frac{\cos[x]}{4 \sqrt{\sin[2x]}}$$

■ **Problem #72**

$$\int \frac{\sin[2x]^{3/2}}{\sin[x]^5} dx = -\frac{1}{5} \operatorname{Csc}[x]^5 \sin[2x]^{5/2}$$

■ **Problem #73**

$$\int \frac{1}{\cos[x]^3 \sqrt{\sin[2x]}} dx = \frac{4}{5} \operatorname{Sec}[x] \sqrt{\sin[2x]} + \frac{1}{5} \operatorname{Sec}[x]^3 \sqrt{\sin[2x]}$$

■ **Problem #74**

$$\int \frac{1}{\sin[x] \sin[2x]^{3/2}} dx = -\frac{2 \cos[x]}{3 \sin[2x]^{3/2}} + \frac{4 \sin[x]}{3 \sqrt{\sin[2x]}}$$

■ **Problem #75**

$$\int \frac{(\cos[2x] - 3 \tan[x]) \cos[x]^3}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}} dx = \frac{33}{32} \operatorname{ArcTanh}\left[\frac{\sqrt{\sin[2x]}}{2 \cos[x]}\right] - \frac{9 \cos[x]}{16 \sqrt{\sin[2x]}} - \frac{5 \cos[x] \cot[x]}{24 \sqrt{\sin[2x]}} + \frac{\cos[x] \cot[x]^2}{20 \sqrt{\sin[2x]}}$$

Problems 76 - 82 (p. 276)

■ Problem #76

$$\int \sqrt{\frac{\sin[x]}{\cos[x]^5}} dx = \frac{2}{3} \cos[x] \sin[x] \sqrt{\sec[x]^4 \tan[x]}$$

■ Problem #77

$$\int \sqrt{\frac{\sin[x]^5}{\cos[x]}} dx = \frac{3}{4\sqrt{2}} \operatorname{ArcTan}\left[\frac{(1 - \cot[x]) \operatorname{Csc}[x]^2 \sqrt{\sin[x]^4 \tan[x]}}{\sqrt{2}}\right] +$$

$$\frac{3}{4\sqrt{2}} \operatorname{Log}\left[\cos[x] + \sin[x] - \sqrt{2} \cot[x] \operatorname{Csc}[x] \sqrt{\sin[x]^4 \tan[x]}\right] - \frac{1}{2} \cot[x] \sqrt{\sin[x]^4 \tan[x]}$$

■ Problem #78

$$\int \left(\frac{\sin[x]^2}{\cos[x]^{14}}\right)^{1/3} dx = \frac{3}{5} \cos[x]^3 \sin[x] (\sec[x]^{12} \tan[x]^2)^{1/3} + \frac{3}{11} \cos[x] \sin[x]^3 (\sec[x]^{12} \tan[x]^2)^{1/3}$$

■ Problem #79

$$\int \frac{1}{(\sin[x]^{13} \cos[x]^{11})^{1/4}} dx = -\frac{4 \cos[x]^5 \sin[x]}{9 (\cos[x]^{11} \sin[x]^{13})^{1/4}} - \frac{8 \cos[x]^3 \sin[x]^3}{(\cos[x]^{11} \sin[x]^{13})^{1/4}} + \frac{4 \cos[x] \sin[x]^5}{7 (\cos[x]^{11} \sin[x]^{13})^{1/4}}$$

■ Problem #80

$$\int \frac{\cos[2x] - \sqrt{\sin[2x]}}{\sqrt{\sin[x] \cos[x]^3}} dx = -\sqrt{2} \operatorname{Log}\left[\cos[x] + \sin[x] - \sqrt{2} \sec[x] \sqrt{\cos[x]^3 \sin[x]}\right] -$$

$$\frac{\operatorname{ArcSin}[\cos[x] - \sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\operatorname{ArcTanh}[\sin[x]] \cos[x] \sqrt{\sin[2x]}}{\sqrt{\cos[x]^3 \sin[x]}} - \frac{\sin[2x]}{\sqrt{\cos[x]^3 \sin[x]}}$$

■ **Problem #81**

$$\int \frac{\sqrt{\sin[x]^3 \cos[x]} - 2 \sin[2x]}{\sqrt{\tan[x]} - \sqrt{\sin[x] \cos[x]^3}} dx = -2\sqrt{2} \operatorname{Arcoth}\left[\frac{\cos[x] (\cos[x] + \sin[x])}{\sqrt{2} \sqrt{\cos[x]^3 \sin[x]}}\right] +$$

$$2^{1/4} \operatorname{Arcoth}\left[\frac{\cos[x] (\sqrt{2} \cos[x] + \sin[x])}{2^{3/4} \sqrt{\cos[x]^3 \sin[x]}}\right] - 2^{1/4} \operatorname{Arcoth}\left[\frac{\sqrt{2} + \tan[x]}{2^{3/4} \sqrt{\tan[x]}}\right] - 2\sqrt{2} \operatorname{Arctan}\left[\frac{\cos[x] (\cos[x] - \sin[x])}{\sqrt{2} \sqrt{\cos[x]^3 \sin[x]}}\right] +$$

$$2^{1/4} \operatorname{Arctan}\left[\frac{\cos[x] (\sqrt{2} \cos[x] - \sin[x])}{2^{3/4} \sqrt{\cos[x]^3 \sin[x]}}\right] - 2^{1/4} \operatorname{Arctan}\left[\frac{\sqrt{2} - \tan[x]}{2^{3/4} \sqrt{\tan[x]}}\right] + 4 \csc[x] \sec[x] \sqrt{\cos[x]^3 \sin[x]} +$$

$$\frac{1}{4} \csc[x]^2 \log[1 + \cos[x]^2] \sec[x]^2 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\cos[x] \sin[x]^3} + \frac{1}{2} \csc[x]^2 \log[\sin[x]] \sec[x]^2 \sqrt{\cos[x]^3 \sin[x]} \sqrt{\cos[x] \sin[x]^3} +$$

$$\frac{4}{\sqrt{\tan[x]}} - \frac{1}{4} \csc[x]^2 \log[1 + \cos[x]^2] \sqrt{\cos[x] \sin[x]^3} \sqrt{\tan[x]} + \frac{1}{2} \csc[x]^2 \log[\sin[x]] \sqrt{\cos[x] \sin[x]^3} \sqrt{\tan[x]}$$

■ **Problem #82**

$$\int \frac{\left(\frac{\sin[x]}{\cos[x]^7}\right)^{1/3} - 3 \tan[x]}{(\sin[x] \cos[x]^5)^{2/3}} dx = -\frac{9 \sin[x]^4}{10 (\cos[x]^5 \sin[x])^{2/3}} - \frac{9}{4} \sec[x]^8 (\cos[x]^5 \sin[x])^{4/3} + \frac{3}{2} (\cos[x]^5 \sin[x])^{1/3} (\sec[x]^6 \tan[x])^{1/3} +$$

$$\frac{3}{4} (\cos[x]^5 \sin[x])^{1/3} \tan[x]^2 (\sec[x]^6 \tan[x])^{1/3} + \frac{3}{14} (\cos[x]^5 \sin[x])^{1/3} \tan[x]^4 (\sec[x]^6 \tan[x])^{1/3}$$

Problems 83 - 92 (p. 288-289)

■ **Problem #83**

$$\int (2 \cos[x]^2 + 1)^{5/2} \sin[x] dx = -\frac{5 \operatorname{Arcsinh}\left[\sqrt{2} \cos[x]\right]}{16 \sqrt{2}} - \frac{5}{16} \cos[x] \sqrt{1 + 2 \cos[x]^2} - \frac{5}{24} \cos[x] (1 + 2 \cos[x]^2)^{3/2} - \frac{1}{6} \cos[x] (1 + 2 \cos[x]^2)^{5/2}$$

■ **Problem #84**

$$\int (5 \cos[x]^2 + \sin[x]^2)^{5/2} \cos[x] dx = \frac{625}{32} \operatorname{Arcsin}\left[\frac{2 \sin[x]}{\sqrt{5}}\right] + \frac{125}{16} \sin[x] \sqrt{5 - 4 \sin[x]^2} + \frac{25}{24} \sin[x] (5 - 4 \sin[x]^2)^{3/2} + \frac{1}{6} \sin[x] (5 - 4 \sin[x]^2)^{5/2}$$

■ **Problem #85**

$$\int (-\cos[x]^2 - 5 \sin[x]^2)^{3/2} \cos[x] dx = \frac{3}{16} \operatorname{Arctan}\left[\frac{2 \sin[x]}{\sqrt{-1 - 4 \sin[x]^2}}\right] - \frac{3}{8} \sin[x] \sqrt{-1 - 4 \sin[x]^2} + \frac{1}{4} \sin[x] (-1 - 4 \sin[x]^2)^{3/2}$$

■ Problem #86

$$\int \frac{\sin[x]}{(5 \cos[x]^2 - 2 \sin[x]^2)^{7/2}} dx = \frac{\cos[x]}{10(-2 + 7 \cos[x]^2)^{5/2}} - \frac{\cos[x]}{15(-2 + 7 \cos[x]^2)^{3/2}} + \frac{\cos[x]}{15 \sqrt{-2 + 7 \cos[x]^2}}$$

■ Problem #87

$$\int \frac{\cos[2x] \cos[x]}{(2 - 5 \sin[x]^2)^{3/2}} dx = \frac{2 \operatorname{ArcSin}\left[\sqrt{\frac{5}{2}} \sin[x]\right]}{5 \sqrt{5}} + \frac{\sin[x]}{10 \sqrt{2 - 5 \sin[x]^2}}$$

■ Problem #88

$$\int \frac{\sin[5x]}{(5 \cos[x]^2 + 9 \sin[x]^2)^{5/2}} dx = -\frac{1}{2} \operatorname{ArcSin}\left[\frac{2 \cos[x]}{3}\right] - \frac{55 \cos[x]}{27(9 - 4 \cos[x]^2)^{3/2}} + \frac{295 \cos[x]}{243 \sqrt{9 - 4 \cos[x]^2}}$$

■ Problem #89

$$\int \frac{\cos[x] \cos[2x] \sin[3x]}{(4 \sin[x]^2 - 5)^{5/2}} dx = -\frac{1}{4(-5 + 4 \sin[x]^2)^{3/2}} - \frac{5}{8 \sqrt{-5 + 4 \sin[x]^2}} + \frac{1}{8} \sqrt{-5 + 4 \sin[x]^2}$$

■ Problem #90

$$\int \frac{\sin[x] \cos[2x] - 2(\sin[x] - 1) \cos[x]^3}{\sin[x]^2 \sqrt{\sin[x]^2 - 5}} dx = 2 \operatorname{ArcTan}\left[\frac{\cos[x]}{\sqrt{\sin[x]^2 - 5}}\right] - \frac{1}{\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{5} \cos[x]}{\sqrt{\sin[x]^2 - 5}}\right] - \frac{2}{\sqrt{5}} \operatorname{ArcTan}\left[\frac{\sqrt{\sin[x]^2 - 5}}{\sqrt{5}}\right] - 2 \operatorname{ArcTanh}\left[\frac{\sin[x]}{\sqrt{\sin[x]^2 - 5}}\right] + \frac{2 \sqrt{\sin[x]^2 - 5}}{5 \sin[x]} + 2 \sqrt{\sin[x]^2 - 5}$$

■ Problem #91

$$\int \frac{\cos[3x]}{\sqrt{3 \cos[x]^2 - \sin[x]^2} \sqrt{8 \cos[x]^2 - 1}} dx = \frac{5}{4 \sqrt{2}} \operatorname{ArcSin}\left[2 \sqrt{\frac{2}{7}} \sin[x]\right] + \frac{3}{4} \operatorname{ArcSin}\left[\frac{2 \sin[x]}{\sqrt{3}}\right] - \frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{4 \cos[x]^2 - 1}}\right] - \frac{3}{4} \operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{8 \cos[x]^2 - 1}}\right] - \frac{1}{2} \sin[x] \sqrt{4 \cos[x]^2 - 1} - \frac{1}{2} \sin[x] \sqrt{8 \cos[x]^2 - 1}$$

■ Problem #92

$$\int (2 - 3 \sin[x]^2)^{3/5} \sin[4x] dx = \frac{5}{36} (2 - 3 \sin[x]^2)^{8/5} - \frac{20}{117} (2 - 3 \sin[x]^2)^{13/5}$$

Problems 93 - 97 (p. 293)

■ Problem #93

$$\int \sqrt{\cos[2x]} \cos[x] dx = \frac{\text{ArcSin}\left[\sqrt{2} \sin[x]\right]}{2\sqrt{2}} + \frac{1}{2} \sin[x] \sqrt{\cos[2x]}$$

■ Problem #94

$$\int \cos[2x]^{3/2} \sin[x] dx = -\frac{3}{8\sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{2} \cos[x]}{\sqrt{\cos[2x]}}\right] + \frac{3}{8} \cos[x] \sqrt{\cos[2x]} - \frac{1}{4} \cos[x] \cos[2x]^{3/2}$$

■ Problem #95

$$\int \frac{\sin[x]}{\cos[2x]^{5/2}} dx = -\frac{\cos[3x]}{3 \cos[2x]^{3/2}}$$

■ Problem #96

$$\int \frac{\cos[2x]^{3/2}}{\cos[x]^3} dx = 2\sqrt{2} \text{ArcSin}\left[\sqrt{2} \sin[x]\right] - \frac{5}{2} \text{ArcTan}\left[\frac{\sin[x]}{\sqrt{\cos[2x]}}\right] - \frac{1}{2} \sec[x] \tan[x] \sqrt{\cos[2x]}$$

■ Problem #97

$$\int \frac{3 \sin[x]^3 - \cos[x] \sin[4x]}{\csc[x]^2 \cos[2x]^{7/2}} dx = -\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \cos[x]}{\sqrt{\cos[2x]}}\right]}{\sqrt{2}} - \frac{11 \cos[x]}{20 \cos[2x]^{3/2}} - \frac{2 \cos[x]^3}{3 \cos[2x]^{3/2}} + \frac{63 \cos[x]}{20 \sqrt{\cos[2x]}} + \frac{3 \cos[x] \sin[x]^2}{10 \cos[2x]^{5/2}}$$

Problems 98 - 103 (p. 297)

■ Problem #98

$$\int (4 - 5 \sec[x]^2)^{3/2} dx = 8 \text{ArcTan}\left[\frac{2 \tan[x]}{\sqrt{-1 - 5 \tan[x]^2}}\right] - \frac{7}{2} \sqrt{5} \text{ArcTan}\left[\frac{\sqrt{5} \tan[x]}{\sqrt{-1 - 5 \tan[x]^2}}\right] - \frac{5}{2} \tan[x] \sqrt{-1 - 5 \tan[x]^2}$$

■ Problem #99

$$\int \frac{1}{(4 - 5 \sec[x]^2)^{3/2}} dx = \frac{1}{8} \text{ArcTan}\left[\frac{2 \tan[x]}{\sqrt{-1 - 5 \tan[x]^2}}\right] - \frac{5 \tan[x]}{4 \sqrt{-1 - 5 \tan[x]^2}}$$

■ **Problem #100**

$$\int \frac{\sin[x] - 2 \cot[x]^2}{(1 + 5 \tan[x]^2)^{3/2}} dx = -\frac{1}{4} \operatorname{ArcTanh}\left[\frac{2 \tan[x]}{\sqrt{1 + 5 \tan[x]^2}}\right] - \frac{\cos[x]}{4 \sqrt{1 + 5 \tan[x]^2}} - \frac{5 \cot[x]}{2 \sqrt{1 + 5 \tan[x]^2}} - \frac{1}{8} \cos[x] \sqrt{1 + 5 \tan[x]^2} + \frac{9}{2} \cot[x] \sqrt{1 + 5 \tan[x]^2}$$

■ **Problem #101**

$$\int \frac{\cos[2x] - 3}{\cos[x]^4 \sqrt{4 - \cot[x]^2}} dx = -\frac{2}{3} \sqrt{4 - \cot[x]^2} \tan[x] - \frac{1}{3} \sqrt{4 - \cot[x]^2} \tan[x]^3$$

■ **Problem #102**

$$\int \frac{(3 + \sin[x]^2) \tan[x]^3}{(\cos[x]^2 - 2) (5 - 4 \sec[x]^2)^{3/2}} dx = -\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{5-4 \sec[x]^2}}{\sqrt{3}}\right]}{6 \sqrt{3}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{5-4 \sec[x]^2}}{\sqrt{5}}\right]}{5 \sqrt{5}} - \frac{2}{15 \sqrt{5 - 4 \sec[x]^2}}$$

■ **Problem #103**

$$\int \frac{\sec[x]^2 - 3 \tan[x] \sqrt{4 \sec[x]^2 + 5 \tan[x]^2}}{\sin[x]^2 (4 \sec[x]^2 + 5 \tan[x]^2)^{3/2}} dx = -\frac{3}{4} \operatorname{Log}[\tan[x]] + \frac{3}{8} \operatorname{Log}[4 + 9 \tan[x]^2] - \frac{\cot[x]}{4 \sqrt{4 + 9 \tan[x]^2}} - \frac{7 \tan[x]}{8 \sqrt{4 + 9 \tan[x]^2}}$$

Problems 104 - 110 (p. 303)

■ **Problem #104**

$$\int (1 + 5 \tan[x]^2)^{5/2} \tan[x] dx = -32 \operatorname{ArcTan}\left[\frac{1}{2} \sqrt{1 + 5 \tan[x]^2}\right] + 16 \sqrt{1 + 5 \tan[x]^2} - \frac{4}{3} (1 + 5 \tan[x]^2)^{3/2} + \frac{1}{5} (1 + 5 \tan[x]^2)^{5/2}$$

■ **Problem #105**

$$\int \frac{\tan[x]}{(1 + 5 \tan[x]^2)^{5/2}} dx = \frac{1}{32} \operatorname{ArcTan}\left[\frac{1}{2} \sqrt{1 + 5 \tan[x]^2}\right] - \frac{1}{12 (1 + 5 \tan[x]^2)^{3/2}} + \frac{1}{16 \sqrt{1 + 5 \tan[x]^2}}$$

■ **Problem #106**

$$\int \frac{\tan[x]}{(a^3 + b^3 \tan[x]^2)^{1/3}} dx = \frac{\sqrt{3}}{2 (a^3 - b^3)^{1/3}} \operatorname{ArcTan}\left[\frac{(a^3 - b^3)^{1/3} + 2 (a^3 + b^3 \tan[x]^2)^{1/3}}{\sqrt{3} (a^3 - b^3)^{1/3}}\right] + \frac{\operatorname{Log}[\cos[x]]}{2 (a^3 - b^3)^{1/3}} + \frac{3 \operatorname{Log}[(a^3 - b^3)^{1/3} - (a^3 + b^3 \tan[x]^2)^{1/3}]}{4 (a^3 - b^3)^{1/3}}$$

■ **Problem #107**

$$\int (1 - 7 \tan[x]^2)^{2/3} \tan[x] \, dx = 2\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + (1 - 7 \tan[x]^2)^{1/3}}{\sqrt{3}}\right] + 2 \operatorname{Log}[\cos[x]] + 3 \operatorname{Log}\left[2 - (1 - 7 \tan[x]^2)^{1/3}\right] + \frac{3}{4} (1 - 7 \tan[x]^2)^{2/3}$$

■ **Problem #108a**

$$\int \frac{\cot[x]}{(a^4 + b^4 \csc[x]^2)^{1/4}} \, dx = -\frac{\operatorname{ArcTan}\left[\frac{(a^4 + b^4 \csc[x]^2)^{1/4}}{a}\right]}{a} + \frac{\operatorname{ArcTanh}\left[\frac{(a^4 + b^4 \csc[x]^2)^{1/4}}{a}\right]}{a}$$

■ **Problem #108b**

$$\int \frac{\cot[x]}{(a^4 - b^4 \csc[x]^2)^{1/4}} \, dx = -\frac{\operatorname{ArcTan}\left[\frac{(a^4 - b^4 \csc[x]^2)^{1/4}}{a}\right]}{a} + \frac{\operatorname{ArcTanh}\left[\frac{(a^4 - b^4 \csc[x]^2)^{1/4}}{a}\right]}{a}$$

■ **Problem #109**

$$\int \frac{(3 \tan[x]^2 + \sin[x]^2 (1 - 3 \sec[x]^2)^{1/3}) \tan[x]}{\cos[x]^2 (1 - 3 \sec[x]^2)^{5/6} (1 - \sqrt{1 - 3 \sec[x]^2})} \, dx = \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2 (1 - 3 \sec[x]^2)^{1/6}}{\sqrt{3}}\right] + \frac{1}{4} \operatorname{Log}[\sec[x]^2] - \frac{3}{2} \operatorname{Log}[1 - (1 - 3 \sec[x]^2)^{1/6}] + \frac{1}{3} \operatorname{Log}\left[1 - \sqrt{1 - 3 \sec[x]^2}\right] - (1 - 3 \sec[x]^2)^{1/6} - \frac{1}{4} (1 - 3 \sec[x]^2)^{2/3} + \frac{1}{2 (1 - \sqrt{1 - 3 \sec[x]^2})}$$

■ **Problem #110**

$$\int \frac{2 \tan[x]^2 - \cos[2x]}{\cos[x]^2 (\tan[x] \tan[2x])^{3/2}} \, dx = 2 \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right] - \frac{11}{4\sqrt{2}} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right] + \frac{\tan[x]}{2 (\tan[x] \tan[2x])^{3/2}} + \frac{2 \tan[x]^3}{3 (\tan[x] \tan[2x])^{3/2}} + \frac{3 \tan[x]}{4 \sqrt{\tan[x] \tan[2x]}}$$

Problems 111 - 113 (p. 305-306)

■ **Problem #111**

$$\int \frac{\tan[x]}{(a^3 - b^3 \cos[x]^n)^{4/3}} \, dx = -\frac{\sqrt{3}}{a^4 n} \operatorname{ArcTan}\left[\frac{a + 2 (a^3 - b^3 \cos[x]^n)^{1/3}}{\sqrt{3} a}\right] - \frac{3}{a^3 n (a^3 - b^3 \cos[x]^n)^{1/3}} + \frac{\operatorname{Log}[\cos[x]]}{2 a^4} - \frac{3 \operatorname{Log}[a - (a^3 - b^3 \cos[x]^n)^{1/3}]}{2 a^4 n}$$

■ **Problem #112**

$$\int (1 + 2 \cos [x]^9)^{5/6} \tan [x] \, dx = \frac{\operatorname{ArcTan} \left[\frac{1 - (1 + 2 \cos [x]^9)^{1/3}}{\sqrt{3} (1 + 2 \cos [x]^9)^{1/6}} \right]}{3 \sqrt{3}} + \frac{1}{3} \operatorname{ArcTanh} \left[(1 + 2 \cos [x]^9)^{1/6} \right] - \frac{1}{9} \operatorname{ArcTanh} \left[\sqrt{1 + 2 \cos [x]^9} \right] - \frac{2}{15} (1 + 2 \cos [x]^9)^{5/6}$$

■ **Problem #113**

$$\int \frac{\sin [x]^9 \cot [x]}{(2 - 5 \sin [x]^3)^{4/3}} \, dx = \frac{4}{125 (2 - 5 \sin [x]^3)^{1/3}} + \frac{2}{125} (2 - 5 \sin [x]^3)^{2/3} - \frac{1}{625} (2 - 5 \sin [x]^3)^{5/3}$$

Problems 114 - 120 (p. 308-309)

■ **Problem #114**

$$\int \frac{(1 + (1 - 8 \tan [x]^2)^{1/3}) \tan [x]}{\cos [x]^2 (1 - 8 \tan [x]^2)^{2/3}} \, dx = -\frac{3}{32} (1 + (1 - 8 \tan [x]^2)^{1/3})^2$$

■ **Problem #115**

$$\int \frac{(1 + (1 - 8 \tan [x]^2)^{1/3}) \cot [x]}{\cos [x]^2 (1 - 8 \tan [x]^2)^{2/3}} \, dx = -\operatorname{Log} [\tan [x]] + \frac{3}{2} \operatorname{Log} [1 - (1 - 8 \tan [x]^2)^{1/3}]$$

■ **Problem #116**

$$\int \frac{(5 \cos [x]^2 - \sqrt{5 \sin [x]^2 - 1}) \tan [x]}{(5 \sin [x]^2 - 1)^{1/4} (2 + \sqrt{5 \sin [x]^2 - 1})} \, dx =$$

$$-\frac{3}{\sqrt{2}} \operatorname{ArcTan} \left[\frac{(-1 + 5 \sin [x]^2)^{1/4}}{\sqrt{2}} \right] - \frac{1}{2 \sqrt{2}} \operatorname{ArcTanh} \left[\frac{(-1 + 5 \sin [x]^2)^{1/4}}{\sqrt{2}} \right] + 2 (-1 + 5 \sin [x]^2)^{1/4} - \frac{(-1 + 5 \sin [x]^2)^{1/4}}{2 (2 + \sqrt{-1 + 5 \sin [x]^2})}$$

■ **Problem #117**

$$\int \cos [x]^4 \cos [2x]^{2/3} \tan [x] \, dx = -\frac{3}{40} \cos [2x]^{5/3} - \frac{3}{64} \cos [2x]^{8/3}$$

■ **Problem #118**

$$\int \frac{\sin [x]^6 \tan [x]}{\cos [2x]^{3/4}} \, dx = \frac{\operatorname{ArcTan} \left[\frac{1 - \sqrt{\cos [2x]}}{\sqrt{2} \cos [2x]^{1/4}} \right]}{\sqrt{2}} - \frac{\operatorname{ArcTanh} \left[\frac{1 + \sqrt{\cos [2x]}}{\sqrt{2} \cos [2x]^{1/4}} \right]}{\sqrt{2}} + \frac{71}{45} \cos [2x]^{1/4} + \frac{13}{45} \cos [2x]^{1/4} \sin [x]^2 + \frac{1}{9} \cos [2x]^{1/4} \sin [x]^4$$

■ **Problem #119**

$$\int \sqrt{\tan[x] \tan[2x]} \, dx = -\operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{\tan[x] \tan[2x]}}\right]$$

■ **Problem #120**

$$\int \sqrt{\frac{\cot[2x]}{\cot[x]}} \, dx = -\frac{\operatorname{ArcSin}[\tan[x]]}{\sqrt{2}} + \operatorname{ArcTan}\left[\frac{\sqrt{2} \tan[x]}{\sqrt{1 - \tan[x]^2}}\right]$$

Chapter 6 (p. 310-330)

Rational and Algebraic Function Integration Problems

Problems 1 - 5 (p. 314)

■ Problem #1

$$\int \frac{1}{x^5 (5 + x^2)} dx = -\frac{1}{20 x^4} + \frac{1}{50 x^2} + \frac{\text{Log}[x]}{125} - \frac{1}{250} \text{Log}[5 + x^2]$$

■ Problem #2

$$\int \frac{1}{x^6 (5 + x^2)} dx = -\frac{1}{25 x^5} + \frac{1}{75 x^3} - \frac{1}{125 x} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{5}}\right]}{125 \sqrt{5}}$$

■ Problem #3

$$\int \frac{1}{x (x^2 - 4)^4} dx = \frac{1}{24 (4 - x^2)^3} + \frac{1}{64 (4 - x^2)^2} + \frac{1}{128 (4 - x^2)} + \frac{\text{Log}[x]}{256} - \frac{1}{512} \text{Log}[4 - x^2]$$

■ Problem #4

$$\int \frac{1}{x (x^2 - 2)^{5/2}} dx = -\frac{1}{6 (x^2 - 2)^{3/2}} + \frac{1}{4 \sqrt{x^2 - 2}} + \frac{\text{ArcTan}\left[\frac{\sqrt{x^2 - 2}}{\sqrt{2}}\right]}{4 \sqrt{2}}$$

■ Problem #5

$$\int \frac{(x^2 - 10)^{5/2}}{x} dx = 100 \sqrt{x^2 - 10} - \frac{10}{3} (x^2 - 10)^{3/2} + \frac{1}{5} (x^2 - 10)^{5/2} - 100 \sqrt{10} \text{ArcTan}\left[\frac{\sqrt{x^2 - 10}}{\sqrt{10}}\right]$$

Problems 6 - 21 (p. 327-328)

■ Problem #6

$$\int x^{2n+1} dx = \frac{x^{2(n+1)}}{2(n+1)}$$

■ **Problem #7**

$$\int \frac{x^7}{(x^2 - 5)^3} dx = \frac{x^2}{2} - \frac{125}{4(5 - x^2)^2} + \frac{75}{2(5 - x^2)} + \frac{15}{2} \text{Log}[5 - x^2]$$

■ **Problem #8**

$$\int \frac{3x^5 - 4x^3}{(x^2 - 1)^5} dx = \frac{1}{8(1 - x^2)^4} + \frac{1}{3(1 - x^2)^3} - \frac{3}{4(1 - x^2)^2}$$

■ **Problem #9**

$$\int (1 + x^2)^{9/14} x^3 dx = -\frac{7}{23} (1 + x^2)^{23/14} + \frac{7}{37} (1 + x^2)^{37/14}$$

■ **Problem #10**

$$\int \frac{x^5}{(x^2 - 4)^{13/6}} dx = -\frac{48}{7(x^2 - 4)^{7/6}} - \frac{24}{(x^2 - 4)^{1/6}} + \frac{3}{5} (x^2 - 4)^{5/6}$$

■ **Problem #11**

$$\int \frac{1}{(1 + 2x^2)^{5/2}} dx = \frac{x}{3(1 + 2x^2)^{3/2}} + \frac{2x}{3\sqrt{1 + 2x^2}}$$

■ **Problem #12**

$$\int \frac{1}{(x^2 - 2x - 1)^{5/2}} dx = \frac{1 - x}{6(x^2 - 2x - 1)^{3/2}} - \frac{1 - x}{6\sqrt{x^2 - 2x - 1}}$$

■ **Problem #13**

$$\int \frac{1}{x^4(x^2 - 8)^{3/2}} dx = \frac{1}{24x^3\sqrt{x^2 - 8}} + \frac{1}{48x\sqrt{x^2 - 8}} - \frac{x}{192\sqrt{x^2 - 8}}$$

■ **Problem #14**

$$\int \frac{(x^2 + 5)^2}{x^4 x^{1/3}} dx = -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2}$$

■ **Problem #15**

$$\int \frac{1}{x^7(1 + x^2)^3} dx = -\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{1}{4(1 + x^2)^2} - \frac{2}{1 + x^2} - 10 \text{Log}[x] + 5 \text{Log}[1 + x^2]$$

■ Problem #16

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}} dx = -\frac{9\left(\frac{2+x^2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

■ Problem #17

$$\int \frac{x^4}{(\sqrt{10-x^2})^{9/2}} dx = \frac{x^5}{7\sqrt{10}(\sqrt{10-x^2})^{7/2}} + \frac{x^5}{175(\sqrt{10-x^2})^{5/2}}$$

■ Problem #18

$$\int \frac{x^2}{(3-x^2)^{3/2}} dx = \frac{x}{\sqrt{3-x^2}} - \text{ArcSin}\left[\frac{x}{\sqrt{3}}\right]$$

■ Problem #19

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \text{ArcSin}\left[\frac{x}{5}\right]$$

■ Problem #20

$$\int \frac{1}{(1-2x^2)^{7/2}} dx = \frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8x}{15\sqrt{1-2x^2}}$$

■ Problem #21

$$\int \frac{1}{(-x^2+6x-7)^{5/2}} dx = -\frac{3-x}{6(-x^2+6x-7)^{3/2}} - \frac{3-x}{6\sqrt{-x^2+6x-7}}$$

Problems 22 - 25 (p. 329)

■ Problem #22

$$\int (-2x^2 - 2x + 1)^3 dx = x - 3x^2 + 2x^3 + 4x^4 - \frac{12x^5}{5} - 4x^6 - \frac{8x^7}{7}$$

■ Problem #23

$$\int (x^2 - x - 1)^2 (5x - 1) dx = -x + \frac{3x^2}{2} + \frac{11x^3}{3} - \frac{3x^4}{4} - \frac{11x^5}{5} + \frac{5x^6}{6}$$

■ Problem #24

$$\int \frac{3x + 1}{(2x^2 - 8x + 1)^{5/2}} dx = \frac{1 - 2x}{6(2x^2 - 8x + 1)^{3/2}} - \frac{2(2 - x)}{21\sqrt{2x^2 - 8x + 1}}$$

■ Problem #25

$$\int \frac{8x^3 - 8x - 1}{(1 + 2x - 4x^2)^{5/2}} dx = -\frac{23 + 38x - 60x^2}{30(1 + 2x - 4x^2)^{3/2}} + \frac{61(1 - 4x)}{150\sqrt{1 + 2x - 4x^2}}$$

Chapter 7 (p. 331-344)

Trigonometric Function Integration Problems

Problems 1 - 4 (p. 334)

■ **Problem #1**

$$\int x^2 \cos[x]^5 dx =$$

$$\frac{16}{15} x \cos[x] + \frac{8}{45} x \cos[x]^3 + \frac{2}{25} x \cos[x]^5 - \frac{298 \sin[x]}{225} + \frac{8}{15} x^2 \sin[x] + \frac{4}{15} x^2 \cos[x]^2 \sin[x] + \frac{1}{5} x^2 \cos[x]^4 \sin[x] + \frac{76 \sin[x]^3}{675} - \frac{2 \sin[x]^5}{125}$$

■ **Problem #2**

$$\int x^3 \sin[x]^3 dx = \frac{40}{9} x \cos[x] - \frac{2}{3} x^3 \cos[x] - \frac{40 \sin[x]}{9} + 2 x^2 \sin[x] + \frac{2}{9} x \cos[x] \sin[x]^2 - \frac{1}{3} x^3 \cos[x] \sin[x]^2 - \frac{2 \sin[x]^3}{27} + \frac{1}{3} x^2 \sin[x]^3$$

■ **Problem #3**

$$\int x^2 \sin[x]^6 dx = -\frac{245 x}{1152} + \frac{5 x^3}{48} + \frac{245 \cos[x] \sin[x]}{1152} - \frac{5}{16} x^2 \cos[x] \sin[x] + \frac{5}{16} x \sin[x]^2 + \frac{65 \cos[x] \sin[x]^3}{1728} - \frac{5}{24} x^2 \cos[x] \sin[x]^3 + \frac{5}{48} x \sin[x]^4 + \frac{1}{108} \cos[x] \sin[x]^5 - \frac{1}{6} x^2 \cos[x] \sin[x]^5 + \frac{1}{18} x \sin[x]^6$$

■ **Problem #4**

$$\int x^2 \sin[x]^2 \cos[x] dx = \frac{4}{9} x \cos[x] - \frac{4 \sin[x]}{9} + \frac{2}{9} x \cos[x] \sin[x]^2 - \frac{2 \sin[x]^3}{27} + \frac{1}{3} x^2 \sin[x]^3$$

Problems 5 - 9 (p. 342-343)

■ **Problem #5**

$$\int \frac{x \cos[x]^4}{\sin[x]^2} dx = -\frac{3 x^2}{4} - \frac{\cos[x]^2}{4} - x \cot[x] + \log[\sin[x]] - \frac{1}{2} x \cos[x] \sin[x]$$

■ **Problem #6**

$$\int \frac{x \sin[x]^3}{\cos[x]^4} dx = \frac{5}{6} \operatorname{ArcTanh}[\sin[x]] - x \sec[x] + \frac{1}{3} x \sec[x]^3 - \frac{1}{6} \sec[x] \tan[x]$$

■ **Problem #7**

$$\int \frac{x \sin[x]}{\cos[x]^3} dx = \frac{1}{2} x \sec[x]^2 - \frac{\tan[x]}{2}$$

■ **Problem #8**

$$\int \frac{x \sin[x]^3}{\cos[x]} dx = \frac{x}{4} + \frac{i x^2}{2} - x \operatorname{Log}[1 + e^{2ix}] + \frac{1}{2} i \operatorname{PolyLog}[2, -e^{2ix}] - \frac{1}{4} \cos[x] \sin[x] - \frac{1}{2} x \sin[x]^2$$

■ **Problem #9**

$$\int \frac{x \sin[x]^3}{\cos[x]^3} dx = \frac{x}{2} - \frac{i x^2}{2} + x \operatorname{Log}[1 + e^{2ix}] - \frac{1}{2} i \operatorname{PolyLog}[2, -e^{2ix}] - \frac{\tan[x]}{2} + \frac{1}{2} x \tan[x]^2$$

Problems 10 - 11 (p. 344)

■ **Problem #10**

$$\int \frac{2x + \sin[2x]}{(x \sin[x] + \cos[x])^2} dx = \frac{2}{1 + \frac{\cot[x]}{x}}$$

■ **Problem #11**

$$\int \left(\frac{x}{x \cos[x] - \sin[x]} \right)^2 dx = \frac{\cos[x] + x \sin[x]}{x \cos[x] - \sin[x]}$$

Chapter 8 (p. 345-378)

Exponential, Hyperbolic and Logarithmic Function Integration Problems

Problems 1 - 5 (p. 346)

■ **Problem #1**

$$\int a^{m x} b^{n x} dx = \frac{a^{m x} b^{n x}}{m \operatorname{Log}[a] + n \operatorname{Log}[b]}$$

■ **Problem #2**

$$\int \frac{(a^x - b^x)^2}{a^x b^x} dx = -2x + \frac{\frac{a^x}{b^x} - \frac{b^x}{a^x}}{\operatorname{Log}[a] - \operatorname{Log}[b]}$$

■ **Problem #3.1**

$$\int (e^x - e^{-x})^1 dx = e^x + e^{-x}$$

■ **Problem #3.2**

$$\int (e^x - e^{-x})^2 dx = -\frac{1}{2 e^{2x}} + \frac{e^{2x}}{2} - 2x$$

■ **Problem #3.3**

$$\int (e^x - e^{-x})^3 dx = \frac{1}{3 e^{3x}} - \frac{3}{e^x} - 3e^x + \frac{e^{3x}}{3}$$

■ **Problem #3.4**

$$\int (e^x - e^{-x})^4 dx = -\frac{1}{4 e^{4x}} + \frac{2}{e^{2x}} - 2e^{2x} + \frac{e^{4x}}{4} + 6x$$

■ **Problem #3.n**

$$\int (e^x - e^{-x})^n dx = -\frac{(-e^{-x} + e^x)^n (1 - e^{2x})}{n} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 1 - \frac{n}{2}, e^{2x}\right]$$

■ **Problem #4**

$$\int (a^{-4x} - a^{2x})^3 dx = 3x - \frac{1}{a^{12x} (12 \operatorname{Log}[a])} + \frac{1}{a^{6x} (2 \operatorname{Log}[a])} - \frac{a^{6x}}{6 \operatorname{Log}[a]}$$

■ **Problem #5a.1**

$$\int (a^{kx} + a^{lx})^1 dx = \frac{a^{kx}}{k \operatorname{Log}[a]} + \frac{a^{lx}}{l \operatorname{Log}[a]}$$

■ **Problem #5a.2**

$$\int (a^{kx} + a^{lx})^2 dx = \frac{a^{2kx}}{2k \operatorname{Log}[a]} + \frac{a^{2lx}}{2l \operatorname{Log}[a]} + \frac{2a^{(k+l)x}}{(k+l) \operatorname{Log}[a]}$$

■ **Problem #5a.3**

$$\int (a^{kx} + a^{lx})^3 dx = \frac{a^{3kx}}{3k \operatorname{Log}[a]} + \frac{a^{3lx}}{3l \operatorname{Log}[a]} + \frac{3a^{(2k+l)x}}{(2k+l) \operatorname{Log}[a]} + \frac{3a^{(k+2l)x}}{(k+2l) \operatorname{Log}[a]}$$

■ **Problem #5a.4**

$$\int (a^{kx} + a^{lx})^4 dx = \frac{a^{4kx}}{4k \operatorname{Log}[a]} + \frac{a^{4lx}}{4l \operatorname{Log}[a]} + \frac{3a^{2(k+l)x}}{(k+l) \operatorname{Log}[a]} + \frac{4a^{(3k+l)x}}{(3k+l) \operatorname{Log}[a]} + \frac{4a^{(k+3l)x}}{(k+3l) \operatorname{Log}[a]}$$

■ **Problem #5a.n**

$$\int (a^{kx} + a^{lx})^n dx = \frac{(1 + a^{(k-l)x}) (a^{kx} + a^{lx})^n}{ln \operatorname{Log}[a]} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{kn}{k-l}, 1 + \frac{ln}{k-l}, -a^{(k-l)x}\right]$$

■ **Problem #5b.1**

$$\int (a^{kx} - a^{lx})^1 dx = \frac{a^{kx}}{k \operatorname{Log}[a]} - \frac{a^{lx}}{l \operatorname{Log}[a]}$$

■ **Problem #5b.2**

$$\int (a^{kx} - a^{lx})^2 dx = \frac{a^{2kx}}{2k \operatorname{Log}[a]} + \frac{a^{2lx}}{2l \operatorname{Log}[a]} - \frac{2a^{(k+l)x}}{(k+l) \operatorname{Log}[a]}$$

■ **Problem #5b.3**

$$\int (a^{kx} - a^{lx})^3 dx = \frac{a^{3kx}}{3k \operatorname{Log}[a]} - \frac{a^{3lx}}{3l \operatorname{Log}[a]} - \frac{3a^{(2k+l)x}}{(2k+l) \operatorname{Log}[a]} + \frac{3a^{(k+2l)x}}{(k+2l) \operatorname{Log}[a]}$$

■ **Problem #5b.4**

$$\int (a^{kx} - a^{lx})^4 dx = \frac{a^{4kx}}{4k \operatorname{Log}[a]} + \frac{a^{4lx}}{4l \operatorname{Log}[a]} + \frac{3a^{2(k+l)x}}{(k+l) \operatorname{Log}[a]} - \frac{4a^{(3k+l)x}}{(3k+l) \operatorname{Log}[a]} - \frac{4a^{(k+3l)x}}{(k+3l) \operatorname{Log}[a]}$$

■ **Problem #5b.n**

$$\int (a^{kx} - a^{lx})^n dx = \frac{(1 - a^{(k-l)x}) (a^{kx} - a^{lx})^n}{ln \operatorname{Log}[a]} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{kn}{k-l}, 1 + \frac{ln}{k-l}, a^{(k-l)x}\right]$$

Problems 6 - 9 (p. 346)

■ Problem #6a.1

$$\int (1 + a^{mx})^{-1} dx = x + \frac{a^{mx}}{m \operatorname{Log}[a]}$$

■ Problem #6a.2

$$\int (1 + a^{mx})^{-2} dx = x + \frac{2 a^{mx}}{m \operatorname{Log}[a]} + \frac{a^{2mx}}{2 m \operatorname{Log}[a]}$$

■ Problem #6a.3

$$\int (1 + a^{mx})^{-3} dx = x + \frac{3 a^{mx}}{m \operatorname{Log}[a]} + \frac{3 a^{2mx}}{2 m \operatorname{Log}[a]} + \frac{a^{3mx}}{3 m \operatorname{Log}[a]}$$

■ Problem #6a.4

$$\int (1 + a^{mx})^{-4} dx = x + \frac{4 a^{mx}}{m \operatorname{Log}[a]} + \frac{3 a^{2mx}}{m \operatorname{Log}[a]} + \frac{4 a^{3mx}}{3 m \operatorname{Log}[a]} + \frac{a^{4mx}}{4 m \operatorname{Log}[a]}$$

■ Problem #6a.n

$$\int (1 + a^{mx})^{-n} dx = - \frac{(1 + a^{mx})^{n+1} \operatorname{Hypergeometric2F1}[1, n+1, n+2, 1 + a^{mx}]}{m (n+1) \operatorname{Log}[a]}$$

■ Problem #6b.1

$$\int (1 - a^{mx})^{-1} dx = x - \frac{a^{mx}}{m \operatorname{Log}[a]}$$

■ Problem #6b.2

$$\int (1 - a^{mx})^{-2} dx = x - \frac{2 a^{mx}}{m \operatorname{Log}[a]} + \frac{a^{2mx}}{2 m \operatorname{Log}[a]}$$

■ Problem #6b.3

$$\int (1 - a^{mx})^{-3} dx = x - \frac{3 a^{mx}}{m \operatorname{Log}[a]} + \frac{3 a^{2mx}}{2 m \operatorname{Log}[a]} - \frac{a^{3mx}}{3 m \operatorname{Log}[a]}$$

■ Problem #6b.4

$$\int (1 - a^{mx})^{-4} dx = x - \frac{4 a^{mx}}{m \operatorname{Log}[a]} + \frac{3 a^{2mx}}{m \operatorname{Log}[a]} - \frac{4 a^{3mx}}{3 m \operatorname{Log}[a]} + \frac{a^{4mx}}{4 m \operatorname{Log}[a]}$$

■ Problem #6b.n

$$\int (1 - a^{mx})^{-n} dx = - \frac{(1 - a^{mx})^{n+1} \operatorname{Hypergeometric2F1}[1, n+1, n+2, 1 - a^{mx}]}{m (n+1) \operatorname{Log}[a]}$$

■ **Problem #7**

$$\int \frac{1}{a e^{nx} + b} dx = \frac{x}{b} - \frac{\text{Log}[b + a e^{nx}]}{bn}$$

■ **Problem #8**

$$\int \frac{e^x}{a e^{3x} + b} dx = -\frac{\text{ArcTan}\left[\frac{b^{1/3} - 2 a^{1/3} e^x}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} a^{1/3} b^{2/3}} + \frac{\text{Log}[b^{1/3} + a^{1/3} e^x]}{2 a^{1/3} b^{2/3}} - \frac{\text{Log}[b + a e^{3x}]}{6 a^{1/3} b^{2/3}}$$

■ **Problem #9**

$$\int \frac{e^x - 1}{e^x + 1} dx = -x + 2 \text{Log}[1 + e^x]$$

Problems 10 - 16 (p. 347)

■ **Problem #10**

$$\int \frac{e^{4x}}{3 e^{4x} - 2 e^{2x} + 1} dx = -\frac{\text{ArcTan}\left[\frac{1-3 e^{2x}}{\sqrt{2}}\right]}{6 \sqrt{2}} + \frac{1}{12} \text{Log}[1 - 2 e^{2x} + 3 e^{4x}]$$

■ **Problem #11**

$$\int \frac{e^{5x} + e^x}{e^{3x} - e^{2x} + e^x - 1} dx = e^x + \frac{e^{2x}}{2} - \text{ArcTan}[e^x] + \text{Log}[1 - e^x] - \frac{1}{2} \text{Log}[1 + e^{2x}]$$

■ **Problem #12**

$$\int (a + b e^{nx})^{r/s} e^{nx} dx = \frac{s (a + b e^{nx})^{\frac{s+r}{s}}}{bn (s+r)}$$

■ **Problem #13**

$$\int (1 - 2 e^{x/3})^{1/4} dx = 12 (1 - 2 e^{x/3})^{1/4} - 6 \text{ArcTan}[(1 - 2 e^{x/3})^{1/4}] - 6 \text{ArcTanh}[(1 - 2 e^{x/3})^{1/4}]$$

■ **Problem #14**

$$\int (a + b e^{nx})^{r/s} dx = -\frac{(a + b e^{nx})^{\frac{r+s}{s}} s \text{Hypergeometric2F1}\left[1, \frac{r+s}{s}, 2 + \frac{r}{s}, \frac{a+b e^{nx}}{a}\right]}{an (r+s)}$$

■ **Problem #15a**

$$\int \frac{e^x}{\sqrt{e^{2x} + a^2}} dx = \text{ArcTanh}\left[\frac{e^x}{\sqrt{a^2 + e^{2x}}}\right]$$

■ **Problem #15b**

$$\int \frac{e^x}{\sqrt{e^{2x} - a^2}} dx = \text{ArcTanh} \left[\frac{e^x}{\sqrt{-a^2 + e^{2x}}} \right]$$

■ **Problem #16**

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}} - 2\right) \sqrt{e^{\frac{3x}{2}} + e^{\frac{3x}{4}} - 2}} dx = \frac{2}{3} \text{ArcTanh} \left[\frac{2 - 5 e^{\frac{3x}{4}}}{4 \sqrt{-2 + e^{\frac{3x}{4}} + e^{\frac{3x}{2}}}} \right]$$

Problems 17 - 18 (p. 348)

■ **Problem #17**

$$\int \frac{(e^{7x} - 3)^{2/3}}{e^{2x}} dx = \frac{1}{6} e^{-2x} (e^{7x} - 3)^{5/3} \text{Hypergeometric2F1} \left[1, \frac{29}{21}, \frac{5}{7}, \frac{e^{7x}}{3} \right]$$

■ **Problem #18**

$$\int \frac{e^{2x}}{(3 - e^{x/2})^{3/4}} dx = -216 (3 - e^{x/2})^{1/4} + \frac{216}{5} (3 - e^{x/2})^{5/4} - 8 (3 - e^{x/2})^{9/4} + \frac{8}{13} (3 - e^{x/2})^{13/4}$$

Problems 19 - 24 (p. 351)

■ **Problem #19**

$$\int \frac{x^3}{e^{x/2}} dx = -\frac{96}{e^{x/2}} - \frac{48x}{e^{x/2}} - \frac{12x^2}{e^{x/2}} - \frac{2x^3}{e^{x/2}}$$

■ **Problem #20**

$$\int \frac{1}{x^3 e^{x/2}} dx = -\frac{1}{e^{x/2} (2x^2)} + \frac{1}{e^{x/2} (4x)} + \frac{1}{8} \text{ExpIntegralEi} \left[-\frac{x}{2} \right]$$

■ **Problem #21**

$$\int x^2 a^{3x} dx = \frac{2 a^{3x}}{27 \text{Log}[a]^3} - \frac{2 a^{3x} x}{9 \text{Log}[a]^2} + \frac{a^{3x} x^2}{3 \text{Log}[a]}$$

■ **Problem #22**

$$\int x (x^2 + 1) e^{x^2} dx = \frac{1}{2} e^{x^2} x^2$$

■ **Problem #23**

$$\int \frac{x}{(e^x + e^{-x})^2} dx = \frac{x}{2} - \frac{x}{2(1 + e^{2x})} - \frac{1}{4} \operatorname{Log}[1 + e^{2x}]$$

■ **Problem #24**

$$\int \frac{(1 - x - x^2) e^x}{\sqrt{1 - x^2}} dx = e^x \sqrt{1 - x^2}$$

Problems 25 - 32 (p. 353)

■ **Problem #25**

$$\int \frac{\operatorname{Cos}[2x]}{e^{3x}} dx = -\frac{3 \operatorname{Cos}[2x]}{13 e^{3x}} + \frac{2 \operatorname{Sin}[2x]}{13 e^{3x}}$$

■ **Problem #26**

$$\int \frac{\operatorname{Sin}\left[\frac{x}{2}\right] + \operatorname{Cos}\left[\frac{x}{2}\right]}{(e^x)^{1/3}} dx = -\frac{30 \operatorname{Cos}\left[\frac{x}{2}\right]}{13 (e^x)^{1/3}} + \frac{6 \operatorname{Sin}\left[\frac{x}{2}\right]}{13 (e^x)^{1/3}}$$

■ **Problem #27**

$$\int \frac{\operatorname{Cos}\left[\frac{3x}{2}\right]}{(3^{3x})^{1/4}} dx = -\frac{4 \operatorname{Cos}\left[\frac{3x}{2}\right] \operatorname{Log}[3]}{3 (3^{3x})^{1/4} (4 + \operatorname{Log}[3]^2)} + \frac{8 \operatorname{Sin}\left[\frac{3x}{2}\right]}{3 (3^{3x})^{1/4} (4 + \operatorname{Log}[3]^2)}$$

■ **Problem #28**

$$\int e^{mx} \operatorname{Cos}[x]^2 dx = \frac{2 e^{mx}}{m(4 + m^2)} + \frac{e^{mx} m \operatorname{Cos}[x]^2}{4 + m^2} + \frac{2 e^{mx} \operatorname{Cos}[x] \operatorname{Sin}[x]}{4 + m^2}$$

■ **Problem #29**

$$\int e^{mx} \operatorname{Sin}[x]^3 dx = -\frac{6 e^{mx} \operatorname{Cos}[x]}{9 + 10m^2 + m^4} + \frac{6 e^{mx} m \operatorname{Sin}[x]}{9 + 10m^2 + m^4} - \frac{3 e^{mx} \operatorname{Cos}[x] \operatorname{Sin}[x]^2}{9 + m^2} + \frac{e^{mx} m \operatorname{Sin}[x]^3}{9 + m^2}$$

■ **Problem #30**

$$\int \frac{\operatorname{Cos}\left[\frac{x}{3}\right]^3}{\sqrt{e^x}} dx = -\frac{48 \operatorname{Cos}\left[\frac{x}{3}\right]}{65 \sqrt{e^x}} - \frac{2 \operatorname{Cos}\left[\frac{x}{3}\right]^3}{5 \sqrt{e^x}} + \frac{32 \operatorname{Sin}\left[\frac{x}{3}\right]}{65 \sqrt{e^x}} + \frac{4 \operatorname{Cos}\left[\frac{x}{3}\right]^2 \operatorname{Sin}\left[\frac{x}{3}\right]}{5 \sqrt{e^x}}$$

■ **Problem #31**

$$\int e^{2x} \operatorname{Sin}[x]^2 \operatorname{Cos}[x]^2 dx = \frac{e^{2x}}{16} - \frac{1}{80} e^{2x} \operatorname{Cos}[4x] - \frac{1}{40} e^{2x} \operatorname{Sin}[4x]$$

■ **Problem #32**

$$\int e^{3x} \sin\left[\frac{3x}{2}\right]^2 \cos\left[\frac{3x}{2}\right]^2 dx = \frac{e^{3x}}{24} - \frac{1}{120} e^{3x} \cos[6x] - \frac{1}{60} e^{3x} \sin[6x]$$

Problems 33 - 36 (p. 355)

■ **Problem #33**

$$\int e^{mx} \tan[x]^2 dx = -\frac{e^{mx}}{m} + \frac{4 e^{(2i+m)x} \text{Hypergeometric2F1}\left[2, 1 - \frac{im}{2}, 2 - \frac{im}{2}, -e^{2ix}\right]}{2i + m}$$

■ **Problem #34**

$$\int \frac{e^{mx}}{\sin[x]^2} dx = -\frac{4 e^{(2i+m)x} \text{Hypergeometric2F1}\left[2, 1 - \frac{im}{2}, 2 - \frac{im}{2}, e^{2ix}\right]}{2i + m}$$

■ **Problem #35**

$$\int \frac{e^{mx}}{\cos[x]^3} dx = \frac{8 e^{(3i+m)x}}{3i + m} \text{Hypergeometric2F1}\left[3, \frac{3 - im}{2}, \frac{5 - im}{2}, -e^{2ix}\right]$$

■ **Problem #36a**

$$\int \frac{e^x}{1 + \cos[x]} dx = (1 - i) e^{(1+i)x} \text{Hypergeometric2F1}\left[2, 1 - i, 2 - i, -e^{ix}\right]$$

■ **Problem #36b**

$$\int \frac{e^x}{1 - \cos[x]} dx = -(1 - i) e^{(1+i)x} \text{Hypergeometric2F1}\left[2, 1 - i, 2 - i, e^{ix}\right]$$

■ **Problem #36c**

$$\int \frac{e^x}{1 + \sin[x]} dx = -(1 - i) e^{x + \frac{1}{2}i(\pi + 2x)} \text{Hypergeometric2F1}\left[1 - i, 2, 2 - i, e^{\frac{1}{2}i(\pi + 2x)}\right]$$

■ **Problem #36d**

$$\int \frac{e^x}{1 - \sin[x]} dx = (1 - i) e^{x + \frac{1}{2}i(\pi + 2x)} \text{Hypergeometric2F1}\left[1 - i, 2, 2 - i, -e^{\frac{1}{2}i(\pi + 2x)}\right]$$

Problems 37 - 44 (p. 356)

■ Problem #37

$$\int \frac{e^x (1 - \sin[x])}{1 - \cos[x]} dx = -\frac{e^x \sin[x]}{1 - \cos[x]}$$

■ Problem #38

$$\int \frac{e^x (1 + \sin[x])}{1 - \cos[x]} dx = \frac{e^x \sin[x]}{1 - \cos[x]} - 2 (1 - i) e^{(1+i)x} \text{Hypergeometric2F1}[2, 1 - i, 2 - i, e^{ix}]$$

■ Problem #39

$$\int \frac{e^x (1 + \sin[x])}{1 + \cos[x]} dx = \frac{e^x \sin[x]}{1 + \cos[x]}$$

■ Problem #40

$$\int \frac{e^x (1 - \sin[x])}{1 + \cos[x]} dx = -\frac{e^x \sin[x]}{1 + \cos[x]} + 2 (1 - i) e^{(1+i)x} \text{Hypergeometric2F1}[2, 1 - i, 2 - i, -e^{ix}]$$

■ Problem #41

$$\int \frac{e^x (1 - \cos[x])}{1 - \sin[x]} dx = -\frac{e^x \cos[x]}{1 - \sin[x]} + 2 (1 + i) e^{(1+i)x} \text{Hypergeometric2F1}[2, 1 - i, 2 - i, -i e^{ix}]$$

■ Problem #42

$$\int \frac{e^x (1 + \cos[x])}{1 - \sin[x]} dx = \frac{e^x \cos[x]}{1 - \sin[x]}$$

■ Problem #43

$$\int \frac{e^x (1 + \cos[x])}{1 + \sin[x]} dx = \frac{e^x \cos[x]}{1 + \sin[x]} - 2 (1 + i) e^{(1+i)x} \text{Hypergeometric2F1}[2, 1 - i, 2 - i, i e^{ix}]$$

■ Problem #44

$$\int \frac{e^x (1 - \cos[x])}{1 + \sin[x]} dx = -\frac{e^x \cos[x]}{1 + \sin[x]}$$

Problems 45 - 49 (p. 357-358)

■ Problem #45

$$\int x e^x \cos[x] dx = \frac{1}{2} e^x x \cos[x] - \frac{1}{2} e^x \sin[x] + \frac{1}{2} e^x x \sin[x]$$

■ **Problem #46**

$$\int x^2 e^x \sin[x] \, dx = -\frac{1}{2} e^x \cos[x] + e^x x \cos[x] - \frac{1}{2} e^x x^2 \cos[x] - \frac{1}{2} e^x \sin[x] + \frac{1}{2} e^x x^2 \sin[x]$$

■ **Problem #47**

$$\int \frac{x^2 \sin[x]}{e^{3x}} \, dx = -\frac{13 \cos[x]}{250 e^{3x}} - \frac{3 x \cos[x]}{25 e^{3x}} - \frac{x^2 \cos[x]}{10 e^{3x}} - \frac{9 \sin[x]}{250 e^{3x}} - \frac{4 x \sin[x]}{25 e^{3x}} - \frac{3 x^2 \sin[x]}{10 e^{3x}}$$

■ **Problem #48**

$$\int e^{x/2} x^2 \cos[x]^3 \, dx = \frac{1}{125} (-132) e^{x/2} \cos[x] + \frac{18}{25} e^{x/2} x \cos[x] + \frac{48}{185} e^{x/2} x^2 \cos[x] + \frac{2}{37} e^{x/2} x^2 \cos[x]^3 - \frac{428 e^{x/2} \cos[3x]}{50 \, 653} + \frac{70 e^{x/2} x \cos[3x]}{1369} - \frac{24}{125} e^{x/2} \sin[x] - \frac{24}{25} e^{x/2} x \sin[x] + \frac{96}{185} e^{x/2} x^2 \sin[x] + \frac{12}{37} e^{x/2} x^2 \cos[x]^2 \sin[x] - \frac{792 e^{x/2} \sin[3x]}{50 \, 653} - \frac{24 e^{x/2} x \sin[3x]}{1369}$$

■ **Problem #49**

$$\int e^{2x} x^2 \sin[4x] \, dx = \frac{1}{250} e^{2x} \cos[4x] + \frac{2}{25} e^{2x} x \cos[4x] - \frac{1}{5} e^{2x} x^2 \cos[4x] - \frac{11}{500} e^{2x} \sin[4x] + \frac{3}{50} e^{2x} x \sin[4x] + \frac{1}{10} e^{2x} x^2 \sin[4x]$$

Problems 50 (p. 359)

■ **Problem #50**

$$\int e^{x/2} x^2 \sin[x]^2 \cos[x] \, dx = \frac{1}{125} (-44) e^{x/2} \cos[x] + \frac{6}{25} e^{x/2} x \cos[x] + \frac{1}{10} e^{x/2} x^2 \cos[x] + \frac{428 e^{x/2} \cos[3x]}{50 \, 653} - \frac{70 e^{x/2} x \cos[3x]}{1369} - \frac{1}{74} e^{x/2} x^2 \cos[3x] - \frac{8}{125} e^{x/2} \sin[x] - \frac{8}{25} e^{x/2} x \sin[x] + \frac{1}{5} e^{x/2} x^2 \sin[x] + \frac{792 e^{x/2} \sin[3x]}{50 \, 653} + \frac{24 e^{x/2} x \sin[3x]}{1369} - \frac{3}{37} e^{x/2} x^2 \sin[3x]$$

Problems 51 - 55 (p. 361)

■ **Problem #51a**

$$\int \cosh[x] \, dx = \sinh[x]$$

■ **Problem #51b**

$$\int \sinh[x] \, dx = \cosh[x]$$

■ **Problem #52a**

$$\int \tanh[x] \, dx = \log[\cosh[x]]$$

■ **Problem #52b**

$$\int \operatorname{Coth}[x] \, dx = \operatorname{Log}[\operatorname{Sinh}[x]]$$

■ **Problem #53a**

$$\int \operatorname{Sech}[x] \, dx = \operatorname{ArcTan}[\operatorname{Sinh}[x]]$$

■ **Problem #53b**

$$\int \operatorname{Csch}[x] \, dx = -\operatorname{ArcTanh}[\operatorname{Cosh}[x]]$$

■ **Problem #54**

$$\int \operatorname{Cosh}[x]^2 \, dx = \frac{x}{2} + \frac{1}{2} \operatorname{Cosh}[x] \operatorname{Sinh}[x]$$

■ **Problem #55**

$$\int \operatorname{Sinh}[x]^5 \, dx = \operatorname{Cosh}[x] - \frac{2 \operatorname{Cosh}[x]^3}{3} + \frac{\operatorname{Cosh}[x]^5}{5}$$

Problems 56 - 60 (p. 365)

■ **Problem #56**

$$\int \operatorname{Tanh}[x]^4 \, dx = x - \operatorname{Tanh}[x] - \frac{\operatorname{Tanh}[x]^3}{3}$$

■ **Problem #57**

$$\int \operatorname{Csch}[x]^3 \, dx = \frac{1}{2} \operatorname{ArcTanh}[\operatorname{Cosh}[x]] - \frac{1}{2} \operatorname{Coth}[x] \operatorname{Csch}[x]$$

■ **Problem #58**

$$\int \frac{1}{\operatorname{Cosh}[x]^5} \, dx = \frac{3}{8} \operatorname{ArcTan}[\operatorname{Sinh}[x]] + \frac{3}{8} \operatorname{Sech}[x] \operatorname{Tanh}[x] + \frac{1}{4} \operatorname{Sech}[x]^3 \operatorname{Tanh}[x]$$

■ **Problem #59**

$$\int \frac{\operatorname{Tanh}[x]^5}{\operatorname{Sech}[x]^4} \, dx = -\operatorname{Cosh}[x]^2 + \frac{\operatorname{Cosh}[x]^4}{4} + \operatorname{Log}[\operatorname{Cosh}[x]]$$

■ **Problem #60**

$$\int \operatorname{Tanh}[x]^5 \operatorname{Sech}[x]^{3/4} \, dx = \frac{1}{3} (-4) \operatorname{Sech}[x]^{3/4} + \frac{8}{11} \operatorname{Sech}[x]^{11/4} - \frac{4}{19} \operatorname{Sech}[x]^{19/4}$$

Problems 61 - 65 (p. 365-366)

■ Problem #61

$$\int \frac{1}{a + b \operatorname{Cosh}[x]} dx = \frac{2 \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}}$$

■ Problem #62

$$\int \frac{1}{(1 + \operatorname{Cosh}[x])^2} dx = \frac{\operatorname{Sinh}[x]}{3(1 + \operatorname{Cosh}[x])^2} + \frac{\operatorname{Sinh}[x]}{3(1 + \operatorname{Cosh}[x])}$$

■ Problem #63

$$\int \frac{1}{a + b \operatorname{Tanh}[x]} dx = \frac{ax}{a^2 - b^2} - \frac{b \operatorname{Log}[a \operatorname{Cosh}[x] + b \operatorname{Sinh}[x]]}{a^2 - b^2}$$

■ Problem #64a

$$\int \frac{1}{a^2 + b^2 \operatorname{Cosh}[x]^2} dx = \frac{\operatorname{ArcTanh}\left[\frac{a \operatorname{Tanh}[x]}{\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2 + b^2}}$$

■ Problem #64b

$$\int \frac{1}{a^2 - b^2 \operatorname{Cosh}[x]^2} dx = \frac{\operatorname{ArcTanh}\left[\frac{a \operatorname{Tanh}[x]}{\sqrt{a^2-b^2}}\right]}{a \sqrt{a^2 - b^2}}$$

■ Problem #65

$$\int \frac{1}{1 - \operatorname{Sinh}[x]^4} dx = \frac{\operatorname{ArcTanh}\left[\sqrt{2} \operatorname{Tanh}[x]\right]}{2\sqrt{2}} + \frac{\operatorname{Tanh}[x]}{2}$$

Problems 66 - 72 (p. 366-367)

■ Problem #66

$$\int \frac{\operatorname{Cosh}[x]^3 - \operatorname{Sinh}[x]^3}{\operatorname{Cosh}[x]^3 + \operatorname{Sinh}[x]^3} dx = -\frac{4 \operatorname{ArcTan}\left[\frac{1-2 \operatorname{Tanh}[x]}{\sqrt{3}}\right]}{3\sqrt{3}} - \frac{1}{3(1 + \operatorname{Tanh}[x])}$$

■ Problem #67

$$\int \operatorname{Cosh}[x] \operatorname{Cosh}[2x] \operatorname{Cosh}[3x] dx = \frac{x}{4} + \frac{1}{8} \operatorname{Sinh}[2x] + \frac{1}{16} \operatorname{Sinh}[4x] + \frac{1}{24} \operatorname{Sinh}[6x]$$

■ **Problem #68**

$$\int \sinh[x] \cosh\left[\frac{3x}{2}\right] \sinh\left[\frac{5x}{2}\right] dx = -\frac{x}{4} + \frac{1}{8} \sinh[2x] - \frac{1}{12} \sinh[3x] + \frac{1}{20} \sinh[5x]$$

■ **Problem #69**

$$\int \frac{(\tanh[x] - \cosh[2x]) \cosh[x]}{(\sinh[2x] + \sinh[x]^2) \sqrt{\sinh[2x]}} dx =$$

$$\sqrt{2} \operatorname{ArcTan}\left[\operatorname{sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{1}{6} \operatorname{ArcTan}\left[\frac{\sinh[x]}{\sqrt{\sinh[2x]}}\right] - \frac{1}{3} \sqrt{2} \operatorname{ArcTanh}\left[\operatorname{sech}[x] \sqrt{\cosh[x] \sinh[x]}\right] + \frac{\cosh[x]}{\sqrt{\sinh[2x]}}$$

■ **Problem #70**

$$\int \frac{\sinh[x]}{(4 \cosh[x]^2 - 9)^{5/2}} dx = -\frac{\cosh[x]}{27(-9 + 4 \cosh[x]^2)^{3/2}} + \frac{2 \cosh[x]}{243 \sqrt{-9 + 4 \cosh[x]^2}}$$

■ **Problem #71**

$$\int \frac{\sinh[x]^2 \sinh[2x]}{(1 - \sinh[x]^2)^{3/2}} dx = \frac{2}{\sqrt{1 - \sinh[x]^2}} + 2 \sqrt{1 - \sinh[x]^2}$$

■ **Problem #72**

$$\int \frac{\cosh[x]}{\sqrt{\cosh[2x]}} dx = \frac{\operatorname{ArcSinh}\left[\sqrt{2} \sinh[x]\right]}{\sqrt{2}}$$

Problems 73 - 75 (p. 368)

■ **Problem #73a**

$$\int x \tanh[x]^2 dx = \frac{x^2}{2} + \operatorname{Log}[\cosh[x]] - x \tanh[x]$$

■ **Problem #73b**

$$\int x \coth[x]^2 dx = \frac{x^2}{2} - x \coth[x] + \operatorname{Log}[\sinh[x]]$$

■ **Problem #74**

$$\int \frac{x + \sinh[x] + \cosh[x]}{\cosh[x] - \sinh[x]} dx = -e^x + \frac{e^{2x}}{2} + e^x x$$

■ **Problem #75**

$$\int \frac{x + \sinh[x] + \cosh[x]}{1 + \cosh[x]} dx = x - (1 - x) \tanh\left[\frac{x}{2}\right]$$

Problems 76 - 82 (p. 373)

■ **Problem #76**

$$\int \frac{e^{2x}}{\sinh[x]^4} dx = \frac{8 e^{6x}}{3 (1 - e^{2x})^3}$$

■ **Problem #77**

$$\int \frac{1}{e^{2x} \cosh[x]^4} dx = -\frac{8}{3 (1 + e^{2x})^3}$$

■ **Problem #78**

$$\int \frac{e^x}{\cosh[x] - \sinh[x]} dx = \frac{e^{2x}}{2}$$

■ **Problem #79a**

$$\int \frac{e^{mx}}{\cosh[x] + \sinh[x]} dx = \frac{e^{(m-1)x}}{m-1}$$

■ **Problem #79b**

$$\int \frac{e^x}{\cosh[x] + \sinh[x]} dx = x$$

■ **Problem #80**

$$\int \frac{e^x}{1 - \cosh[x]} dx = -\frac{2}{1 - e^x} - 2 \operatorname{Log}[1 - e^x]$$

■ **Problem #81**

$$\int \frac{e^x (1 + \sinh[x])}{1 + \cosh[x]} dx = e^x + \frac{2}{1 + e^x}$$

■ **Problem #82**

$$\int \frac{e^x (1 - \sinh[x])}{1 - \cosh[x]} dx = e^x - \frac{2}{1 - e^x}$$

Problems 83 - 87 (p. 375)

■ Problem #83

$$\int x^m \operatorname{Log}[x] \, dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m} \operatorname{Log}[x]}{1+m}$$

■ Problem #84

$$\int x^m \operatorname{Log}[x]^2 \, dx = \frac{2 x^{1+m}}{(1+m)^3} - \frac{2 x^{1+m} \operatorname{Log}[x]}{(1+m)^2} + \frac{x^{1+m} \operatorname{Log}[x]^2}{1+m}$$

■ Problem #85

$$\int \frac{\operatorname{Log}[x]^2}{x^{5/2}} \, dx = -\frac{16}{27 x^{3/2}} - \frac{8 \operatorname{Log}[x]}{9 x^{3/2}} - \frac{2 \operatorname{Log}[x]^2}{3 x^{3/2}}$$

■ Problem #86

$$\int (a + b x) \operatorname{Log}[x] \, dx = -a x - \frac{b x^2}{4} - \frac{a^2 \operatorname{Log}[x]}{2 b} + \frac{(a + b x)^2 \operatorname{Log}[x]}{2 b}$$

■ Problem #87

$$\int (a + b x)^3 \operatorname{Log}[x] \, dx = -a^3 x - \frac{3}{4} a^2 b x^2 - \frac{1}{3} a b^2 x^3 - \frac{b^3 x^4}{16} - \frac{a^4 \operatorname{Log}[x]}{4 b} + \frac{(a + b x)^4 \operatorname{Log}[x]}{4 b}$$

Problems 88 - 89 (p. 375)

■ Problem #88

$$\int (3 \operatorname{Log}[x]^3 - 8 \operatorname{Log}[x]^2 - 1) \, dx = -35 x + 34 x \operatorname{Log}[x] - 17 x \operatorname{Log}[x]^2 + 3 x \operatorname{Log}[x]^3$$

■ Problem #89

$$\int (x^4 + 1) (\operatorname{Log}[x]^3 - 2 \operatorname{Log}[x] + 1) \, dx = -3 x + \frac{169 x^5}{625} + 4 x \operatorname{Log}[x] - \frac{44}{125} x^5 \operatorname{Log}[x] - 3 x \operatorname{Log}[x]^2 - \frac{3}{25} x^5 \operatorname{Log}[x]^2 + x \operatorname{Log}[x]^3 + \frac{1}{5} x^5 \operatorname{Log}[x]^3$$

Problems 90 - 92 (p. 376)

■ Problem #90

$$\int \frac{1}{x^3 \operatorname{Log}[x]^4} \, dx = \frac{1}{3} (-4) \operatorname{ExpIntegralEi}[-2 \operatorname{Log}[x]] - \frac{1}{3 x^2 \operatorname{Log}[x]^3} + \frac{1}{3 x^2 \operatorname{Log}[x]^2} - \frac{2}{3 x^2 \operatorname{Log}[x]}$$

■ **Problem #91**

$$\int \frac{\text{Log}[x]}{a + b x} dx = \frac{\text{Log}[x] \text{Log}\left[\frac{a+bx}{a}\right]}{b} + \frac{\text{PolyLog}\left[2, -\frac{bx}{a}\right]}{b}$$

■ **Problem #92**

$$\int \frac{\text{Log}[x]}{(a + b x)^2} dx = \frac{\text{Log}[x]}{a b} - \frac{\text{Log}[x]}{b(a + b x)} - \frac{\text{Log}[a + b x]}{a b}$$

Problems 93 - 97 (p. 377)

■ **Problem #93a**

$$\int \frac{\text{Log}[x]^n}{x} dx = \frac{\text{Log}[x]^{1+n}}{1+n}$$

■ **Problem #93b**

$$\int \frac{(a + b \text{Log}[x])^n}{x} dx = \frac{(a + b \text{Log}[x])^{1+n}}{b(1+n)}$$

■ **Problem #94a**

$$\int \frac{1}{x(a + b \text{Log}[x])} dx = \frac{\text{Log}[a + b \text{Log}[x]]}{b}$$

■ **Problem #94b**

$$\int \frac{1}{x(a + b \text{Log}[x])^n} dx = \frac{(a + b \text{Log}[x])^{1-n}}{b(1-n)}$$

■ **Problem #95a1**

$$\int \frac{1}{x \sqrt{\text{Log}[x]^2 + a^2}} dx = \text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{\text{Log}[x]^2 + a^2}}\right]$$

■ **Problem #95a2**

$$\int \frac{1}{x \sqrt{\text{Log}[x]^2 - a^2}} dx = \text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{\text{Log}[x]^2 - a^2}}\right]$$

■ **Problem #95b**

$$\int \frac{1}{x \sqrt{a^2 - \text{Log}[x]^2}} dx = \text{ArcTan}\left[\frac{\text{Log}[x]}{\sqrt{a^2 - \text{Log}[x]^2}}\right]$$

■ **Problem #96a1**

$$\int \frac{1}{x \operatorname{Log}[x] \sqrt{a^2 + \operatorname{Log}[x]^2}} dx = -\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a^2 + \operatorname{Log}[x]^2}}{a}\right]}{a}$$

■ **Problem #96a2**

$$\int \frac{1}{x \operatorname{Log}[x] \sqrt{a^2 - \operatorname{Log}[x]^2}} dx = -\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a^2 - \operatorname{Log}[x]^2}}{a}\right]}{a}$$

■ **Problem #96b**

$$\int \frac{1}{x \operatorname{Log}[x] \sqrt{\operatorname{Log}[x]^2 - a^2}} dx = \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-a^2 + \operatorname{Log}[x]^2}}{a}\right]}{a}$$

■ **Problem #97.1**

$$\int \frac{\operatorname{Log}[\operatorname{Log}[x]]}{x} dx = -\operatorname{Log}[x] (1 - \operatorname{Log}[\operatorname{Log}[x]])$$

■ **Problem #97.2**

$$\int \frac{\operatorname{Log}[\operatorname{Log}[x]]^2}{x} dx = 2 \operatorname{Log}[x] - 2 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]] + \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^2$$

■ **Problem #97.3**

$$\int \frac{\operatorname{Log}[\operatorname{Log}[x]]^3}{x} dx = -6 \operatorname{Log}[x] + 6 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]] - 3 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^2 + \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^3$$

■ **Problem #97.4**

$$\int \frac{\operatorname{Log}[\operatorname{Log}[x]]^4}{x} dx = 24 \operatorname{Log}[x] - 24 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]] + 12 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^2 - 4 \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^3 + \operatorname{Log}[x] \operatorname{Log}[\operatorname{Log}[x]]^4$$

■ **Problem #97.n**

$$\int \frac{\operatorname{Log}[\operatorname{Log}[x]]^n}{x} dx = \frac{\operatorname{Gamma}[1 + n, -\operatorname{Log}[\operatorname{Log}[x]]] \operatorname{Log}[\operatorname{Log}[x]]^n}{(-\operatorname{Log}[\operatorname{Log}[x]])^n}$$

Problems 98 - 103 (p. 377-378)

■ Problem #98

$$\int \frac{\text{Cot}[x]}{\text{Log}[\text{Sin}[x]]} dx = \text{Log}[\text{Log}[\text{Sin}[x]]]$$

■ Problem #99

$$\int (e^{\text{Log}[\text{Cos}[x]]} + e^{-\text{Log}[\text{Cos}[x]]}) \text{Tan}[x] dx = -\text{Cos}[x] + \text{Sec}[x]$$

■ Problem #100

$$\int \text{Sinh}[x] \text{Log}[\text{Cosh}[x]] dx = -\text{Cosh}[x] + \text{Cosh}[x] \text{Log}[\text{Cosh}[x]]$$

■ Problem #101

$$\int \text{Tanh}[x] \text{Log}[\text{Cosh}[x]] dx = \frac{1}{2} \text{Log}[\text{Cosh}[x]]^2$$

■ Problem #102

$$\int \text{Log}[x - \sqrt{1+x^2}] dx = \sqrt{1+x^2} + x \text{Log}[x - \sqrt{1+x^2}]$$

■ Problem #103

$$\int \frac{\text{Log}[x-1]}{x^3} dx = \frac{1}{2x} + \frac{1}{2} \text{Log}[1-x] - \frac{\text{Log}[-1+x]}{2x^2} - \frac{\text{Log}[x]}{2}$$

Problems 104 - 109 (p. 378)

■ Problem #104

$$\int (e^x - e^{-x}) \text{Log}[e^{2x} + 1] dx = -2e^x + \frac{\text{Log}[1 + e^{2x}]}{e^x} + e^x \text{Log}[1 + e^{2x}]$$

■ Problem #105

$$\int e^{\frac{3x}{2}} \text{Log}[e^x - 1] dx = -\frac{1}{3} (4e^{x/2}) - \frac{4}{9} e^{\frac{3x}{2}} + \frac{4}{3} \text{ArcTanh}[e^{x/2}] + \frac{2}{3} e^{\frac{3x}{2}} \text{Log}[-1 + e^x]$$

■ Problem #106

$$\int \text{Cos}[x]^3 \text{Log}[\text{Sin}[x]] dx = -\text{Sin}[x] + \text{Log}[\text{Sin}[x]] \text{Sin}[x] + \frac{\text{Sin}[x]^3}{9} - \frac{1}{3} \text{Log}[\text{Sin}[x]] \text{Sin}[x]^3$$

■ **Problem #107**

$$\int \frac{\text{Log}[\text{Tan}[x]]}{\text{Cos}[x]^4} dx = -\text{Tan}[x] + \text{Log}[\text{Tan}[x]] \text{Tan}[x] - \frac{\text{Tan}[x]^3}{9} + \frac{1}{3} \text{Log}[\text{Tan}[x]] \text{Tan}[x]^3$$

■ **Problem #108**

$$\int \frac{\text{Log}[\text{Cos}[\frac{x}{2}]]}{1 + \text{Cos}[x]} dx = -\frac{x}{2} + \frac{\text{Log}[\text{Cos}[\frac{x}{2}]] \text{Sin}[x]}{1 + \text{Cos}[x]} + \text{Tan}\left[\frac{x}{2}\right]$$

■ **Problem #109**

$$\int \frac{\text{Cos}[x] \text{Log}[\text{Sin}[x]]}{(1 + \text{Cos}[x])^2} dx = -\frac{1}{3} (2x) - \frac{\text{Sin}[x]}{9(1 + \text{Cos}[x])^2} + \frac{8 \text{Sin}[x]}{9(1 + \text{Cos}[x])} - \frac{\text{Log}[\text{Sin}[x]] \text{Sin}[x]}{3(1 + \text{Cos}[x])^2} + \frac{2 \text{Log}[\text{Sin}[x]] \text{Sin}[x]}{3(1 + \text{Cos}[x])}$$

Chapter 9 (p. 379-427)

Inverse Trigonometric Function Integration Problems

Problems 1 - 6 (p. 391-392)

■ **Problem #1**

$$\int \frac{\text{ArcCos}[x]^2}{x^5} dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2} \text{ArcCos}[x]}{6x^3} + \frac{\sqrt{1-x^2} \text{ArcCos}[x]}{3x} - \frac{\text{ArcCos}[x]^2}{4x^4} + \frac{\text{Log}[x]}{3}$$

■ **Problem #2**

$$\int x^2 \text{ArcSin}[x]^2 dx = -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9} \sqrt{1-x^2} \text{ArcSin}[x] + \frac{2}{9} x^2 \sqrt{1-x^2} \text{ArcSin}[x] + \frac{1}{3} x^3 \text{ArcSin}[x]^2$$

■ **Problem #3**

$$\int \text{ArcTan}[x]^2 x^3 dx = \frac{x^2}{12} + \frac{1}{2} x \text{ArcTan}[x] - \frac{1}{6} x^3 \text{ArcTan}[x] - \frac{\text{ArcTan}[x]^2}{4} + \frac{1}{4} x^4 \text{ArcTan}[x]^2 - \frac{1}{3} \text{Log}[1+x^2]$$

■ **Problem #4**

$$\int \frac{\text{ArcTan}[x]^2}{x^5} dx = -\frac{1}{12x^2} - \frac{\text{ArcTan}[x]}{6x^3} + \frac{\text{ArcTan}[x]}{2x} + \frac{\text{ArcTan}[x]^2}{4} - \frac{\text{ArcTan}[x]^2}{4x^4} - \frac{2 \text{Log}[x]}{3} + \frac{1}{3} \text{Log}[1+x^2]$$

■ **Problem #5**

$$\int \text{ArcCsc}[x]^2 x^3 dx = \frac{x^2}{12} + \frac{1}{3} \sqrt{1-\frac{1}{x^2}} x \text{ArcCsc}[x] + \frac{1}{6} \sqrt{1-\frac{1}{x^2}} x^3 \text{ArcCsc}[x] + \frac{1}{4} x^4 \text{ArcCsc}[x]^2 + \frac{\text{Log}[x]}{3}$$

■ **Problem #6**

$$\int \frac{\text{ArcSec}[x]^4}{x^5} dx = -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}} \text{ArcSec}[x]}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}} \text{ArcSec}[x]}{64x} - \frac{45 \text{ArcSec}[x]^2}{128} +$$

$$\frac{3 \text{ArcSec}[x]^2}{16x^4} + \frac{9 \text{ArcSec}[x]^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}} \text{ArcSec}[x]^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}} \text{ArcSec}[x]^3}{8x} + \frac{3 \text{ArcSec}[x]^4}{32} - \frac{\text{ArcSec}[x]^4}{4x^4}$$

Problems 7 - 18 (p. 397-398)

■ Problem #7a

$$\int \text{ArcSin}[x] \sqrt{1-x^2} \, dx = -\frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \text{ArcSin}[x] + \frac{\text{ArcSin}[x]^2}{4}$$

■ Problem #7b

$$\int \text{ArcCos}[x] \sqrt{1-x^2} \, dx = \frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \text{ArcCos}[x] - \frac{\text{ArcCos}[x]^2}{4}$$

■ Problem #8

$$\int \text{ArcCos}[x] x \sqrt{1-x^2} \, dx = -\frac{x}{3} + \frac{x^3}{9} - \frac{1}{3} (1-x^2)^{3/2} \text{ArcCos}[x]$$

■ Problem #9

$$\int \text{ArcSin}[x] (1-x^2)^{3/2} \, dx = -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8} x \sqrt{1-x^2} \text{ArcSin}[x] + \frac{1}{4} x (1-x^2)^{3/2} \text{ArcSin}[x] + \frac{3 \text{ArcSin}[x]^2}{16}$$

■ Problem #10

$$\int \text{ArcSin}[x] x (1-x^2)^{3/2} \, dx = \frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5} (1-x^2)^{5/2} \text{ArcSin}[x]$$

■ Problem #11

$$\int \text{ArcCos}[x] x^3 (1-x^2)^{3/2} \, dx = -\frac{2x}{35} - \frac{x^3}{105} + \frac{8x^5}{175} - \frac{x^7}{49} - \frac{1}{5} (1-x^2)^{5/2} \text{ArcCos}[x] + \frac{1}{7} (1-x^2)^{7/2} \text{ArcCos}[x]$$

■ Problem #12

$$\int \frac{\text{ArcCos}[x] (1-x^2)^{3/2}}{x} \, dx =$$

$$\frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \text{ArcCos}[x] + \frac{1}{3} (1-x^2)^{3/2} \text{ArcCos}[x] + 2i \text{ArcCos}[x] \text{ArcTan}[e^{i \text{ArcCos}[x]}] - i \text{PolyLog}[2, -i e^{i \text{ArcCos}[x]}] + i \text{PolyLog}[2, i e^{i \text{ArcCos}[x]}]$$

■ Problem #13

$$\int \frac{\text{ArcSin}[x] (1-x^2)^{3/2}}{x^6} \, dx = -\frac{1}{20x^4} + \frac{1}{5x^2} - \frac{(1-x^2)^{5/2} \text{ArcSin}[x]}{5x^5} + \frac{\text{Log}[x]}{5}$$

■ Problem #14

$$\int \frac{\text{ArcSin}[x] x^2}{\sqrt{1-x^2}} \, dx = \frac{x^2}{4} - \frac{1}{2} x \sqrt{1-x^2} \text{ArcSin}[x] + \frac{\text{ArcSin}[x]^2}{4}$$

■ Problem #15

$$\int \frac{\text{ArcSin}[x] x^4}{\sqrt{1-x^2}} dx = \frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8} x \sqrt{1-x^2} \text{ArcSin}[x] - \frac{1}{4} x^3 \sqrt{1-x^2} \text{ArcSin}[x] + \frac{3 \text{ArcSin}[x]^2}{16}$$

■ Problem #16a

$$\int \frac{\text{ArcSin}[x] x}{(1-x^2)^{3/2}} dx = \frac{\text{ArcSin}[x]}{\sqrt{1-x^2}} - \text{ArcTanh}[x]$$

■ Problem #16b

$$\int \frac{\text{ArcCos}[x] x}{(1-x^2)^{3/2}} dx = \frac{\text{ArcCos}[x]}{\sqrt{1-x^2}} + \text{ArcTanh}[x]$$

■ Problem #17

$$\int \frac{\text{ArcSin}[x]}{(1-x^2)^{5/2}} dx = -\frac{1}{6(1-x^2)} + \frac{x \text{ArcSin}[x]}{3(1-x^2)^{3/2}} + \frac{2x \text{ArcSin}[x]}{3\sqrt{1-x^2}} + \frac{1}{3} \text{Log}[1-x^2]$$

■ Problem #18

$$\int \frac{\text{ArcSin}[x] x^3}{(1-x^2)^{3/2}} dx = -x + \frac{\text{ArcSin}[x]}{\sqrt{1-x^2}} + \sqrt{1-x^2} \text{ArcSin}[x] - \text{ArcTanh}[x]$$

Problems 19 - 22 (p. 401)

■ Problem #19

$$\int \frac{\text{ArcSin}[x]}{x(1-x^2)^{3/2}} dx = \frac{\text{ArcSin}[x]}{\sqrt{1-x^2}} - 2 \text{ArcSin}[x] \text{ArcTanh}[e^{i \text{ArcSin}[x]}] - \text{ArcTanh}[x] + i \text{PolyLog}[2, -e^{i \text{ArcSin}[x]}] - i \text{PolyLog}[2, e^{i \text{ArcSin}[x]}]$$

■ Problem #20

$$\int \frac{\text{ArcCos}[x]}{x^4 \sqrt{1-x^2}} dx = \frac{1}{6x^2} - \frac{\sqrt{1-x^2} \text{ArcCos}[x]}{3x^3} - \frac{2\sqrt{1-x^2} \text{ArcCos}[x]}{3x} - \frac{2 \text{Log}[x]}{3}$$

■ Problem #21

$$\int \text{ArcCos}[x]^2 x \sqrt{1-x^2} dx = \frac{4}{9} \sqrt{1-x^2} + \frac{2}{27} (1-x^2)^{3/2} - \frac{2}{3} x \text{ArcCos}[x] + \frac{2}{9} x^3 \text{ArcCos}[x] - \frac{1}{3} (1-x^2)^{3/2} \text{ArcCos}[x]^2$$

■ Problem #22

$$\int \frac{\text{ArcSin}[x]^3 x^2}{\sqrt{1-x^2}} dx = -\frac{3x^2}{8} + \frac{3}{4} x \sqrt{1-x^2} \text{ArcSin}[x] - \frac{3 \text{ArcSin}[x]^2}{8} + \frac{3}{4} x^2 \text{ArcSin}[x]^2 - \frac{1}{2} x \sqrt{1-x^2} \text{ArcSin}[x]^3 + \frac{\text{ArcSin}[x]^4}{8}$$

Problems 23 - 26 (p. 404-405)

■ Problem #23

$$\int \frac{\text{ArcTan}[x] x}{(1+x^2)^2} dx = \frac{x}{4(1+x^2)} + \frac{\text{ArcTan}[x]}{4} - \frac{\text{ArcTan}[x]}{2(1+x^2)}$$

■ Problem #24

$$\int \frac{\text{ArcTan}[x] x}{(1+x^2)^3} dx = \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3\text{ArcTan}[x]}{32} - \frac{\text{ArcTan}[x]}{4(1+x^2)^2}$$

■ Problem #25

$$\int \frac{\text{ArcTan}[x] x^2}{1+x^2} dx = x \text{ArcTan}[x] - \frac{\text{ArcTan}[x]^2}{2} - \frac{1}{2} \text{Log}[1+x^2]$$

■ Problem #26

$$\int \frac{\text{ArcTan}[x] x^3}{1+x^2} dx = -\frac{x}{2} + \frac{\text{ArcTan}[x]}{2} + \frac{1}{2} x^2 \text{ArcTan}[x] + \frac{1}{2} i \text{ArcTan}[x]^2 + \text{ArcTan}[x] \text{Log}\left[\frac{2i}{i-x}\right] + \frac{1}{2} i \text{PolyLog}\left[2, -\frac{i+x}{i-x}\right]$$

Problems 27 - 32 (p. 407-408)

■ Problem #27

$$\int \frac{\text{ArcTan}[x] x^2}{(1+x^2)^2} dx = -\frac{1}{4(1+x^2)} - \frac{x \text{ArcTan}[x]}{2(1+x^2)} + \frac{\text{ArcTan}[x]^2}{4}$$

■ Problem #28

$$\int \frac{\text{ArcTan}[x] x^3}{(1+x^2)^2} dx = -\frac{x}{4(1+x^2)} - \frac{\text{ArcTan}[x]}{4} + \frac{\text{ArcTan}[x]}{2(1+x^2)} - \frac{1}{2} i \text{ArcTan}[x]^2 - \text{ArcTan}[x] \text{Log}\left[\frac{2i}{i-x}\right] - \frac{1}{2} i \text{PolyLog}\left[2, -\frac{i+x}{i-x}\right]$$

■ Problem #29

$$\int \frac{\text{ArcTan}[x] x^5}{(1+x^2)^2} dx = -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3\text{ArcTan}[x]}{4} + \frac{1}{2} x^2 \text{ArcTan}[x] - \frac{\text{ArcTan}[x]}{2(1+x^2)} + i \text{ArcTan}[x]^2 + 2 \text{ArcTan}[x] \text{Log}\left[\frac{2i}{i-x}\right] + i \text{PolyLog}\left[2, -\frac{i+x}{i-x}\right]$$

■ Problem #30

$$\int \frac{\text{ArcTan}[x] (1+x^2)}{x^2} dx = -\frac{\text{ArcTan}[x]}{x} + x \text{ArcTan}[x] + \text{Log}[x] - \text{Log}[1+x^2]$$

■ Problem #31

$$\int \frac{\text{ArcTan}[x] (1+x^2)}{x^5} dx = -\frac{1}{12x^3} - \frac{1}{4x} - \frac{(1+x^2)^2 \text{ArcTan}[x]}{4x^4}$$

■ Problem #32

$$\int \frac{\text{ArcTan}[x] (1+x^2)^2}{x^5} dx = -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3 \text{ArcTan}[x]}{4} - \frac{\text{ArcTan}[x]}{4x^4} - \frac{\text{ArcTan}[x]}{x^2} + \frac{1}{2} i \text{PolyLog}[2, -i x] - \frac{1}{2} i \text{PolyLog}[2, i x]$$

Problems 33 - 36 (p. 409)

■ Problem #33

$$\int \frac{\text{ArcTan}[x]}{x^2 (1+x^2)} dx = -\frac{\text{ArcTan}[x]}{x} - \frac{\text{ArcTan}[x]^2}{2} + \text{Log}[x] - \frac{1}{2} \text{Log}[1+x^2]$$

■ Problem #34

$$\int \frac{\text{ArcTan}[x]^2}{x^3} dx = -\frac{\text{ArcTan}[x]}{x} - \frac{\text{ArcTan}[x]^2}{2} - \frac{\text{ArcTan}[x]^2}{2x^2} + \text{Log}[x] - \frac{1}{2} \text{Log}[1+x^2]$$

■ Problem #35

$$\int \frac{\text{ArcTan}[x]^2 (1+x^2)}{x^5} dx = -\frac{1}{12x^2} - \frac{\text{ArcTan}[x]}{6x^3} - \frac{\text{ArcTan}[x]}{2x} - \frac{(1+x^2)^2 \text{ArcTan}[x]^2}{4x^4} + \frac{\text{Log}[x]}{3} - \frac{1}{6} \text{Log}[1+x^2]$$

■ Problem #36

$$\int \frac{\text{ArcTan}[x]^2 x^3}{(1+x^2)^3} dx = -\frac{1}{32(1+x^2)^2} + \frac{5}{32(1+x^2)} + \frac{x^3 \text{ArcTan}[x]}{8(1+x^2)^2} + \frac{3x \text{ArcTan}[x]}{16(1+x^2)} - \frac{3 \text{ArcTan}[x]^2}{32} + \frac{x^4 \text{ArcTan}[x]^2}{4(1+x^2)^2}$$

Problems 37 - 43 (p. 412-414)

■ Problem #37

$$\int \frac{\text{ArcSec}[x] \sqrt{x^2-1}}{x^2} dx = -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{x^2-1} \text{ArcSec}[x]}{x} - \frac{2i\sqrt{x^2} \text{ArcSec}[x] \text{ArcTan}[e^{i \text{ArcSec}[x]}]}{x} + \frac{i\sqrt{x^2} \text{PolyLog}[2, -ie^{i \text{ArcSec}[x]}]}{x} - \frac{i\sqrt{x^2} \text{PolyLog}[2, ie^{i \text{ArcSec}[x]}]}{x}$$

■ **Problem #38**

$$\int \frac{\text{ArcCsc}[x] (x^2 - 1)^{5/2}}{x^3} dx = \frac{3 + 2x^4}{12x\sqrt{x^2}} - \frac{5(x^2 - 1)^{3/2} \text{ArcCsc}[x]}{3x^2} - \frac{5\sqrt{x^2 - 1} \text{ArcCsc}[x]}{2x^2} + \frac{(x^2 - 1)^{5/2} \text{ArcCsc}[x]}{3x^2} - \frac{5x \text{ArcCsc}[x]^2}{4\sqrt{x^2}} - \frac{7x \text{Log}[x]}{3\sqrt{x^2}}$$

■ **Problem #39**

$$\int \frac{\text{ArcSec}[x] \sqrt{x^2 - 1}}{x^4} dx = \frac{1}{3\sqrt{x^2}} - \frac{1}{9(x^2)^{3/2}} + \frac{(x^2 - 1)^{3/2} \text{ArcSec}[x]}{3x^3}$$

■ **Problem #40**

$$\int \frac{\text{ArcSec}[x]}{(x^2 - 1)^{5/2}} dx = \frac{\sqrt{x^2}}{6(1 - x^2)} - \frac{x \text{ArcSec}[x]}{3(x^2 - 1)^{3/2}} + \frac{2x \text{ArcSec}[x]}{3\sqrt{x^2 - 1}} + \frac{5 \text{ArcCoth}[\sqrt{x^2}]}{6}$$

■ **Problem #41**

$$\int \frac{\text{ArcSec}[x] x^2}{(x^2 - 1)^{5/2}} dx = \frac{\sqrt{x^2}}{6(1 - x^2)} - \frac{x^3 \text{ArcSec}[x]}{3(x^2 - 1)^{3/2}} - \frac{\text{ArcCoth}[\sqrt{x^2}]}{6}$$

■ **Problem #42**

$$\int \frac{\text{ArcSec}[x] x^3}{(x^2 - 1)^{5/2}} dx = \frac{x}{6\sqrt{x^2}(1 - x^2)} - \frac{\text{ArcSec}[x]}{3(x^2 - 1)^{3/2}} - \frac{\text{ArcSec}[x]}{\sqrt{x^2 - 1}} - \frac{2x \text{Log}[x]}{3\sqrt{x^2}} + \frac{x \text{Log}[x^2 - 1]}{3\sqrt{x^2}}$$

■ **Problem #43**

$$\int \frac{\text{ArcSec}[x] x^6}{(x^2 - 1)^{5/2}} dx = \frac{\sqrt{x^2}(2 - 3x^2)}{6(x^2 - 1)} - \frac{13 \text{ArcCoth}[\sqrt{x^2}]}{6} - \frac{5x^3 \text{ArcSec}[x]}{6(x^2 - 1)^{3/2}} + \frac{x^5 \text{ArcSec}[x]}{2(x^2 - 1)^{3/2}} - \frac{5x \text{ArcSec}[x]}{2\sqrt{x^2 - 1}} - \frac{5i\sqrt{x^2} \text{ArcSec}[x] \text{ArcTan}[e^{i \text{ArcSec}[x]}]}{x} + \frac{5i\sqrt{x^2} \text{PolyLog}[2, -ie^{i \text{ArcSec}[x]}]}{2x} - \frac{5i\sqrt{x^2} \text{PolyLog}[2, ie^{i \text{ArcSec}[x]}]}{2x}$$

Problems 44 - 48 (p. 416-417)

■ **Problem #44**

$$\int \frac{\text{ArcSec}[x]}{x^2 \sqrt{x^2 - 1}} dx = \frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2 - 1} \text{ArcSec}[x]}{x}$$

■ **Problem #45**

$$\int \frac{\text{ArcCsc}[x]}{x^2 (x^2 - 1)^{5/2}} dx = -\frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2}}{6(x^2 - 1)} + \frac{(3 - 12x^2 + 8x^4) \text{ArcCsc}[x]}{3x(x^2 - 1)^{3/2}} - \frac{11 \text{ArcCoth}[\sqrt{x^2}]}{6}$$

■ **Problem #46**

$$\int \frac{\text{ArcCsc}[x]^4}{x^2 \sqrt{x^2 - 1}} dx = \frac{24 \sqrt{x^2 - 1}}{x} + \frac{24 \text{ArcCsc}[x]}{\sqrt{x^2}} - \frac{12 \sqrt{x^2 - 1} \text{ArcCsc}[x]^2}{x} - \frac{4 \text{ArcCsc}[x]^3}{\sqrt{x^2}} + \frac{\sqrt{x^2 - 1} \text{ArcCsc}[x]^4}{x}$$

■ **Problem #47**

$$\int \frac{\text{ArcSec}[x]^2 (x^2 - 1)^{3/2}}{x^5} dx = \frac{\sqrt{x^2 - 1} (17x^2 - 2)}{64x^4} - \frac{3 \text{ArcSec}[x]}{8x\sqrt{x^2}} + \frac{9x \text{ArcSec}[x]}{64\sqrt{x^2}} + \frac{(x^2 - 1)^2 \text{ArcSec}[x]}{8x^3\sqrt{x^2}} - \frac{3\sqrt{x^2 - 1} \text{ArcSec}[x]^2}{8x^2} - \frac{(x^2 - 1)^{3/2} \text{ArcSec}[x]^2}{4x^4} + \frac{x \text{ArcSec}[x]^3}{8\sqrt{x^2}}$$

■ **Problem #48**

$$\int \frac{\text{ArcSec}[x]^3 \sqrt{x^2 - 1}}{x^4} dx = \frac{2(1 - 21x^2)}{27x^2\sqrt{x^2}} - \frac{4\sqrt{x^2 - 1} \text{ArcSec}[x]}{3x} - \frac{2(x^2 - 1)^{3/2} \text{ArcSec}[x]}{9x^3} + \frac{2 \text{ArcSec}[x]^2}{3\sqrt{x^2}} + \frac{(x^2 - 1) \text{ArcSec}[x]^2}{3x^2\sqrt{x^2}} + \frac{(x^2 - 1)^{3/2} \text{ArcSec}[x]^3}{3x^3}$$

Problems 49 - 56 (p. 422)

■ **Problem #49**

$$\int \text{ArcSin}\left[\sqrt{\frac{x-a}{x+a}}\right] dx = -\frac{\sqrt{2} a \sqrt{\frac{x-a}{x+a}}}{\sqrt{\frac{a}{x+a}}} + (x+a) \text{ArcSin}\left[\sqrt{\frac{x-a}{x+a}}\right]$$

■ **Problem #50**

$$\int \text{ArcTan}\left[\sqrt{\frac{x-a}{x+a}}\right] dx = x \text{ArcTan}\left[\sqrt{-\frac{a-x}{a+x}}\right] - a \text{ArcTanh}\left[\sqrt{-\frac{a-x}{a+x}}\right]$$

■ **Problem #51**

$$\int \frac{\text{ArcTan}[x]}{(1+x)^3} dx = -\frac{1}{4(1+x)} - \frac{\text{ArcTan}[x]}{2(1+x)^2} + \frac{1}{4} \text{Log}[1+x] - \frac{1}{8} \text{Log}[1+x^2]$$

■ **Problem #52**

$$\int \frac{\text{ArcTan}[x - a]}{x + a} dx = \frac{1}{2} i \text{Log}[1 + i(a - x)] \text{Log}\left[-\frac{a + x}{i - 2a}\right] - \frac{1}{2} i \text{Log}[1 - i(a - x)] \text{Log}\left[\frac{a + x}{i + 2a}\right] - \frac{1}{2} i \text{PolyLog}\left[2, \frac{1 - i(a - x)}{1 - 2ia}\right] + \frac{1}{2} i \text{PolyLog}\left[2, \frac{1 + i(a - x)}{1 + 2ia}\right]$$

■ **Problem #53**

$$\int \frac{\text{ArcSin}[\sqrt{1 - x^2}]}{\sqrt{1 - x^2}} dx = \frac{1}{2} \pi \text{ArcSin}[x] - \frac{x}{2\sqrt{x^2}} \text{ArcSin}[x]^2$$

■ **Problem #54**

$$\int \frac{\text{ArcTan}[\sqrt{1 + x^2}] x}{\sqrt{1 + x^2}} dx = \sqrt{1 + x^2} \text{ArcTan}[\sqrt{1 + x^2}] - \frac{1}{2} \text{Log}[2 + x^2]$$

■ **Problem #55**

$$\int \frac{\text{ArcSin}[x]}{(1 - x)^{5/2}} dx = -\frac{\sqrt{1 + x}}{3(1 - x)} + \frac{2 \text{ArcSin}[x]}{3(1 - x)^{3/2}} - \frac{1}{3\sqrt{2}} \text{ArcTanh}\left[\frac{\sqrt{1 + x}}{\sqrt{2}}\right]$$

■ **Problem #56**

$$\int \text{ArcCsc}[x] (x - 1)^{5/2} dx = \frac{4x(83 - 19x + 3x^2)\sqrt{x^2 - 1}}{105\sqrt{x - 1}\sqrt{x^2}} + \frac{2}{7}(x - 1)^{7/2} \text{ArcCsc}[x] + \frac{4x}{7\sqrt{x^2}} \text{ArcTanh}\left[\frac{\sqrt{x^2 - 1}}{\sqrt{x - 1}}\right]$$

Problems 57 - 59 (p. 427)

■ **Problem #57**

$$\int \text{ArcSin}[\text{Sinh}[x]] \text{Sech}[x]^4 dx = \frac{-2}{3} \text{ArcSin}\left[\frac{\text{Cosh}[x]}{\sqrt{2}}\right] + \frac{1}{6} \sqrt{1 - \text{Sinh}[x]^2} \text{Sech}[x] + \text{ArcSin}[\text{Sinh}[x]] \text{Tanh}[x] - \frac{1}{3} \text{ArcSin}[\text{Sinh}[x]] \text{Tanh}[x]^3$$

■ **Problem #58**

$$\int \frac{\text{ArcCot}[\text{Cosh}[x]] \text{Cosh}[x]}{\text{Sinh}[x]^4} dx = \frac{1}{6\sqrt{2}} \text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{2}}\right] + \frac{\text{Coth}[x]}{6} - \frac{1}{3} \text{ArcCot}[\text{Cosh}[x]] \text{Csch}[x]^3$$

■ **Problem #59**

$$\int \text{ArcSin}[\text{Tanh}[x]] e^x dx = e^x \text{ArcSin}[\text{Tanh}[x]] - \text{Cosh}[x] \text{Log}[1 + e^{2x}] \sqrt{\text{Sech}[x]^2}$$