Western Science

Department of Applied Mathematics

APPLIED MATHEMATICS COLLOQUIUM

Date: Friday April 7, 2017
Time: 2:30 – 3:30 pm
Location: MC Room 204

A Hamiltonian for the zeros of the Riemann zeta function

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Abstract: According to the Riemann hypothesis, all non-trivial zeros of the Riemann zeta function lie on the critical line, i.e. have real part 1/2. A suggestion by Hilbert and Polya is to find an operator with eigenvalues which correspond to the (suitably shifted) zeros of the zeta function, such that self-adjointness of this operator would entail the Riemann hypothesis. More concretely, Berry and Keating conjectured that this operator should be a suitable quantization of the classical Hamiltonian xp. In this talk, I describe some recent work with Carl Bender and Dorje Brody (arXiv:1608.03679, to appear in PRL) that succeeds in constructing a linear operator whose eigenvalues correspond exactly to the zeros of the Riemann zeta function, and they are all real if and only if the Riemann hypothesis is true. Moreover, the classical limit of the operator is xp, in accordance with the Berry-Keating conjecture. While we are not able to define a suitable underlying Hilbert space that would make this operator self-adjoint, we give some formal non-rigorous calculations which show, among other things, that the operator (times "i") is PT-symmetric. I also discuss some ideas where to go from there, and show how the operator is related to the question of "how to define summation series with a non-integer number of terms".