The University of Western Ontario
Department of Applied Mathematics

<u>Course Outline Winter 2012</u>
Applied Mathematics 4617b: Numerical Solution of PDE's

Description: Finite difference methods, stability analysis for time-dependent problems.

Prerequisite(s): permission of the instructor + an ability to program in a compiled language.

Book: "Finite Difference Schemes and Partial Differential Equations", John C. Strikwerda, Second edition, SIAM, 2004 (required)

Instructor: Dr. A. MacIsaac (allanb@uwo.ca), Middlesex College, Room 270

Lectures: Monday, Wednesday, Friday 3:30pm - 4:20pm WSC 240

Office Hours: TBD for both TA and instructor.

Note this course has joint lectures with AM9566, however the Graduate student evaluation will include unique assignments and exams and will require additional reading on topics not covered in class. Graduate assignment questions may be completed for additional credit by undergraduate students, but undergraduate students must complete all undergraduate questions before attempting graduate level questions.

Marking Scheme: 40% Assignments
25% Midterm Exam
35% Final Exam

We will try to cover the following chapters at various depths.

Chapter 1:Hyperbolic Partial Differential Equations.

1. Overview of Hyperbolic Partial Differential Equations.
2. Boundary Conditions.
3. Introduction to Finite Difference Schemes.
4. Convergence and Consistency.
5. Stability.
6. The Courant-Friedrichs-Lewy Condition.

Chapter 2:Analysis of Finite Difference Schemes.

1. Fourier Analysis.
2. Von Neumann Analysis.
3. Comments on Instability and Stability

Chapter 3:Order of Accuracy of Finite Difference Schemes.

1. Order of Accuracy.
2. Stability of Lax-Wendroff and Crank-Nicolson Schemes.
3. Difference Notation and Difference Calculus.
4. Boundary Conditions for Finite Difference Schemes.
5. Solving Tridiagonal Systems.

Chapter 4: Stability for Multistep Schemes.

1. Stability for the Leapfrog Scheme.

2. Stability for General Multistep Schemes.

Chapter 5:Dissipation and Dispersion.

1. Dissipation.
2. Dispersion.
3. Group Velocity and the Propagation of Wave Packets.

Chapter 6:Parabolic Partial Differential Equations.

1. Overview of Parabolic Differential Equations.
2. Parabolic Systems and Boundary Conditions.
3. Finite Difference Schemes for Parabolic Equations.
4. The Convection-Diffusion equation.
5. Variable Coefficients

Chapter 7:Systems of Partial Differential Equations.

1. Stability of Finite Difference Schemes for Systems.
2. Finite Difference Schemes in Two and Three Dimensions.
3. The Alternating Direction Implicit Method.

Chapter 10: Convergence Estimates for Initial Value Problems.

1. Convergence Estimates for Smooth Initial Functions.
2. Related Topics.
3. Convergence Estimates for Non-smooth Initial Functions.
4. Convergence Estimates for Parabolic Differential Equations.
5. The Lax-Richtmyer Equivalence Theorem.
6. Analysis of Multistep Schemes.
7. Convergence Estimates for Second-Order Differential Equations.

Chapter 12: Elliptic Partial Differential Equations and Difference Schemes.

1. Overview of Elliptic Partial Differential Equations.
2. Regularity Estimates for Elliptic Equations.
3. Maximum Principles.
4. Boundary Conditions for Elliptic Equations.
5. Finite Difference Schemes for Poisson's Equations.
6. Coordinate Changes and Finite Differences.

Chapter 13: Linear Iterative Methods.

1. Solving Finite Difference Schemes for Laplace's Equation in a Rectangle.
2. Eigenvalues of the Discrete Laplacian
3. Analysis of the Jacobi and Gauss-Seidel Methods.
4. Convergence Analysis of Point SOR.
5. Consistently Ordered Matrices.
6. Linear Iterative Methods for Symmetric, Postive Definite Matrices.
7. The Neumann Boundary Value Problem.

Chapter 14: The Method of Steepest Descent and the Conjugate Gradient Method.

1. The Method of Steepest Decent.
2. The Conjugate Gradient Method.
3. Implementing the Conjugate Gradient Method.
4. A convergence Estimate for the Conjugate Gradient Method.
5. The Preconditioned Conjugate Gradient Method.